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Fuzzy semiotic sets of second order (Triples of morphisms for sign sets IV)

1. The point de départ for this and three previous studies (Toth 2008a, b, c) is Arin's analysis of semiotic catastrophes (Arin 1981, pp. 353 ss.) for which he considered a sign class to be consisting of three semioses. On this basis, it is possible to analyze each sign class and reality thematic as a triple of semiotic morphisms, f. ex.

(3.1 2.1 1.3) → [[β°, id1], [α°, βα], [α°β°, βα]]
(3.1 1.2 1.3) → [[α°β°, α], [id1, β], [α°β°, βα]]

Generally, given a semiotic dual system (DS) in its most abstract form

DS := (a.b c.d e.f) × (f.e d.c b.a),

we assign semiotic morphisms (cf. Toth 1997, pp. 21 ss.) to the following cross-relations of pairs of each semiotic triple, i.e. sign classes (SCl) and reality thematics (RTh):

SCl := [[a.c, b.d], [c.e, d.f], [a.e, b.f]]
RTh:= [[f.d, e.c], [d.b c.a], [f.b, e.a]]

2. Therefore, the system of the 10 sign classes and their dual reality thematics can be assigned to the following triples of semiotic morphisms:

1.	(3.1 2.1 1.1)	→	[[β°, id1], [α°, id1], [α°β°, id1]]	→	((3.2 1.1), (2.1 1.1), (3.1 1.1))
	(1.1 1.2 1.3)	→	[[id1, α], [id1, β], [id1, βα]]	→	((1.1 1.2), (1.1 2.3), (1.1 1.3))
2.	(3.1 2.1 1.2)	→	[[β°, id1], [α°, α], [α°β°, α]]	→	((3.2 1.1), (2.1 1.2), (3.1 1.2))
	(2.1 1.2 1.3)	→	[[α°, α], [id1, β], [α°, βα]]	→	((2.1 1.2), (1.1 2.3), (2.1 1.3))
3.	(3.1 2.1 1.3)	→	[[β°, id1], [α°, βα], [α°β°, βα]]	→	((3.2 1.1), (2.1 1.3), (3.1 1.3))
	(3.1 1.2 1.3)	→	[[α°β°, α], [id1, β], [α°β°, βα]]	→	((3.1 1.2), (1.1 2.3), (3.1 1.3))
4.	(3.1 2.2 1.2)	→	[[β°, α], [α°, id2], [α°β°, α]]	→	((3.2 1.2), (2.1 2.2), (3.1 1.2))
	(2.1 2.2 1.3)	→	[[id2, α], [α°, β], [α°, βα]]	→	((2.2 1.2), (2.1 2.3), (2.1 1.3))
5.	(3.1 2.2 1.3)	→	[[β°, α], [α°, β], [α°β°, βα]]	→	((3.2 1.2), (2.1 2.3), (3.1 1.3))
	(3.1 2.2 1.3)	→	[[β°, α], [α°, β], [α°β°, βα]]	→	((3.2 1.2), (2.1 2.3), (3.1 1.3))
6.	(3.1 2.3 1.3)	→	[[β°, βα], [α°, id3], [α°β°, βα]]	→	((3.2 1.3), (2.1 3.3), (3.1 1.3))
	(3.1 3.2 1.3)	→	[[id3, α], [α°β°, β], [α°β°, βα]]	→	((3.3 1.2), (3.1 2.3), (3.1 1.3))

7.	(3.2 2.2 1.2)	$\rightarrow$	[[$\beta°$, id2], [$\alpha°$, id2], [$\alpha°\beta°$, id2]]	$\rightarrow$	((3.2 2.2), (2.1 2.2), (3.1 2.2))
	(2.1 2.2 2.3)	$\rightarrow$	[[id2, α], [id2, β], [id2, $\beta\alpha$]]	$\rightarrow$	((2.2 1.2), (2.2 2.3), (2.2 1.3))
8.	(3.2 2.2 1.3)	$\rightarrow$	[[$\beta°$, id2], [$\alpha°$, β], [$\alpha°\beta°$, β]]	$\rightarrow$	((3.2 2.2), (2.1 2.3), (3.1 2.3))
	(3.1 2.2 2.3)	$\rightarrow$	[[$\beta°$, α], [id2, β], [$\beta°$, $\beta\alpha$]]	$\rightarrow$	((3.2 1.2), (2.2 2.3), (3.2 1.3))
9.	(3.2 2.3 1.3)	$\rightarrow$	[[$\beta°$, β], [$\alpha°$, id3], [$\alpha°\beta°$, β]]	$\rightarrow$	((3.2 2.3), (2.1 3.3), (3.1 2.3))
	(3.1 3.2 2.3)	$\rightarrow$	[[id3, α], [$\beta°$, β], [$\beta°$, $\beta\alpha$]]	$\rightarrow$	((3.3 1.2), (3.2 2.3), (3.2 1.3))
10.	(3.3 2.3 1.3)	$\rightarrow$	[[$\beta°$, id3], [$\alpha°$, id3], [$\alpha°\beta°$, id3]]	$\rightarrow$	((3.2 3.3), (2.1 3.3), (3.1 3.3))
	(3.1 3.2 3.3)	$\rightarrow$	[[id3, α], [id3, β], [id3, $\beta\alpha$]]	$\rightarrow$	((3.3 1.2), (3.3 2.3), (3.3 1.3))

We recognize that although numerical sign class and numerical reality thematic are dual to one another, the category theoretic sets are not, and neither are the respective numerical triples of semiotic morphisms. Thus, not only does the notation of semiotic triples add more semiotic information not present in the original sign classes and reality thematics; because of their non-duality, both the triple sign sets and the triple reality sets contain additional semiotic information respective to one another. Therefore, we apparently have in the triple-notation of sign classes and reality thematics a semiotic case of Zadeh's Fuzzy Extension Principle (FEP): "The extension principle addresses the following fundamental problem: If there is some relationship between nonfuzzy entities, then what is its equivalent between fuzzy entities? The extension principle makes it therefore possible to extend some known models or algorithms involving nonfuzzy variables to the case of fuzzy variables (Zadeh 1975, p. 16).

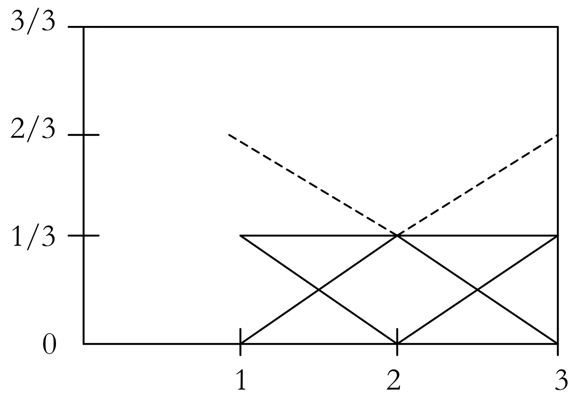
If we agree that X is a sign class or a reality thematic and A its category theoretic notation, then we may say that f(A) turns a sign class or reality thematic into a triple of morphisms and f(X) into the respective numerical notation of this triple. Therefore, we may write the semiotic FEP in the notation given in Zadeh (2002, p. 12):

$$\frac{X \text{ is } A}{f(X) \text{ is } f(A)}$$

3. However, the semiotic FEP is even more intricate than the corresponding mathematical principle. The mathematical FEP can be understood as a fibering of crisp sets into fuzzy sets, while the semiotic FEP is a fibering of already fuzzy semiotic sets (cf. Toth 2008 d, e, f) of first order into fuzzy semiotic sets of second order (cf. Böhme 1993, pp. 150 ss.). It is therefore the aim of this study to display the graphs of the semiotic fuzzy sets of second order – in order to show both the different extensions and intensions of the range of semiotic values of the semiotic membership functions for the complete semiotic dual systems (CDS), i.e. for the triples of morphisms for sign classes (straight) as well as for reality thematics (dashed).

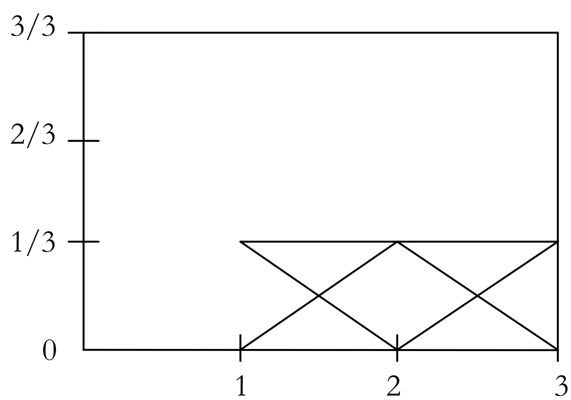
1. (3.1 2.1 1.1) $\rightarrow$ [[β°, id1], [α°, id1], [α°β°, id1]] $\rightarrow$ ((3.2 1.1), (2.1 1.1), (3.1 1.1))
(1.1 1.2 1.3) $\rightarrow$ [[id1, α], [id1, β], [id1, βα]] $\rightarrow$ ((1.1 1.2), (1.1 2.3), (1.1 1.3))

Semiotic fuzzy set for CDS1



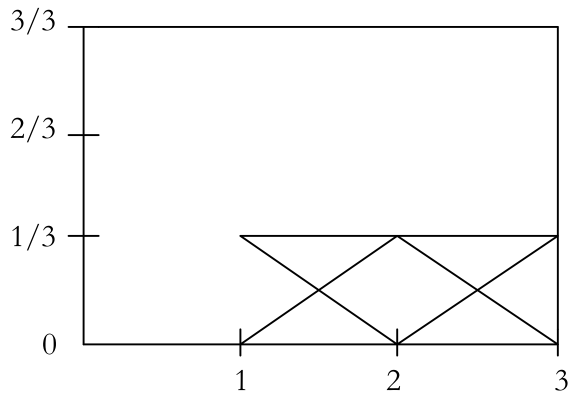
2. (3.1 2.1 1.2) $\rightarrow$ [[β°, id1], [α°, α], [α°β°, α]] $\rightarrow$ ((3.2 1.1), (2.1 1.2), (3.1 1.2))
(2.1 1.2 1.3) $\rightarrow$ [[α°, α], [id1, β], [α°, βα]] $\rightarrow$ ((2.1 1.2), (1.1 2.3), (2.1 1.3))

Semiotic fuzzy set for CDS2



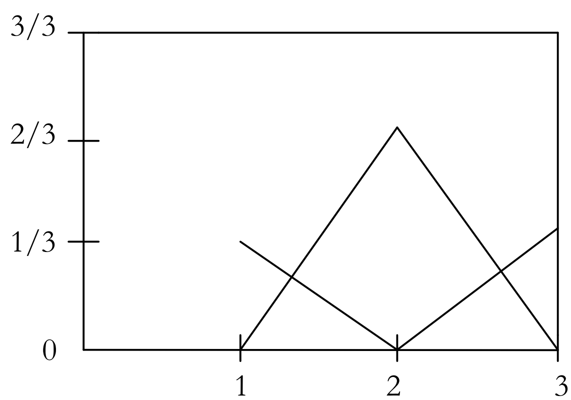
3. (3.1 2.1 1.3) → [[β°, id1], [α°, βα], [α°β°, βα]] → ((3.2 1.1), (2.1 1.3), (3.1 1.3))
(3.1 1.2 1.3) → [[α°β°, α], [id1, β], [α°β°, βα]] → ((3.1 1.2), (1.1 2.3), (3.1 1.3))

Semiotic fuzzy set for CDS3



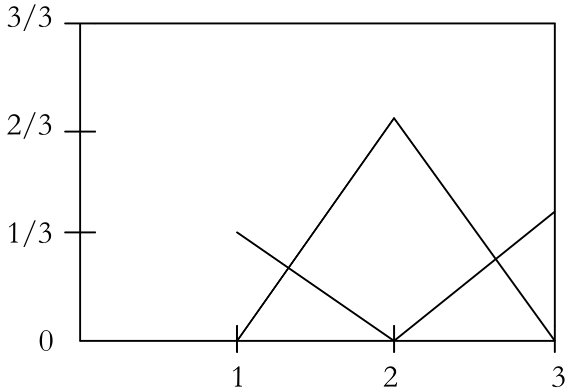
4. (3.1 2.2 1.2) → [[β°, α], [α°, id2], [α°β°, α]] → ((3.2 1.2), (2.1 2.2), (3.1 1.2))
(2.1 2.2 1.3) → [[id2, α], [α°, β], [α°, βα]] → ((2.2 1.2), (2.1 2.3), (2.1 1.3))

Semiotic fuzzy set for CDS4



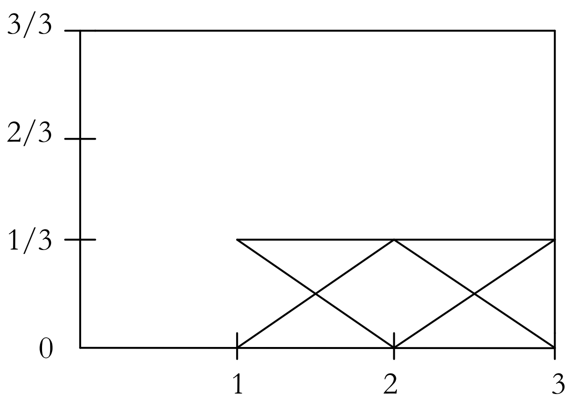
5. (3.1 2.2 1.3) $\rightarrow$ $[[\beta^\circ, \alpha], [\alpha^\circ, \beta], [\alpha^\circ\beta^\circ, \beta\alpha]]$ $\rightarrow$ ((3.2 1.2), (2.1 2.3), (3.1 1.3))
(3.1 2.2 1.3) $\rightarrow$ $[[\beta^\circ, \alpha], [\alpha^\circ, \beta], [\alpha^\circ\beta^\circ, \beta\alpha]]$ $\rightarrow$ ((3.2 1.2), (2.1 2.3), (3.1 1.3))

Semiotic fuzzy set for CDS5



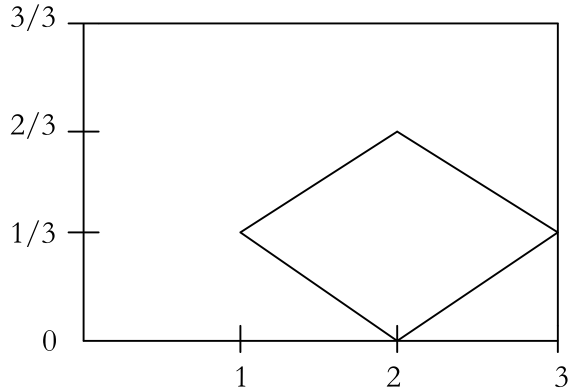
6. (3.1 2.3 1.3) $\rightarrow$ $[[\beta^\circ, \beta\alpha], [\alpha^\circ, \mathrm{id}3], [\alpha^\circ\beta^\circ, \beta\alpha]]$ $\rightarrow$ ((3.2 1.3), (2.1 3.3), (3.1 1.3))
(3.1 3.2 1.3) $\rightarrow$ $[[\mathrm{id}3, \alpha], [\alpha^\circ\beta^\circ, \beta], [\alpha^\circ\beta^\circ, \beta\alpha]]$ $\rightarrow$ ((3.3 1.2), (3.1 2.3), (3.1 1.3))

Semiotic fuzzy set for CDS6



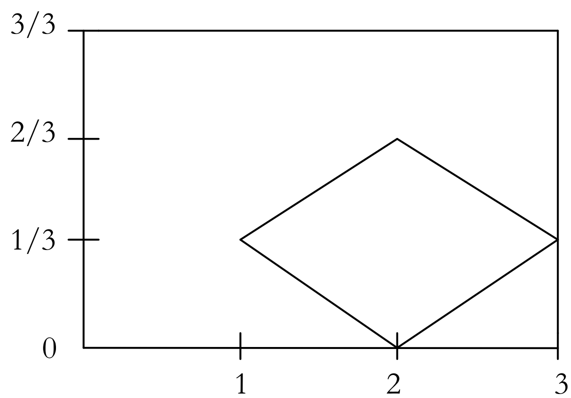
7. (3.2 2.2 1.2) $\rightarrow$ [[β°, id2], [α°, id2], [α°β°, id2]] $\rightarrow$ ((3.2 2.2), (2.1 2.2), (3.1 2.2))
(2.1 2.2 2.3) $\rightarrow$ [[id2, α], [id2, β], [id2, βα]] $\rightarrow$ ((2.2 1.2), (2.2 2.3), (2.2 1.3))

Semiotic fuzzy set for CDS7



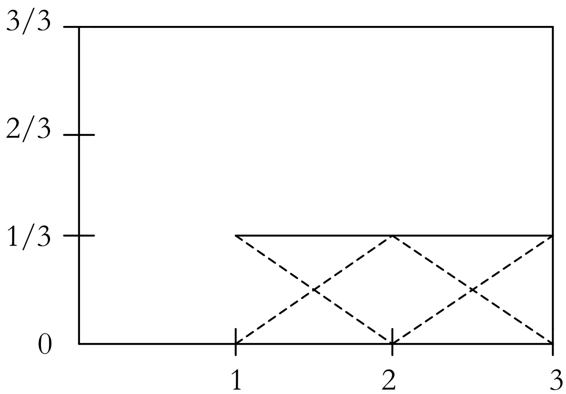
8. (3.2 2.2 1.3) $\rightarrow$ [[β°, id2], [α°, β], [α°β°, β]] $\rightarrow$ ((3.2 2.2), (2.1 2.3), (3.1 2.3))
(3.1 2.2 2.3) $\rightarrow$ [[β°, α], [id2, β], [β°, βα]] $\rightarrow$ ((3.2 1.2), (2.2 2.3), (3.2 1.3))

Semiotic fuzzy set for CDS8



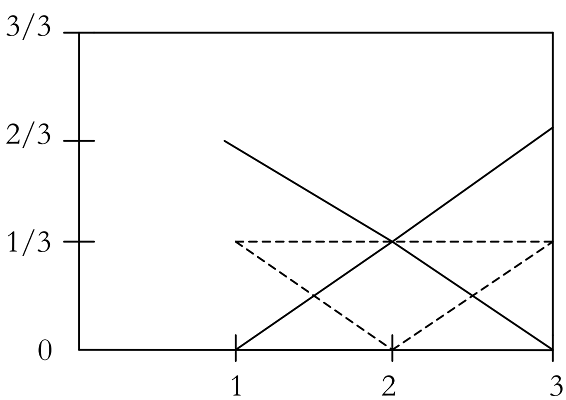
9. (3.2 2.3 1.3) $\rightarrow$ $[[\beta^\circ, \beta], [\alpha^\circ, \mathrm{id}3], [\alpha^\circ\beta^\circ, \beta]]$ $\rightarrow$ ((3.2 2.3), (2.1 3.3), (3.1 2.3))
(3.1 3.2 2.3) $\rightarrow$ $[[\mathrm{id}3, \alpha], [\beta^\circ, \beta], [\beta^\circ, \beta\alpha]]$ $\rightarrow$ ((3.3 1.2), (3.2 2.3), (3.2 1.3))

Semiotic fuzzy set for CDS9



10.(3.3 2.3 1.3) $\rightarrow$ $[[\beta^\circ, \mathrm{id}3], [\alpha^\circ, \mathrm{id}3], [\alpha^\circ\beta^\circ, \mathrm{id}3]]$ $\rightarrow$ ((3.2 3.3), (2.1 3.3), (3.1 3.3))
(3.1 3.2 3.3) $\rightarrow$ $[[\mathrm{id}3, \alpha], [\mathrm{id}3, \beta], [\mathrm{id}3, \beta\alpha]]$ $\rightarrow$ ((3.3 1.2), (3.3 2.3), (3.3 1.3))

Semiotic fuzzy set for CDS10



Applying the **Semiotic Fuzzy Extension Principle**, we recognize that there is not one graph whose part-graphs do not overlap one another. The graphs with exclusively straight lines, as, f. ex., nos. 7 and 8, show also that there are graphs whose fuzzy sign or reality function is completely superimposed on its respective fuzzy reality or sign function. Therefore, we can divide the fuzzy semiotic graphs of second order semiotic sets into the following four discrete groups:

1. Fuzzy semiotic meanders: 2, 3, 6, 9
2. Fuzzy semiotic meanders with cuspic extension: 1, 10
3. Fuzzy semiotic rhombi: 7, 8
4. Fuzzy semiotic triangles (both with partly embedded cuspic sub-graph): 4, 5

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