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Triples of morphisms for sign sets I

1. As I have shown in previous studies (cf. Toth 2008a), there are two possibilities to transform numerical sign sets into category theoretic notation:

1.1. A semiotic morphism is associated to each sub-sign of a given sign set (cf. Bense 1981, p. p. 124 ss.), f.ex.:

(3.1 2.1 1.3) → [α°β°, α°, βα]
(3.1 2.2 1.3) → [α°β°, id2, βα]
(3.1 2.3 1.3) → [α°β°, β, βα]

1.2. A semiotic morphism is associated to each semiosis between every two sub-signs of a given sign set (cf. Toth 2008a, pp. 159 ss.), f.ex.:

(3.1 2.1 1.3) → [[β°, id1], [α°, βα]]
(3.1 2.2 1.3) → [[β°, α], [α°, β]]
(3.1 2.3 1.3) → [[β°, βα], [α°, id3]]

1.3. In his analysis of semiotic catastrophes, Arin (1981, pp. 353 ss.) considered a sign class to be consisting of three semioses. On this basis, it is possible to analyze the above given three sign classes as triples instead of pairs of morphisms:

(3.1 2.1 1.3) → [[β°, id1], [α°, βα], [α°β°, βα]]
(3.1 2.2 1.3) → [[β°, α], [α°, β], [α°β°, βα]]
(3.1 2.3 1.3) → [[β°, βα], [α°, id3], [α°β°, βα]]

2. In the following, I shall first present the triple-morphismic structure of the 10 sign classes, i.e. those triadic sign sets that obey both the Law of Tradicity and the Law of Inclusive Trichotomic Order (cf. Toth 2008b). Each sign class is first given in its numerical notation and then in its triple-morphismic notation, which will then be re-transformed in its numerical notation:

(3.1 2.1 1.1) → [β°, id1], [α°, id1], [α°β°, id1] → (3.2 1.1) (2.1 1.1) (3.1 1.1)
(3.1 2.1 1.2) → [β°, id1], [α°, α], [α°β°, α] → (3.2 1.1) (2.1 1.2) (3.1 1.2)
(3.1 2.1 1.3) → [β°, id1], [α°, βα], [α°β°, βα] → (3.2 1.1) (2.1 1.3) (3.1 1.3)
(3.1 2.2 1.2) → [β°, α], [α°, id2], [α°β°, α] → (3.2 1.2) (2.1 2.2) (3.1 1.2)
(3.1 2.2 1.3) → [β°, α], [α°, β], [α°β°, βα] → (3.2 1.2) (2.1 2.3) (3.1 1.3)
(3.1 2.3 1.3) → [β°, βα], [α°, id3], [α°β°, βα] → (3.2 1.3) (2.1 3.3) (3.1 1.3)
(3.2 2.2 1.2) → [β°, id2], [α°, id2], [α°β°, id2] → (3.2 2.2) (2.1 2.2) (3.1 2.2)

(3.2 2.2 1.3) → [β°, id2], [α°, β], [α°β°, β] → (3.2 2.2) (2.1 2.3) (3.1 2.3)
(3.2 2.3 1.3) → [β°, β], [α°, id3], [α°β°, β] → (3.2 2.3) (2.1 3.3) (3.1 2.3)

(3.3 2.3 1.3) → [β°, id3], [α°, id3], [α°β°, id3] → (3.2 3.3) (2.1 3.3) (3.1 3.3)

Thus the 10 sign classes can be summed up in 6 groups according by their common third morphisms:

(3.1 2.1 1.1) → [β°, id1], [α°, id1],	[α°β°, id1]	→ (3.2 1.1) (2.1 1.1)	(3.1 1.1)
(3.1 2.1 1.2) → [β°, id1], [α°, α],	[α°β°, α]	→ (3.2 1.1) (2.1 1.2)	(3.1 1.2)
(3.1 2.2 1.2) → [β°, α], [α°, id2],	[α°β°, α]	→ (3.2 1.2) (2.1 2.2)	(3.1 1.2)
(3.1 2.1 1.3) → [β°, id1], [α°, βα],	[α°β°, βα]	→ (3.2 1.1) (2.1 1.3)	(3.1 1.3)
(3.1 2.2 1.3) → [β°, α], [α°, β],	[α°β°, βα]	→ (3.2 1.2) (2.1 2.3)	(3.1 1.3)
(3.1 2.3 1.3) → [β°, βα], [α°, id3],	[α°β°, βα]	→ (3.2 1.3) (2.1 3.3)	(3.1 1.3)
(3.2 2.2 1.2) → [β°, id2], [α°, id2],	[α°β°, id2]	→ (3.2 2.2) (2.1 2.2)	(3.1 2.2)
(3.2 2.2 1.3) → [β°, id2], [α°, β],	[α°β°, β]	→ (3.2 2.2) (2.1 2.3)	(3.1 2.3)
(3.2 2.3 1.3) → [β°, β], [α°, id3],	[α°β°, β]	→ (3.2 2.3) (2.1 3.3)	(3.1 2.3)
(3.3 2.3 1.3) → [β°, id3], [α°, id3],	[α°β°, id3]	→ (3.2 3.3) (2.1 3.3)	(3.1 3.3)

3. We thus find

(3.2 1.1) ○ (2.1 1.1) = (3.1 1.1)
(3.2 1.1) ○ (2.1 1.2) = (3.2 1.2) ○ (2.1 2.2) = (3.1 1.2)
(3.2 1.1) ○ (2.1 1.3) = (3.2 1.2) ○ (2.1 2.3) = (3.2 1.3) ○ (2.1 3.3) = (3.1 1.3), etc.,

which allows us to construct the following multiplication table:

○	1	2	3
1	1	1	3
2	1	2	3
3	3	3	3,

which is also valid, as we shall show now, for the 17 additional sign classes that form together with the 10 presented hitherto the complete semiotic representation system of the 27 sign classes in which the Law of Inclusive Trichotomic Order is abolished:

*(3.1 2.2 1.1) → [β°, α], [α°, α°],	[α°β°, id1]	→ (3.2 1.2) (2.1 2.1)	(3.1 1.1)
*(3.1 2.3 1.1) → [β°, βα], [α°, α°β°],	[α°β°, id1]	→ (3.2 1.3) (2.1 3.1)	(3.1 1.1)

*(3.1 2.3 1.2) → [β°, βα], [α°, β°],	[α°β°, α]	→ (3.2 1.3) (2.1 3.2)	(3.1 1.2)
*(3.2 2.1 1.1) → [β°, α°], [α°, id1],	[α°β°, α°]	→ (3.2 2.1) (2.1 1.1)	(3.1 2.1)
*(3.2 2.2 1.1) → [β°, id2], [α°, α°],	[α°β°, α°]	→ (3.2 2.2) (2.1 2.1)	(3.1 2.1)
*(3.2 2.3 1.1) → [β°, β], [α°, α°β°],	[α°β°, α°]	→ (3.2 2.3) (2.1 3.1)	(3.1 2.1)
*(3.2 2.1 1.2) → [β°, α°], [α°, α],	[α°β°, id2]	→ (3.2 2.1) (2.1 1.2)	(3.1 2.2)
*(3.2 2.3 1.2) → [β°, β], [α°, β°],	[α°β°, id2]	→ (3.2 2.3) (2.1 3.2)	(3.1 2.2)
*(3.2 2.1 1.3) → [β°, α°], [α°, βα],	[α°β°, β]	→ (3.2 2.1) (2.1 1.3)	(3.1 2.3)
*(3.3 2.1 1.1) → [β°, α°β°], [α°, id1],	[α°β°, α°β°]	→ (3.2 3.1) (2.1 1.1)	(3.1 3.1)
*(3.3 2.2 1.1) → [β°, β°], [α°, α°],	[α°β°, α°β°]	→ (3.2 3.2) (2.1 2.1)	(3.1 3.1)
*(3.3 2.3 1.1) → [β°, id3], [α°, α°β°],	[α°β°, α°β°]	→ (3.2 3.3) (2.1 3.1)	(3.1 3.1)
*(3.3 2.1 1.2) → [β°, α°β°], [α°, α],	[α°β°, β°]	→ (3.2 3.1) (2.1 1.2)	(3.1 3.2)
*(3.3 2.2 1.2) → [β°, β°], [α°, id2],	[α°β°, β°]	→ (3.2 3.2) (2.1 2.2)	(3.1 3.2)
*(3.3 2.3 1.2) → [β°, id3], [α°, β°],	[α°β°, β°]	→ (3.2 3.3) (2.1 3.2)	(3.1 3.2)
*(3.3 2.1 1.3) → [β°, α°β°], [α°, βα],	[α°β°, id3]	→ (3.2 3.1) (2.1 1.3)	(3.1 3.3)
*(3.3 2.2 1.3) → [β°, β°], [α°, β],	[α°β°, id3]	→ (3.2 3.2) (2.1 2.3)	(3.1 3.3)

Therefore, all 27 sign classes of the complete semiotic representation system can be summed up in triples of morphisms whose third triple has the category theoretic and numeric structure

[α°β°, X] → [3.1 X],

in which X covers the whole range of the sub-signs from the semiotic matrix, i.e. {(1.1), (1.2), (1.3), ..., (3.3)}. In the part-system of the 10 sign-classes, X ≠ (2.1), (3.1), (3.2) and in the part-system of the 17 sign classes, i.e. the complete system without the 10 sign class that obey the Law of Inclusive Trichotomic Order, X ≠ (1.3).

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