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Triples of morphisms for sign sets III

1. In Toth (2008b), we started from the most abstract notation of a sign class as (a.b c.d e.f), obtained its 6 possible transpositions and replaced the variables (a, b, c, d, e, f) by the natural numbers (1, 2, 3, 4, 5, 6). Since this mapping is bijective and the semiotic relations involved are preserved, the following general system of a sign class together with its transpositions is isomorphic to the original system using variables:

$(1.2\ 3.4\ 5.6) \rightarrow [\underline{1}3, \underline{\underline{2}}4], [\underline{3}5, \underline{\underline{4}}6], [1\underline{5}, 2\underline{\underline{6}}]$	(1. → 3. → 5.)	(2. → 4. → 6.)
$(1.2\ 5.6\ 3.4) \rightarrow [\underline{1}5, \underline{\underline{2}}6], [\underline{5}3, \underline{\underline{6}}4], [1\underline{3}, 2\underline{\underline{4}}]$	(1. → 5. → 3.)	(2. → 6. → 4.)
$(3.4\ 1.2\ 5.6) \rightarrow [\underline{3}1, \underline{\underline{4}}2], [\underline{1}5, \underline{\underline{2}}6], [3\underline{5}, 4\underline{\underline{6}}]$	(3. → 1. → 5.)	(4. → 2. → 6.)
$(3.4\ 5.6\ 1.2) \rightarrow [\underline{3}5, \underline{\underline{4}}6], [\underline{5}1, \underline{\underline{6}}2], [3\underline{1}, 4\underline{\underline{2}}]$	(3. → 5. → 1.)	(4. → 6. → 2.)
$(5.6\ 1.2\ 3.4) \rightarrow [\underline{5}1, \underline{\underline{6}}2], [\underline{1}3, \underline{\underline{2}}4], [5\underline{3}, 6\underline{\underline{4}}]$	(5. → 1. → 3.)	(6. → 2. → 4.)
$(5.6\ 3.4\ 1.2) \rightarrow [\underline{5}3, \underline{\underline{6}}4], [\underline{3}1, \underline{\underline{4}}2], [5\underline{1}, 6\underline{\underline{2}}]$	(5. → 3. → 1.)	(6. → 4. → 2.)

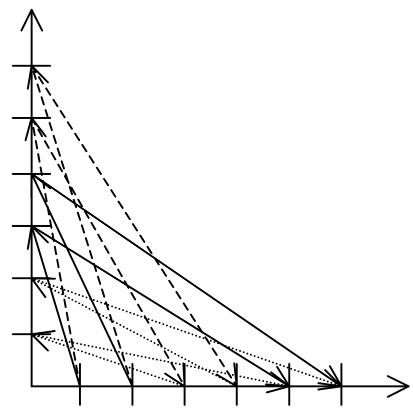
$(6.5\ 4.3\ 2.1) \rightarrow [\underline{6}4, \underline{\underline{5}}3], [\underline{4}2, \underline{\underline{3}}1], [6\underline{2}, 5\underline{\underline{1}}]$	(6. → 4. → 2.)	(5. → 3. → 1.)
$(4.3\ 6.5\ 2.1) \rightarrow [\underline{4}6, \underline{\underline{3}}5], [\underline{6}2, \underline{\underline{5}}1], [4\underline{2}, 3\underline{\underline{1}}]$	(4. → 6. → 2.)	(3. → 5. → 1.)
$(6.5\ 2.1\ 4.3) \rightarrow [\underline{6}2, \underline{\underline{5}}1], [\underline{2}4, \underline{\underline{1}}3], [6\underline{4}, 5\underline{\underline{3}}]$	(6. → 2. → 4.)	(5. → 1. → 3.)
$(2.1\ 6.5\ 4.3) \rightarrow [\underline{2}6, \underline{\underline{1}}5], [\underline{6}4, \underline{\underline{5}}3], [2\underline{4}, 1\underline{\underline{3}}]$	(2. → 6. → 4.)	(1. → 5. → 3.)
$(4.3\ 2.1\ 6.5) \rightarrow [\underline{4}2, \underline{\underline{3}}1], [\underline{2}6, \underline{\underline{1}}5], [4\underline{6}, 3\underline{\underline{5}}]$	(4. → 2. → 6.)	(3. → 1. → 5.)
$(2.1\ 4.3\ 6.5) \rightarrow [\underline{2}4, \underline{\underline{1}}3], [\underline{4}6, \underline{\underline{3}}5], [2\underline{6}, 1\underline{\underline{5}}]$	(2. → 4. → 6.)	(1. → 3. → 5.)

2. One recognizes that the two columns to the right show two orders of semiotic natural numbers. These numbers had been introduced as "Peirce numbers" in Toth (2008a, p. 157). They thus correspond to the triadic and the trichotomic prime signs in the sign classes (upper block of transpositions) and in the reality thematics (lower block of transpositions). Obviously, prime signs show two different kinds of successors, namely odd numbers of the structure (n, (n+2), ((n+2)+2)) for triadic prime signs and even numbers of the structure (m, (m+2), ((m+2)+2)) for trichotomic prime signs:

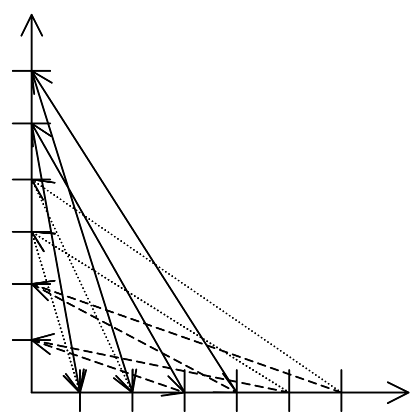
Scheme of successors of triadic prime signs:	**Scheme of successors of trichotomic prime signs:**
(1. → 3. → 5.)	(6. → 4. → 2.)
(1. → 5. → 3.)	(4. → 6. → 2.)
(3. → 1. → 5.)	(6. → 2. → 4.)
(3. → 5. → 1.)	(2. → 6. → 4.)
(5. → 1. → 3.)	(4. → 2. → 6.)
(5. → 3. → 1.)	(2. → 4. → 6.)

3. It shows that we can group the system of transpositions of a sign class and its reality thematic in 4 part-systems of 3 pairs of transpositions each. Thereby, we will draw the transpositions of the first pair straight, the second ones dashed and the third ones round dotted:

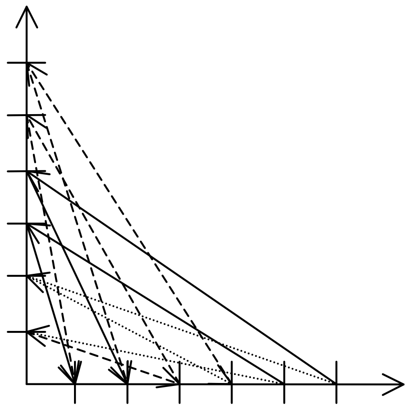
(1. → 3. → 5.)	(1. → 5. → 3.)	(3. → 1. → 5.)
(2. → 4. → 6.)	(2. → 6. → 4.)	(4. → 2. → 6.)



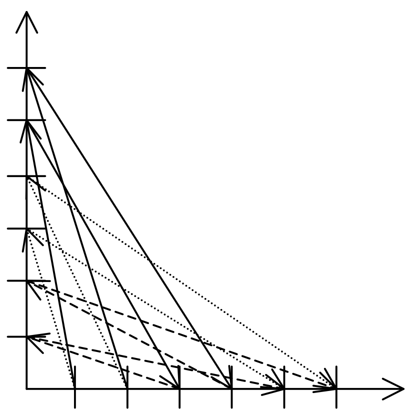
(3. → 5. → 1.)	(5. → 1. → 3.)	(5. → 3. → 1.)
(4. → 6. → 2.)	(6. → 2. → 4.)	(6. → 4. → 2.)



(6. → 4. → 2.)　　(4. → 6. → 2.)　　(6. → 2. → 4.)
(5. → 3. → 1.)　　(3. → 5. → 1.)　　(5. → 1. → 3.)

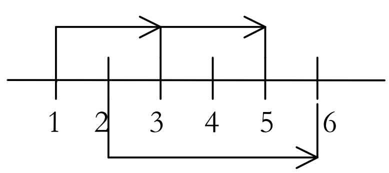


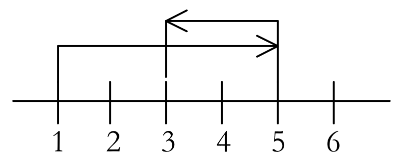
(2. → 6. → 4.)　　(4. → 2. → 6.)　　(2. → 4. → 6.)
(1. → 5. → 3.)　　(3. → 1. → 5.)　　(1. → 3. → 5.)



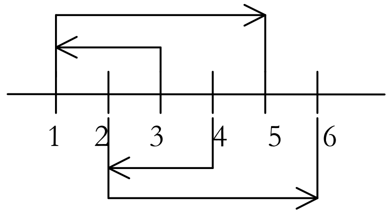
As we recognize, we get four times the same system of two-dimensional counting of the Peirce numbers (1, 3, 5) and (2, 4, 6). Therefore, for Peirce numbers it applies what Kronthaler remarked on trito-numbers: "The genesis of a new character (number) does not iterate the old structure, but leads to an enlargement of structure" (1986, p. 32). Hence we may also display the Peirce numbers as follows:

(1 → 3 → 5)　　(1 → 5 → 3)
(2 → 4 → 6)　　(2 → 6 → 4)

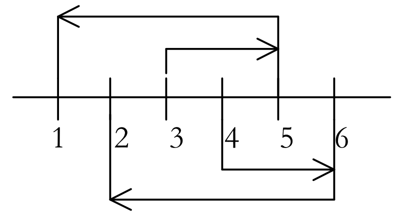




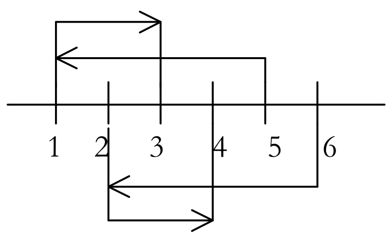
$(3 \to 1 \to 5)$
$(4 \to 2 \to 6)$



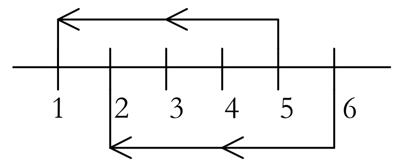
$(3 \to 5 \to 1)$
$(4 \to 6 \to 2)$



$(5 \to 1 \to 3)$
$(6 \to 2 \to 4)$



$(5 \to 3 \to 1)$
$(6 \to 4 \to 2)$



The other pairs of transpositions just repeat the above structures of Peirce numbers. At the hand of these pseudo-linear diagrams, we can learn easily what the semiotic meaning of the "sideward motion" of natural number is, which Barkley Rosser (1942) had mentioned in connection with logical paradoxes. As we notice, in Peirce numbers, there is non-linearity in most cases of these per se two-dimensional numbers; the only exception is the numerical representation of the non-transposed sign-class and its dual (non-transposed) reality thematic. It thus seems that Peirce numbers even transcend the structural possibilities of trito-numbers, the structurally most complex polycontextural numbers.

Bibliography

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