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Triples of morphisms for sign sets II

1. The aim of this contribution is to investigate the general triadic and trichotomic structure of sign classes, reality thematics and their transpositions when written as triples of dynamic morphisms (cf. Toth 2008a, pp. 159 ss.; 2008b). Let us first have a look at the transpositions of the sign class (3.1 2.1 1.3) and its reality thematic (3.1 1.2 1.3):

(3.1 2.1 1.3) → $[\beta^\circ, id1], [\alpha^\circ, \beta\alpha], [\alpha^\circ\beta^\circ, \beta\alpha]$
(3.1 1.3 2.1) → $[\alpha^\circ\beta^\circ, \beta\alpha], [\alpha, \alpha^\circ\beta^\circ], [\beta^\circ, id1]$
(2.1 3.1 1.3) → $[\beta, id1], [\alpha^\circ\beta^\circ, \beta\alpha], [\alpha^\circ, \beta\alpha]$
(2.1 1.3 3.1) → $[\alpha^\circ, \beta\alpha], [\beta\alpha, \alpha^\circ\beta^\circ], [\beta, id1]$
(1.3 3.1 2.1) → $[\beta\alpha, \alpha^\circ\beta^\circ], [\beta^\circ, id1], [\alpha, \alpha^\circ\beta^\circ]$
(1.3 2.1 3.1) → $[\alpha, \alpha^\circ\beta^\circ], [\beta, id1], [\beta\alpha, \alpha^\circ\beta^\circ]$

(3.1 1.2 1.3) → $[[\alpha^\circ\beta^\circ, \alpha], [id1, \beta], [\alpha^\circ\beta^\circ, \beta\alpha]$
(1.2 3.1 1.3) → $[\beta\alpha, \alpha^\circ], [\alpha^\circ\beta^\circ, \beta\alpha], [id1, \beta]$
(3.1 1.3 1.2) → $[\alpha^\circ\beta^\circ, \beta\alpha], [id1, \beta^\circ], [\alpha^\circ\beta^\circ, \alpha]$
(1.3 3.1 1.2) → $[\beta\alpha, \alpha^\circ\beta^\circ], [\alpha^\circ\beta^\circ, \alpha], [id1, \beta^\circ]$
(1.2 1.3 3.1) → $[id1, \beta], [\beta\alpha, \alpha^\circ\beta^\circ], [\beta\alpha, \alpha^\circ]$
(1.3 1.2 3.1) → $[id1, \beta^\circ], [\beta\alpha, \alpha^\circ], [\beta\alpha, \alpha^\circ\beta^\circ]$

Apparently, in the category theoretic notation, there seems to be no semiotic structure that shows that each block of 6 transpositions belongs to the same sign class and reality thematic.

2. We shall now define a sign class abstractly as (a.b c.d e.f), therefore its dual reality thematic is (f.e d.c b.a). We then get the following general system of all transpositions of a sign class and a reality thematic as well as their triadic and trichotomic order:

(a.b c.d e.f) → $[\underline{a}c, \underline{\underline{b}}d], [\underline{c}e, \underline{\underline{d}}f], [a\underline{e}, b\underline{\underline{f}}]$	(a. → c. → e.)	(b. → d. → f.)
(a.b e.f c.d) → $[\underline{a}e, \underline{\underline{b}}f], [\underline{e}c, \underline{\underline{f}}d], [a\underline{c}, b\underline{\underline{d}}]$	(a. → e. → c.)	(b. → f. → d.)
(c.d a.b e.f) → $[\underline{c}a, \underline{\underline{d}}b], [\underline{a}e, \underline{\underline{b}}f], [c\underline{e}, d\underline{\underline{f}}]$	(c. → a. → e.)	(d. → b. → f.)
(c.d e.f a.b) → $[\underline{c}e, \underline{\underline{d}}f], [\underline{e}a, \underline{\underline{f}}b], [c\underline{a}, d\underline{\underline{b}}]$	(c. → e. → a.)	(d. → f. → b.)
(e.f a.b c.d) → $[\underline{e}a, \underline{\underline{f}}b], [\underline{a}c, \underline{\underline{b}}d], [e\underline{c}, f\underline{\underline{d}}]$	(e. → a. → c.)	(f. → b. → d.)
(e.f c.d a.b) → $[\underline{e}c, \underline{\underline{f}}d], [\underline{c}a, \underline{\underline{d}}b], [e\underline{a}, f\underline{\underline{b}}]$	(e. → c. → a.)	(f. → d. → b.)

(f.e d.c b.a) → $[\underline{f}d, \underline{\underline{e}}c], [\underline{d}b, \underline{\underline{c}}a], [f\underline{b}, e\underline{\underline{a}}]$	(f. → d. → b.)	(e. → c. → a.)
(d.c f.e b.a) → $[\underline{d}f, \underline{\underline{c}}e], [\underline{f}b, \underline{\underline{e}}a], [d\underline{b}, c\underline{\underline{a}}]$	(d. → f. → b.)	(c. → e. → a.)
(f.e b.a d.c) → $[\underline{f}b, \underline{\underline{e}}a], [\underline{b}d, \underline{\underline{a}}c], [f\underline{d}, e\underline{\underline{c}}]$	(f. → b. → d.)	(e. → a. → c.)
(b.a f.e d.c) → $[\underline{b}f, \underline{\underline{a}}e], [\underline{f}d, \underline{\underline{e}}c], [b\underline{d}, a\underline{\underline{c}}]$	(b. → f. → d.)	(a. → e. → c.)
(d.c b.a f.e) → $[\underline{d}b, \underline{\underline{c}}a], [\underline{b}f, \underline{\underline{a}}e], [d\underline{f}, c\underline{\underline{e}}]$	(d. → b. → f.)	(c. → a. → e.)
(b.a d.c f.e) → $[\underline{b}d, \underline{\underline{a}}c], [\underline{d}f, \underline{\underline{c}}e], [b\underline{f}, a\underline{\underline{e}}]$	(b. → d. → f.)	(a. → c. → e.)

Using the general notation for sign relations, we can see that both the triadic and the trichotomic structure of both sign class and reality thematic are conserved in the notation of sign relations as triples of morphisms, f. ex.:

$(a.b\ c.d\ e.f) \rightarrow [\underline{a}c, \underline{\underline{b}}d], [\underline{c}e, \underline{\underline{d}}f], [a\underline{e}, b\underline{\underline{f}}]$
$(b.a\ d.c\ f.e) \rightarrow [\underline{b}d, \underline{\underline{a}}c], [\underline{d}f, \underline{\underline{c}}e], [b\underline{f}, a\underline{\underline{e}}]$

In these two examples, we thus have [ac] · [ce] = [ae]; [bd] · [df] = [bf]. In general: The third natural transformation of sign classes is simply obtained by multiplication of the first two, whereby different morphisms are juxtaposed and the product of identical morphisms is 0; thus:

[XY] · [XY] = [YX] · [YX] = 0
[XY] · [YX] = [XY, YX]
[XY] · [XZ] = [YZ] ≠ [XZ] · [XY] = ZY, etc.

3. Now we shall ascribe the natural numbers 1, ..., 6 to the letters a, b, c, d, e, f in this order. Then, we can rewrite the above general system of transpositions in the following form:

$(1.2\ 3.4\ 5.6) \rightarrow [\underline{1}3, \underline{\underline{2}}4], [\underline{3}5, \underline{\underline{4}}6], [1\underline{5}, 2\underline{\underline{6}}]$	(1. → 3. → 5.)	(2. → 4. → 6.)
$(1.2\ 5.6\ 3.4) \rightarrow [\underline{1}5, \underline{\underline{2}}6], [\underline{5}3, \underline{\underline{6}}4], [1\underline{3}, 2\underline{\underline{4}}]$	(1. → 5. → 3.)	(2. → 6. → 4.)
$(3.4\ 1.2\ 5.6) \rightarrow [\underline{3}1, \underline{\underline{4}}2], [\underline{1}5, \underline{\underline{2}}6], [3\underline{5}, 4\underline{\underline{6}}]$	(3. → 1. → 5.)	(4. → 2. → 6.)
$(3.4\ 5.6\ 1.2) \rightarrow [\underline{3}5, \underline{\underline{4}}6], [\underline{5}1, \underline{\underline{6}}2], [3\underline{1}, 4\underline{\underline{2}}]$	(3. → 5. → 1.)	(4. → 6. → 2.)
$(5.6\ 1.2\ 3.4) \rightarrow [\underline{5}1, \underline{\underline{6}}2], [\underline{1}3, \underline{\underline{2}}4], [5\underline{3}, 6\underline{\underline{4}}]$	(5. → 1. → 3.)	(6. → 2. → 4.)
$(5.6\ 3.4\ 1.2) \rightarrow [\underline{5}3, \underline{\underline{6}}4], [\underline{3}1, \underline{\underline{4}}2], [5\underline{1}, 6\underline{\underline{2}}]$	(5. → 3. → 1.)	(6. → 4. → 2.)

$(6.5\ 4.3\ 2.1) \rightarrow [\underline{6}4, \underline{\underline{5}}3], [\underline{4}2, \underline{\underline{3}}1], [6\underline{2}, 5\underline{\underline{1}}]$	(6. → 4. → 2.)	(5. → 3. → 1.)
$(4.3\ 6.5\ 2.1) \rightarrow [\underline{4}6, \underline{\underline{3}}5], [\underline{6}2, \underline{\underline{5}}1], [4\underline{2}, 3\underline{\underline{1}}]$	(4. → 6. → 2.)	(3. → 5. → 1.)
$(6.5\ 2.1\ 4.3) \rightarrow [\underline{6}2, \underline{\underline{5}}1], [\underline{2}4, \underline{\underline{1}}3], [6\underline{4}, 5\underline{\underline{3}}]$	(6. → 2. → 4.)	(5. → 1. → 3.)
$(2.1\ 6.5\ 4.3) \rightarrow [\underline{2}6, \underline{\underline{1}}5], [\underline{6}4, \underline{\underline{5}}3], [2\underline{4}, 1\underline{\underline{3}}]$	(2. → 6. → 4.)	(1. → 5. → 3.)
$(4.3\ 2.1\ 6.5) \rightarrow [\underline{4}2, \underline{\underline{3}}1], [\underline{2}6, \underline{\underline{1}}5], [4\underline{6}, 3\underline{\underline{5}}]$	(4. → 2. → 6.)	(3. → 1. → 5.)
$(2.1\ 4.3\ 6.5) \rightarrow [\underline{2}4, \underline{\underline{1}}3], [\underline{4}6, \underline{\underline{3}}5], [2\underline{6}, 1\underline{\underline{5}}]$	(2. → 4. → 6.)	(1. → 3. → 5.)

Since the elements of the original sign class, reality thematic and their transpositions have been substituted by bijective mapping and since the sign relations (triadic, trichotomic, semiotic inclusion) have been conserved, the above two general systems of semiotic transpositions are isomorphic. From the latter system, we recognize that in semiotic transpositions, triadic order has the general structure:

(ODD NUMBER) → (ODD NUMBER ± 2) → ((ODD NUMBER ± 2) ± 2))

and trichotomic order has the following general structure:

(EVEN NUMBER) → (EVEN NUMBER ± 2) → ((EVEN NUMBER ± 2) ± 2))

Bibliography

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)
Toth, Alfred, Triples of morphisms for sign sets I. Ch. 2 (2008b)