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Triadic trichotomies and trichotomic triads

1. In his chapter "fundamental sign-internal and sign-external semioses" (1975, pp. 102 ss.), Max Bense proposed the two following ways to construct semiotic matrices:

1. $_{T}(t \times t)$: the Cartesian product over the triadic trichotomies
2. $(t \times t)_{T}$: the Cartesian product over the trichotomic triades

Bense then constructed the following three semiotic matrices of the Cartesian products over the triadic trichotomies:

$_{T}(t_1 \times t_1)$:

	1.1	1.2	1.3
1.1	1.1 1.1	1.1 1.2	1.1 1.3
1.2	1.2 1.1	1.2 1.2	1.2 1.3
1.3	1.3 1.1	1.3 1.2	1.3 1.3

$_{T}(t_2 \times t_2)$:

	2.1	2.2	2.3
2.1	2.1 2.1	2.1 2.2	2.1 2.3
2.2	2.2 2.1	2.2 2.2	2.2 2.3
2.3	2.3 2.1	2.3 2.2	2.3 2.3

$_{T}(t_3 \times t_3)$:

	3.1	3.2	3.3
3.1	3.1 3.1	3.1 3.2	3.1 3.3
3.2	3.2 3.1	3.2 3.2	3.2 3.3
3.3	3.3 3.1	3.3 3.2	3.3 3.3

and the following three over the trichotomic triads:

$(t_1 \times t_1)_T$:

	1.1	2.1	3.1
1.1	1.1 1.1	1.1 2.1	1.1 3.1
2.1	2.1 1.1	2.1 2.1	2.1 3.1
3.1	3.1 1.1	3.1 2.1	3.1 3.1

$(t_2 \times t_2)_T$:

	1.2	2.2	3.2
1.2	1.2 1.2	1.2 2.2	1.2 3.2
2.2	2.2 1.2	2.2 2.2	2.2 3.2
3.2	3.2 1.2	3.2 2.2	3.2 3.2

$(t_3 \times t_3)_T$:

	1.3	2.3	3.3
1.3	1.3 1.3	1.3 2.3	1.3 3.3
2.3	2.3 1.3	2.3 2.3	2.3 3.3
3.3	3.3 1.3	3.3 2.3	3.3 3.3

By these 6 matrices, $6 \cdot 9 = 54$ combinations of pairs of sub-signs are obtained, and thus 27 obtained by 3 more semiotic matrices are lacking in order to construct Bense's "triadic-trichotomic matrix" (1975, p. 105), which is therefore defective.

2. Each triadic sign class or reality thematic can be written in the form $(\alpha_1, \alpha_2, \alpha_3)$, the α_i's standing either for the sub-signs or the respective semioses of the sign-relations. Since this set of 3 elements allows 6 permutations, we get the following general semiotic permutation matrices:

$$\begin{bmatrix} \alpha_1, \alpha_2, \alpha_3 \\ \alpha_1, \alpha_2, \alpha_3 \end{bmatrix} \begin{bmatrix} \alpha_1, \alpha_2, \alpha_3 \\ \alpha_1, \alpha_3, \alpha_2 \end{bmatrix} \begin{bmatrix} \alpha_1, \alpha_2, \alpha_3 \\ \alpha_2, \alpha_3, \alpha_3 \end{bmatrix} \begin{bmatrix} \alpha_1, \alpha_2, \alpha_3 \\ \alpha_2, \alpha_1, \alpha_3 \end{bmatrix} \begin{bmatrix} \alpha_1, \alpha_2, \alpha_3 \\ \alpha_3, \alpha_2, \alpha_1 \end{bmatrix} \begin{bmatrix} \alpha_1, \alpha_2, \alpha_3 \\ \alpha_3, \alpha_1, \alpha_2 \end{bmatrix}$$

If we assign the sub-signs (1.1), (1.2), (1.3), ..., (3.3) to the α_i's, we obtain $6 \cdot 3^3 = 162$ triadic sets of sub-signs which are all elements of cyclic semiotic groups (cf. Toth 2008b). Since each triadic relation can be written in six (ordered) pairs of dyads, e.g., (1.2 2.1 3.1) = (1.2 2.1), (2.1 3.1), (1.2 3.1), (3.1 2.1), (2.1. 1.2), (3.1 1.2), we get by this method the total amount of $6 \cdot 162 = 972$ (not necessarily different) pairs of sub-signs which both belong to mixed triadic-trichotomic and trichotomic-triadic matrices not reachable by Bense's above method.

3. Another way of constructing pairs of sub-signs we get by semiotic matrices that are not strictly either triadic-trichotomic or trichotomic-triads, but also allow mixed forms. A semiotic matrix has the following general form:

	.b	.d	.f
a.	a.b	a.d	a.f
c.	c.b	c.d	c.f
e.	e.b	e.d	e.f

Since (a.b) ≠ (a.d) ≠ (a.f), etc., i.e. the Cartesian products must be pairwise different, we can set all sub-signs from (1.1) to (3.3) for (a.b) and then continue according to increasing (or decreasing) semiotic order, i.e. (1.1) (1.2) (1.3), (1.2 1.3 2.1), (1.3 2.1 2.2), ..., or (1.1 3.3 3.2), (1.2 1.1 3.3), (1.3 1.2 1.1), etc. Yet, in doing so, we get only 9 semiotic matrices of cyclic groups. However, we can use the fact that the matrices can be rotated either clockwise or counter-clockwise (0° = 360°, 90°, 180°, 270°) and the rotations also form cyclic groups (cf. Wolf/Wolff (1956, pp. 7 ss.). In this way, we get the following 4 · 9 = 36 semiotic matrices:

$$\begin{pmatrix} 1.1 & 1.2 & 1.3 \\ 2.1 & 2.2 & 2.3 \\ 3.1 & 3.2 & 3.3 \end{pmatrix} \begin{pmatrix} 3.1 & 2.1 & 1.1 \\ 3.2 & 2.2 & 1.2 \\ 3.3 & 2.3 & 1.3 \end{pmatrix} \begin{pmatrix} 3.3 & 3.2 & 3.1 \\ 2.3 & 2.2 & 2.1 \\ 1.3 & 1.2 & 1.1 \end{pmatrix} \begin{pmatrix} 1.3 & 2.3 & 3.3 \\ 1.2 & 2.2 & 3.2 \\ 1.1 & 2.1 & 3.1 \end{pmatrix}$$

$$\begin{pmatrix} 1.2 & 1.3 & 2.1 \\ 2.2 & 2.3 & 3.1 \\ 3.2 & 3.3 & 1.1 \end{pmatrix} \begin{pmatrix} 3.2 & 2.2 & 1.2 \\ 3.3 & 2.3 & 1.3 \\ 1.1 & 3.1 & 2.1 \end{pmatrix} \begin{pmatrix} 1.1 & 3.3 & 3.2 \\ 3.1 & 2.3 & 2.2 \\ 2.1 & 1.3 & 1.2 \end{pmatrix} \begin{pmatrix} 2.1 & 3.1 & 1.1 \\ 1.3 & 2.3 & 3.3 \\ 1.2 & 2.2 & 3.2 \end{pmatrix}$$

$$\begin{pmatrix} 1.3 & 2.1 & 2.2 \\ 2.3 & 3.1 & 3.2 \\ 3.3 & 1.1 & 1.2 \end{pmatrix} \begin{pmatrix} 3.3 & 2.3 & 1.3 \\ 1.1 & 3.1 & 2.1 \\ 1.2 & 3.2 & 2.2 \end{pmatrix} \begin{pmatrix} 1.2 & 1.1 & 3.3 \\ 3.2 & 3.1 & 2.3 \\ 2.2 & 2.1 & 1.3 \end{pmatrix} \begin{pmatrix} 2.2 & 3.2 & 1.2 \\ 2.1 & 3.1 & 1.1 \\ 1.3 & 2.3 & 3.3 \end{pmatrix}$$

$$\begin{pmatrix} 2.1 & 2.2 & 2.3 \\ 3.1 & 3.2 & 3.3 \\ 1.1 & 1.2 & 1.3 \end{pmatrix} \begin{pmatrix} 1.1 & 3.1 & 2.1 \\ 1.2 & 3.2 & 2.2 \\ 1.3 & 3.3 & 2.3 \end{pmatrix} \begin{pmatrix} 1.3 & 1.2 & 1.1 \\ 3.3 & 3.2 & 3.1 \\ 2.3 & 2.2 & 2.1 \end{pmatrix} \begin{pmatrix} 2.3 & 3.3 & 1.3 \\ 2.2 & 3.2 & 1.2 \\ 2.1 & 3.1 & 1.1 \end{pmatrix}$$

$$\begin{pmatrix} 2.2 & 2.3 & 3.1 \\ 3.2 & 3.3 & 1.1 \\ 1.2 & 1.3 & 2.1 \end{pmatrix} \begin{pmatrix} 1.2 & 3.2 & 2.2 \\ 1.3 & 3.3 & 2.3 \\ 2.1 & 1.1 & 3.1 \end{pmatrix} \begin{pmatrix} 2.1 & 1.3 & 1.2 \\ 1.1 & 3.3 & 3.2 \\ 3.1 & 2.3 & 2.2 \end{pmatrix} \begin{pmatrix} 3.1 & 1.1 & 2.1 \\ 2.3 & 3.3 & 1.3 \\ 2.2 & 3.2 & 1.2 \end{pmatrix}$$

$$\begin{pmatrix} 2.3 & 3.1 & 3.2 \\ 3.3 & 1.1 & 1.2 \\ 1.3 & 2.1 & 2.2 \end{pmatrix} \begin{pmatrix} 1.3 & 3.3 & 2.3 \\ 2.1 & 1.1 & 3.1 \\ 2.2 & 1.2 & 3.2 \end{pmatrix} \begin{pmatrix} 2.2 & 2.1 & 1.3 \\ 1.2 & 1.1 & 3.3 \\ 3.2 & 3.1 & 2.3 \end{pmatrix} \begin{pmatrix} 3.2 & 1.2 & 2.2 \\ 3.1 & 1.1 & 2.1 \\ 2.3 & 3.3 & 1.3 \end{pmatrix}$$

$$\begin{pmatrix} 3.1 & 3.2 & 3.3 \\ 1.1 & 1.2 & 1.3 \\ 2.1 & 2.2 & 2.3 \end{pmatrix} \begin{pmatrix} 2.1 & 1.1 & 3.1 \\ 2.2 & 1.2 & 3.2 \\ 2.3 & 1.3 & 3.3 \end{pmatrix} \begin{pmatrix} 2.3 & 2.2 & 2.1 \\ 1.3 & 1.2 & 1.1 \\ 3.3 & 3.2 & 3.1 \end{pmatrix} \begin{pmatrix} 3.3 & 1.3 & 2.3 \\ 3.2 & 1.2 & 2.2 \\ 3.1 & 1.1 & 2.1 \end{pmatrix}$$

$$\begin{pmatrix} 3.2 & 3.3 & 1.1 \\ 1.2 & 1.3 & 2.1 \\ 2.2 & 2.3 & 3.1 \end{pmatrix} \begin{pmatrix} 2.2 & 1.2 & 3.2 \\ 2.3 & 1.3 & 3.3 \\ 3.1 & 2.1 & 1.1 \end{pmatrix} \begin{pmatrix} 3.1 & 2.3 & 2.2 \\ 2.1 & 1.3 & 1.2 \\ 1.1 & 3.3 & 3.2 \end{pmatrix} \begin{pmatrix} 1.1 & 2.1 & 3.1 \\ 3.3 & 1.3 & 2.3 \\ 3.2 & 1.2 & 2.2 \end{pmatrix}$$

$$\begin{pmatrix} 3.3 & 1.1 & 1.2 \\ 1.3 & 2.1 & 2.2 \\ 2.3 & 3.1 & 3.2 \end{pmatrix} \begin{pmatrix} 2.3 & 1.3 & 3.3 \\ 3.1 & 2.1 & 1.1 \\ 3.2 & 2.2 & 1.2 \end{pmatrix} \begin{pmatrix} 3.2 & 3.1 & 2.3 \\ 2.2 & 2.1 & 1.3 \\ 1.2 & 1.1 & 3.3 \end{pmatrix} \begin{pmatrix} 1.2 & 2.2 & 3.2 \\ 1.1 & 2.1 & 3.1 \\ 3.3 & 1.3 & 2.3 \end{pmatrix}$$

Thus, the 36 semiotic matrices contain all $2^9 = 512$ possible pairs of sub-signs and thus all ordered dyadic subsets of the sets of triadic trichotomies and trichotomic triads. Like the "general" semiotic matrix, all other semiotic matrices contain the "main" sign classes (3.1 2.1 1.1), (3.2 2.2 1.2) and (3.3 2.3 1.3), their dual reality thematics, yet also their transpositions (cf. Toth 2008a, pp. 223 ss.). According to the types of rotation of the "general" matrix, either the main or the side diagonal contains the Genuine Category Class (3.3 2.2 1.1) and the eigenreal sign class (3.1 2.2 1.3), cf. Bense (1992, pp. 27 ss.), yet also their transpositions.

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