Prof. Dr. Alfred Toth

Trichotomic triads as fuzzy sets

1. The aim of the present study is to apply the concept of sign relation as fuzzy set, introduced in Toth (2008), to Trichotomic Triads, introduced by Walther (1981, 1982). Under a Trichotomic Triad one simply understands every union of three reality thematics (or trichotomies) to a triad, whereby the semiotic connections between the three trichotomies and the system of structural realities presented by the trichotomies are focussed. Therefore, there are two main types of Trichotomic Triads, such whose structural realities present a complete triadic sign relation, f. ex.

$$\left.\begin{array}{ll} (3.1\ 2.1\ 1.1) \times (1.1\ \underline{1.2\ 1.3}) & \text{M-them. M} \\ (3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3}) & \text{M-them. O} \\ (3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3}) & \text{M-them. I} \end{array}\right\} \quad \text{Trichotomic Triad}$$

and all other types, f. ex.

$$\left.\begin{array}{ll} (3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3}) & \text{M-them. I} \\ (3.1\ 2.2\ 1.3) \times (3.1\ \underline{2.2\ 1.3}) & \text{O, M-them. I} \\ (3.1\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 1.3) & \text{I-them. M} \end{array}\right\} \quad \text{Trichotomic Triad}$$

2. Walther (1979, p. 108) introduced a classification of the system of the 10 reality thematics (and thus also of their dual sign classes) according to the different degrees of reality share of their structural realities:

1. (3.1 2.1 1.1) × (1.1 $\underline{1.2\ 1.3}$)	3/3 M	—	—
2. (3.1 2.1 1.2) × (2.1 $\underline{1.2\ 1.3}$)	2/3 M	1/3 O	—
3. (3.1 2.1 1.3) × (3.1 $\underline{1.2\ 1.3}$)	2/3 M	—	1/3 I
4. (3.1 2.2 1.2) × ($\underline{2.1\ 2.2}$ 1.3)	—	2/3 O	1/3 M
5. (3.1 2.2 1.3) × ($\underline{3.1}$ $\underline{2.2}$ $\underline{1.3}$)	1/3 M	1/3 O	1/3 I
6. (3.1 2.3 1.3) × ($\underline{3.1\ 3.2}$ 1.3)	1/3 M	—	2/3 I
7. (3.2 2.2 1.2) × (2.1 $\underline{2.2\ 2.3}$)	—	3/3 O	—
8. (3.2 2.2 1.3) × (3.1 $\underline{2.2\ 2.3}$)	—	2/3 O	1/3 I
9. (3.2 2.3 1.3) × ($\underline{3.1\ 3.2}$ 2.3)	—	1/3 O	2/3 I
10. (3.3 2.3 1.3) × (3.1 $\underline{3.2\ 3.3}$)	—	—	3/3 I

Maximally, a reality thematic can have 3/3 of reality share of either M, O or I, or a combination of two, and only in the case of the dual-invariant sign class (3.1 2.2 1.3) × (3.1 2.2 1.3) of all three triadic realities. If a reality thematic presents 3/3 of the same structural reality, it is called homogeneous, in all other cases heterogeneous. Besides the dual-invariant sign class, in all structural realities, 2/3 of reality X thematizes 1/3 of reality Y, whereby X, Y $\in$ {1., 2., 3.}, in the case of the homogeneous realities, X = Y, so that all but the dual-invariant sign class have dyadic structural realities. The structural reality of the sign class (3.1

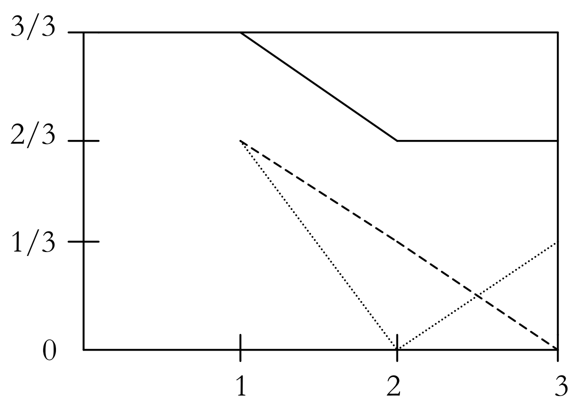
2.2 1.3) is triadic and shows thus the three structural cases ($\underline{3.1\ 2.2}$ 1.3), (3.1 $\underline{2.2\ 1.3}$) and ($\underline{3.1}$ 2.2 $\underline{1.3}$). This has to be considered wherever the dual-invariant sign class is part of a Trichotomic Triad like in the second example above in which thus two more Trichotomic Triads are "hidden" in the given Trichotomic Triad.

3. Therefore, the classification of a reality thematic by thirds of reality share of their presented structural realities is unambiguous, the mapping of the "classification by thirds" onto the reality thematics bijective. Moreover, since this "classification by thirds" is nothing else than an informal way to express the membership functions of the three trichotomies of the trichotomic triads in their respective reality sets, the mapping of the reality thematics onto their respective fuzzy sets is bijective, too (cf. Toth 2008).

In the present study, we shall thus investigate the 18 main Trichotomic Triads presented in Walther (1981, pp. 34 s.) and sketch afterward a fuzzy set theoretic construction of Walther's "determinant-symmetric duality system" (1982). In the following graphs, the M-sets are straight, the O-sets dashed, and the I-sets dotted.

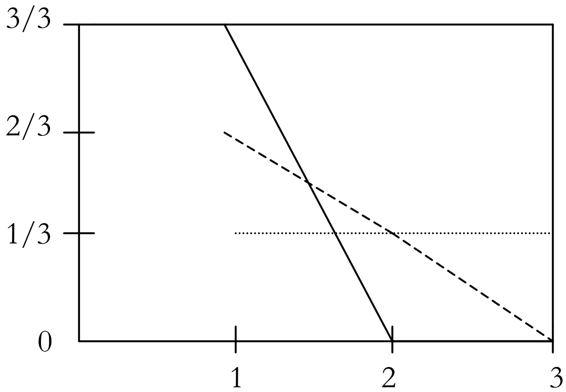
(3.1 2.1 1.1) × (1.1 $\underline{1.2\ 1.3}$)	M-them. M	3/3 M	—	—	TrTr1
(3.1 2.1 1.2) × (2.1 $\underline{1.2\ 1.3}$)	O-them. O	2/3 M	1/3 O	—	
(3.1 2.1 1.3) × (3.1 $\underline{1.2\ 1.3}$)	I-them. I	2/3 M	—	1/3 I	

Fuzzy set for TrTr1



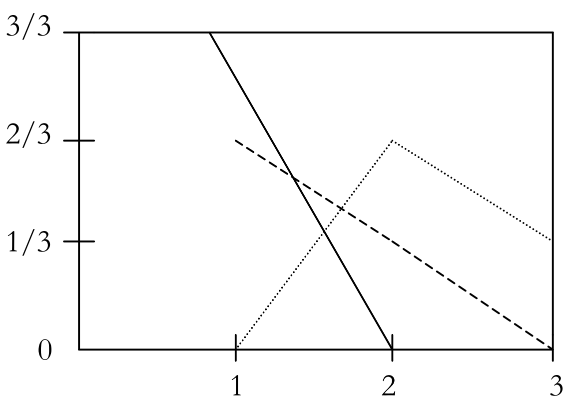
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.1 2.1 1.2) × (2.1 1.2 1.3)	M-them. O	2/3 M	1/3 O	—	TrTr2
(3.1 2.2 1.3) × (3.1 2.2 1.3)	M, O-them. I	1/3 M	1/3 O	1/3 I	

Fuzzy set for TrTr2



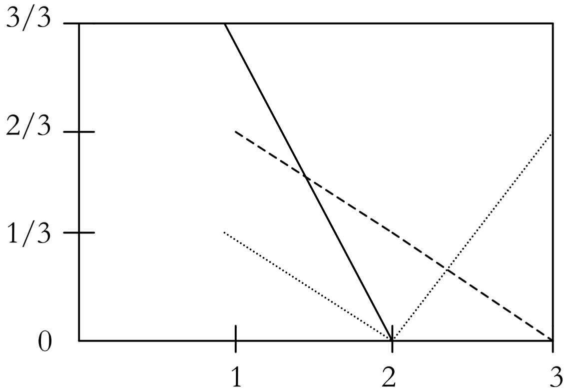
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.1 2.1 1.2) × (2.1 1.2 1.3)	M-them. O	2/3 M	1/3 O	—	TrTr3
(3.2 2.2 1.3) × (3.1 2.2 2.3)	M-them. I	—	2/3 O	1/3 I	

Fuzzy set for TrTr3



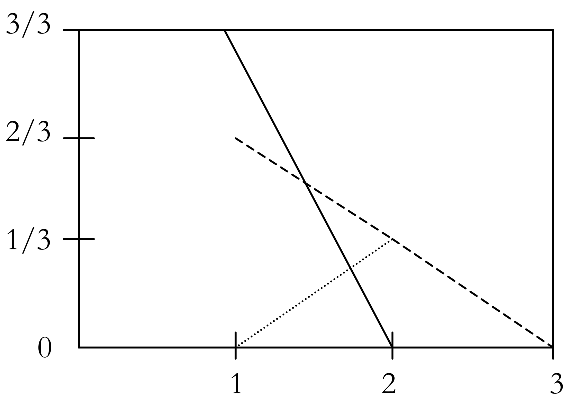
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.1 2.1 1.2) × (2.1 1.2 1.3)	M-them. O	2/3 M	1/3 O	—	TrTr4
(3.1 2.3 1.3) × (3.1 3.2 1.3)	I-them. M	1/3 M	—	2/3 I	

Fuzzy set for TrTr4



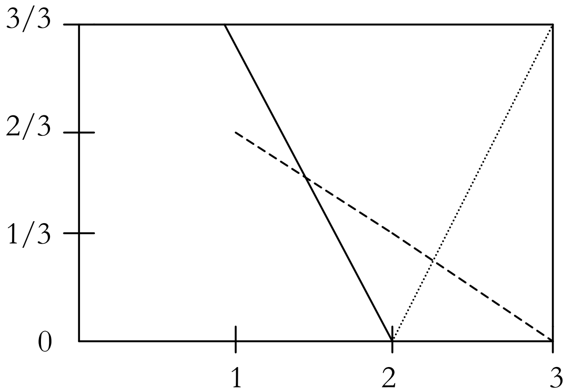
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.1 2.1 1.2) × (2.1 1.2 1.3)	M-them. O	2/3 M	1/3 O	—	TrTr5
(3.2 2.3 1.3) × (3.1 3.2 2.3)	I-them. O	—	1/3 O	2/3 I	

Fuzzy set for TrTr5



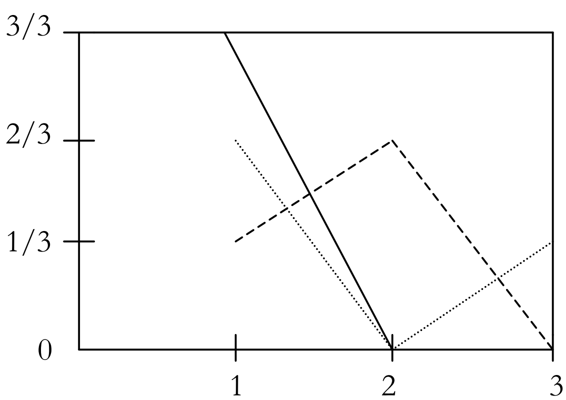
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.1 2.1 1.2) × (2.1 1.2 1.3)	M-them. O	2/3 M	1/3 O	—	TrTr6
(3.3 2.3 1.3) × (3.1 3.2 3.3)	I-them. I	—	—	3/3 I	

Fuzzy set for TrTr6



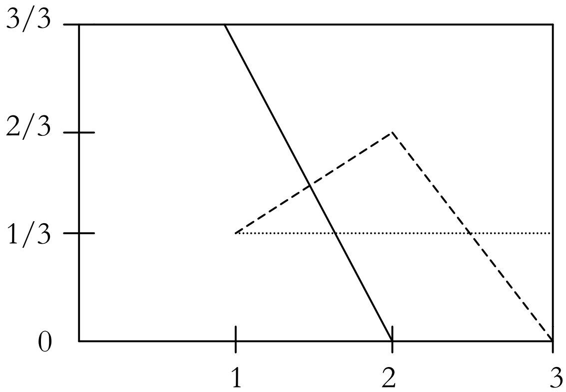
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.1 2.2 1.2) × (2.1 2.2 1.3)	O-them. M	1/3 M	2/3 O	—	TrTr7
(3.1 2.1 1.3) × (3.1 1.2 1.3)	M-them. I	2/3 M	—	1/3 I	

Fuzzy set for TrTr7



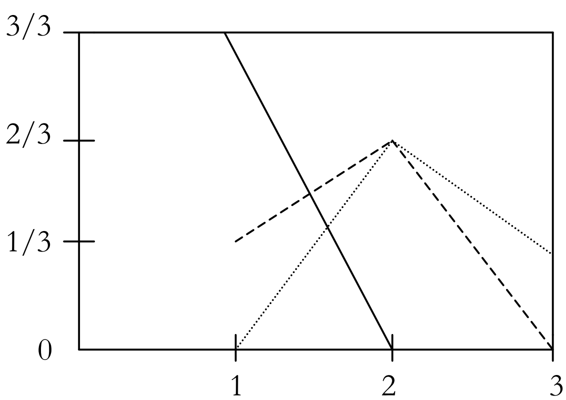
(3.1 2.1 1.1) × (1.1 $\underline{1.2\ 1.3}$)	M-them. M	3/3 M	—	—	
(3.1 2.2 1.2) × ($\underline{2.1\ 2.2}$ 1.3)	O-them. M	1/3 M	2/3 O	—	TrTr8
(3.1 2.2 1.3) × (3.1 $\underline{2.2\ 1.3}$)	M, O-them. I	1/3 M	1/3 O	1/3 I	

Fuzzy set for TrTr8



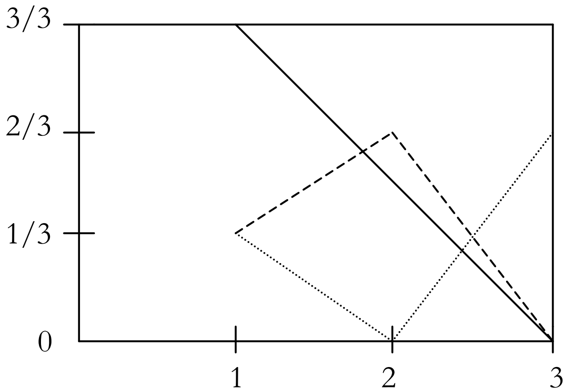
(3.1 2.1 1.1) × (1.1 $\underline{1.2\ 1.3}$)	M-them. M	3/3 M	—	—	
(3.1 2.2 1.2) × ($\underline{2.1\ 2.2}$ 1.3)	O-them. M	1/3 M	2/3 O	—	TrTr9
(3.2 2.2 1.3) × (3.1 $\underline{2.2\ 2.3}$)	O-them. I	—	2/3 O	1/3 I	

Fuzzy set for TrTr9



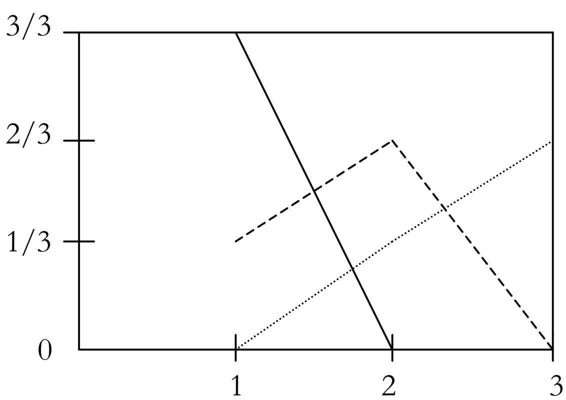
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.1 2.2 1.2) × (2.1 2.2 1.3)	O-them. M	1/3 M	2/3 O	—	TrTr10
(3.1 2.3 1.3) × (3.1 3.2 1.3)	I-them. M	1/3 M	—	2/3 I	

Fuzzy set for TrTr10



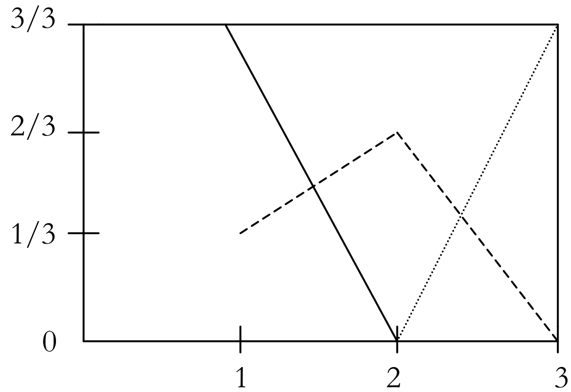
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.1 2.2 1.2) × (2.1 2.2 1.3)	O-them. M	1/3 M	2/3 O	—	TrTr11
(3.2 2.3 1.3) × (3.1 3.2 2.3)	I-them. O	—	1/3 O	2/3 I	

Fuzzy set for TrTr11



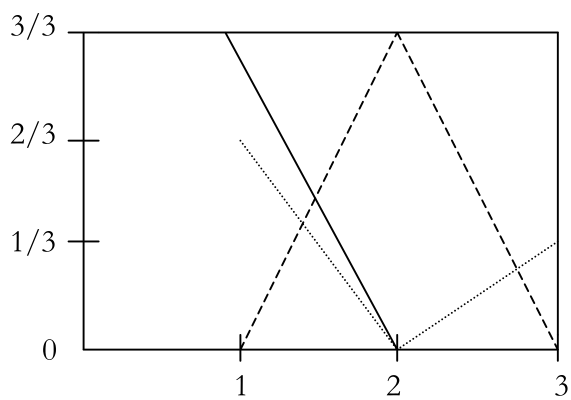
(3.1 2.1 1.1) × (1.1 $\underline{1.2\ 1.3}$)	M-them. M	3/3 M	—	—	
(3.1 2.2 1.2) × ($\underline{2.1\ 2.2}$ 1.3)	O-them. M	1/3 M	2/3 O	—	TrTr12
(3.3 2.3 1.3) × (3.1 $\underline{3.2\ 3.3}$)	I-them. I	—	—	3/3 I	

Fuzzy set for TrTr12



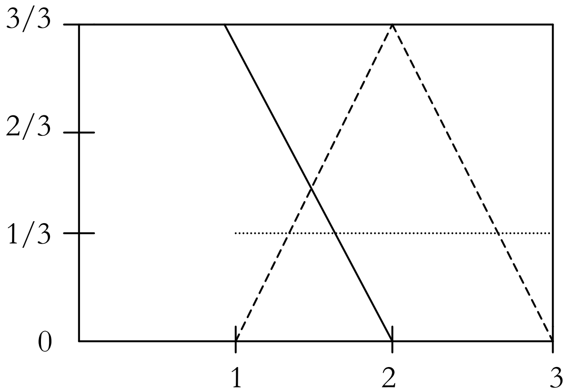
(3.1 2.1 1.1) × (1.1 $\underline{1.2\ 1.3}$)	M-them. M	3/3 M	—	—	
(3.2 2.2 1.2) × (2.1 $\underline{2.2\ 2.3}$)	O-them. O	—	3/3 O	—	TrTr13
(3.1 2.1 1.3) × (3.1 $\underline{1.2\ 1.3}$)	M-them. I	2/3 M	—	1/3 I	

Fuzzy set for TrTr13



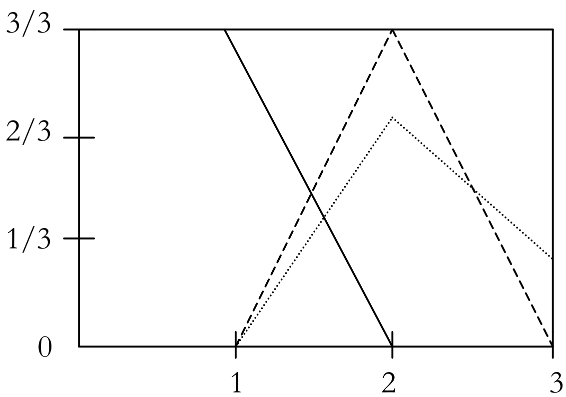
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.2 2.2 1.2) × (2.1 2.2 2.3)	O-them. O	—	3/3 O	—	TrTr14
(3.1 2.2 1.3) × (3.1 2.2 1.3)	M, O-them. I	1/3 M	1/3 O	1/3 I	

Fuzzy set for TrTr14



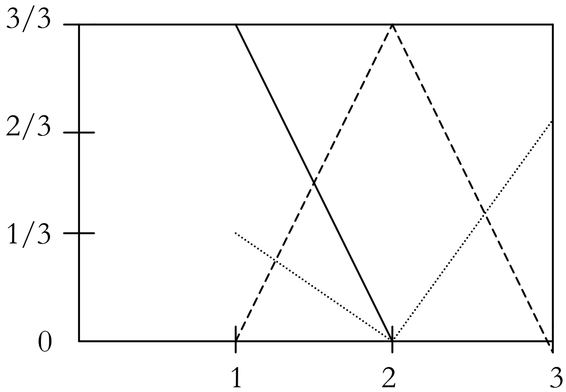
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.2 2.2 1.2) × (2.1 2.2 2.3)	O-them. O	—	3/3 O	—	TrTr15
(3.2 2.2 1.3) × (3.1 2.2 2.3)	O-them. I	—	2/3 O	1/3 I	

Fuzzy set for TrTr15



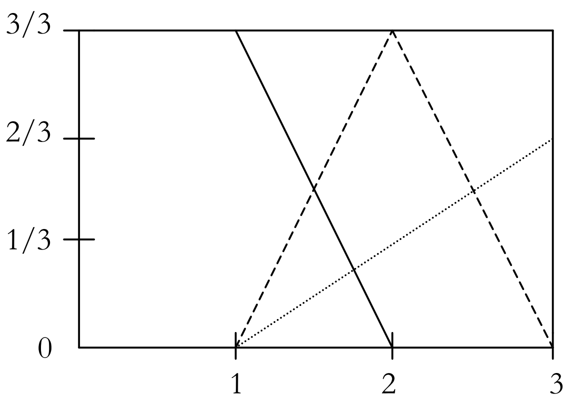
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.2 2.2 1.2) × (2.1 2.2 2.3)	O-them. O	—	3/3 O	—	TrTr16
(3.1 2.3 1.3) × (3.1 3.2 1.3)	I-them. M	1/3 M	—	2/3 I	

Fuzzy set for TrTr16



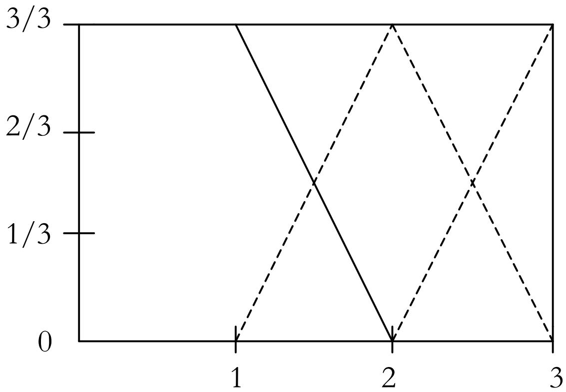
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	
(3.2 2.2 1.2) × (2.1 2.2 2.3)	O-them. O	—	3/3 O	—	TrTr17
(3.2 2.3 1.3) × (3.1 3.2 2.3)	I-them. O	—	1/3 O	2/3 I	

Fuzzy set for TrTr17



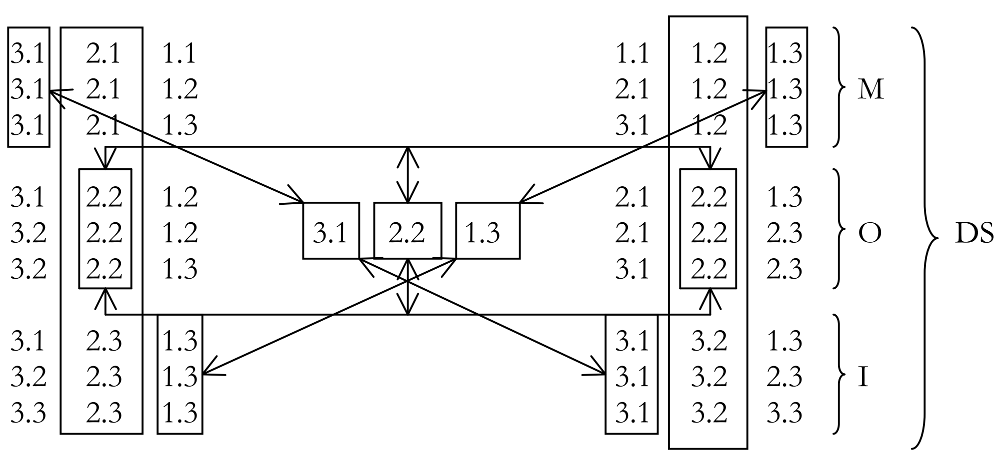
(3.1 2.1 1.1) × (1.1 1.2 1.3)	M-them. M	3/3 M	—	—	TrTr18
(3.2 2.2 1.2) × (2.1 2.2 2.3)	O-them. O	—	3/3 O	—	
(3.3 2.3 1.3) × (3.1 3.2 3.3)	I-them. I	—	—	3/3 I	

Fuzzy set for TrTr18



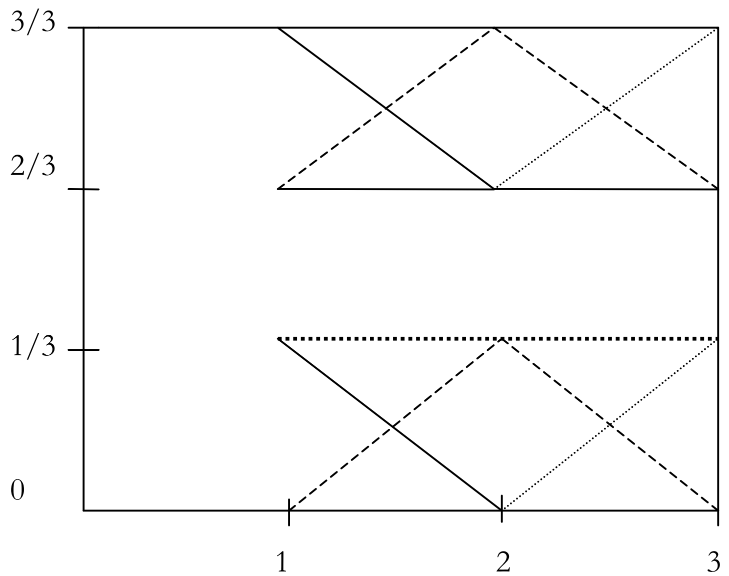
The graphs thus show nicely the mutual embeddings, crossings and superpositions of the M-, O- and I-subsets of the single trichotomic triads.

4. To conclude this study, we will now sketch a fuzzy theoretic construction of Walther's (1982) Determinant-Symmetric Duality System (DS). Roughly speaking, we are dealing here with a display of the system of the 10 sign classes and reality thematics that is based on the semiotic theorem that the eigenreal sign class (3.1 2.2 1.3) is connected with each other sign class and reality thematic by at least one and maximally two sub-signs. Therefore, the eigenreal sign class determines the dual semiotic system of the sign classes and the reality thematics, which can be shown by the following diagram from Walther (1982, p. 18):



We shall now turn this diagram into a graph. As we did before, the M-sub graphs are straight, the O-sub graphs dashed and the I-sub graphs dotted; in addition, the graph of the reality thematic (3.1 2.2 1.3) is bold:

Fuzzy set for DS (determinant-symmetric duality system)



As one can see, the lower and the upper sub-graphs are symmetric, and the structural reality of the eigenreal sign class (3.1 2.2 1.3) serves as **reality theoretic fuzzy equilibrium** (1/3 M, 1/3 O, 1/3 I) of both semiotic presentation (system of the reality thematics) and representation (system of the sign classes).

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