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Tetratomic tetrads from an extension of the set of the pre-semiotic sign classes

1. Unlike the "classic" semiotic sign relation $SR_{3,3}$ = (3.a 2.b 1.c), which is triadic-trichotomic, the "trans-classic" pre-semiotic sign relation $SR_{4,3}$ = (3.a 2.b 1.c 0.d) is tetradic-trichotomic. As a tetradic-trichotomic sign relation, $SR_{4,3}$ thus can be considered an expansion of $SR_{3,3}$. However, at the same time, $SR_{4,3}$ is also a fragment of the tetradic-tetratomic sign relation $SR_{4,4}$ (cf. Toth 2007, pp. 214 ss.), which can be seen best if we have a look at the structural realities presented by the reality thematics of the 15 pre-semiotic sign classes:

1 $(3.1\ 2.1\ 1.1\ 0.1) \times (1.0\ \underline{1.1\ 1.2}\ 1.3)$

2 $(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2}\ 1.3)$

3 $(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2}\ 1.3)$

4 $(3.1\ 2.1\ 1.2\ 0.2) \times (2.0\ \underline{2.1\ 1.2}\ 1.3)$

5 $(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 1.2}\ 1.3)$

6 $(3.1\ 2.1\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 1.2}\ 1.3)$

7 $(3.1\ 2.2\ 1.2\ 0.2) \times (2.0\ \underline{2.1\ 2.2}\ 1.3)$

8 $(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$

9 $(3.1\ 2.2\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 2.2}\ 1.3)$

10 $(3.1\ 2.3\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 3.2}\ 1.3)$

11 $(3.2\ 2.2\ 1.2\ 0.2) \times (2.0\ \underline{2.1\ 2.2}\ 2.3)$

12 $(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 2.3)$

13 $(3.2\ 2.2\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 2.2}\ 2.3)$

14 $(3.2\ 2.3\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 3.2}\ 2.3)$

15 $(3.3\ 2.3\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 3.2}\ 3.3)$

Thus, the reality thematic of the first trichotomic triad is characterized by (1.1 1.2). It turns out that each of the 15 reality thematics can be embedded into a trichotomic triads characterized by a pair of sub-signs. However, in order to do that, we have to reconstruct a semiotic system whose part SS15 is. As one easily sees, it is not SS35, which is built from the tetradic-tetratomic sign relation $SR_{4,4}$ = (3.a 2.b 1.c 0.d) and the semiotic inclusion order $a \leq b \leq c \leq d$, since in the pre-semiotic system SS15, a, b, c, d $\in$ {1, 2, 3} and thus $\neq 0$ (cf. Bense 1975, p. 65; Toth 2008a).

In the following table, we reconstruct the lacking reality thematics to build trichotomic triads by asterisk (*, **):

1	(3.1 2.1 1.1 0.1) × (1.0 $\underline{1.1\ 1.2}$ 1.3)
2	(3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2}$ 1.3)
3	(3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2}$ 1.3)
*	(3.1 2.1 1.2 0.1) × (1.0 $\underline{2.1\ 1.2}$ 1.3)
4	(3.1 2.1 1.2 0.2) × (2.0 $\underline{2.1\ 1.2}$ 1.3)
5	(3.1 2.1 1.2 0.3) × (3.0 $\underline{2.1\ 1.2}$ 1.3)
*	(3.1 2.1 1.3 0.1) × (1.0 $\underline{3.1\ 1.2}$ 1.3)
**	(3.1 2.1 1.3 0.2) × (2.0 $\underline{3.1\ 1.2}$ 1.3)
6	(3.1 2.1 1.3 0.3) × (3.0 $\underline{3.1\ 1.2}$ 1.3)
*	(3.1 2.2 1.2 0.1) × (1.0 $\underline{2.1\ 2.2}$ 1.3)
7	(3.1 2.2 1.2 0.2) × (2.0 $\underline{2.1\ 2.2}$ 1.3)
8	(3.1 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 1.3)
*	(3.1 2.2 1.3 0.1) × (1.0 $\underline{3.1\ 2.2}$ 1.3)
**	(3.1 2.2 1.3 0.2) × (2.0 $\underline{3.1\ 2.2}$ 1.3)
9	(3.1 2.2 1.3 0.3) × (3.0 $\underline{3.1\ 2.2}$ 1.3)
*	(3.1 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2}$ 1.3)
**	(3.1 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2}$ 1.3)
10	(3.1 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 1.3)
*	(3.2 2.2 1.2 0.1) × (1.0 $\underline{2.1\ 2.2}$ 2.3)
11	(3.2 2.2 1.2 0.2) × (2.0 $\underline{2.1\ 2.2}$ 2.3)
12	(3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 2.3)
*	(3.2 2.2 1.3 0.1) × (1.0 $\underline{3.1\ 2.2}$ 2.3)
**	(3.2 2.2 1.3 0.2) × (2.0 $\underline{3.1\ 2.2}$ 2.3)
13	(3.2 2.2 1.3 0.3) × (3.0 $\underline{3.1\ 2.2}$ 2.3)
*	(3.2 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2}$ 2.3)
**	(3.2 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2}$ 2.3)
14	(3.2 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 2.3)
*	(3.3 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2}$ 3.3)
**	(3.3 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2}$ 3.3)
15	(3.3 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 3.3)

2. Hence, we get 10 trichotomic triads and thus a system of 30 pre-semiotic sign classes (SS30). However, the set SS30\SS15 contains sign classes that are not built according to the inclusion order ($a \leq b \leq c \leq d$), which is valid for SS15. But note that this "violation" of semiotic inclusion touches only trichotomic zeroness, i.e. d, so that SS30 can be characterized by the following pre-semiotic inclusion orders:

$a \leq b \leq c < d$, e.g. (3.1 2.1 1.2 0.3)
$a \leq b \leq c = d$, e.g. (3.1 2.1 1.2 0.2)
$a \leq b \leq c > d$, e.g. (3.1 2.1 1.2 0.1)

Without this constraint that is based on Bense's distinction between relational and categorial numbers (cf. Toth 2008a), the maximal amount of sign classes from $SR_{4,3}$ would be $4^3 = 64$.

Moreover, if we look, e.g. at the reality thematic of the following pre-semiotic dual system:

13 $(3.2\ 2.2\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 2.2}\ 2.3)$

we recognize that the pair of sub-signs characteristic for the embedding of no. 13 into a trichotomic triads (3.1 2.2) belongs partly to the thematizing and partly to the thematized group of sub-signs in the following structural reality:

$(\underline{3.0\ 3.1}\ \underline{2.2\ 2.3}) \equiv 3^2 \leftrightarrow 2^2$,

which thus can be interpreted both as object-thematized interpretant ($3^2 \leftarrow 2^2$) and as interpretant-thematized object ($3^2 \rightarrow 2^2$).

3. We will now order the 30 pre-semiotic sign classes over this extension of $SR_{4,3}$, which we shall call $SR_{4,3}$*, according to their types of thematizations introduced in Toth (2007, pp. 214 ss.).

1. Homogeneous thematizations:

1 $(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$ 1^4
11 $(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$ 2^4
15 $(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$ 3^4

2. Dyadic thematizations:

2 $(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$ $2^1 \leftarrow 1^3$
3 $(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$ $3^1 \leftarrow 1^3$
4 $(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$ $2^2 \leftrightarrow 1^2$
6 $(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$ $3^2 \leftrightarrow 1^2$
7 $(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$ $2^3 \rightarrow 1^1$
10 $(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$ $3^3 \rightarrow 1^1$

*	(3.2 2.2 1.2 0.1) × (1.0 $\underline{2.1\ 2.2\ 2.3}$)	$1^1 \leftarrow 2^3$
12	(3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2\ 2.3}$)	$3^1 \leftarrow 2^3$
13	(3.2 2.2 1.3 0.3) × ($\underline{3.0\ 3.1}$ $\underline{2.2\ 2.3}$)	$3^2 \leftrightarrow 2^2$
14	(3.2 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2}$ 2.3)	$3^3 \rightarrow 2^1$
*	(3.3 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2\ 3.3}$)	$1^1 \leftarrow 3^3$
*	(3.3 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2\ 3.3}$)	$2^1 \leftarrow 3^3$

3. Triadic thematizations:

*	(3.1 2.1 1.2 0.1) × (1.0 2.1 $\underline{1.2\ 1.3}$)	$1^1 \leftrightarrow 2^1 \leftarrow 1^2$
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 $\underline{1.2\ 1.3}$)	$3^1 \leftrightarrow 2^1 \leftarrow 1^2$
*	(3.1 2.1 1.3 0.1) × (1.0 3.1 $\underline{1.2\ 1.3}$)	$1^1 \leftrightarrow 3^1 \leftarrow 1^2$
*	(3.1 2.1 1.3 0.2) × (2.0 3.1 $\underline{1.2\ 1.3}$)	$2^1 \leftrightarrow 3^1 \leftarrow 1^2$
*	(3.1 2.2 1.2 0.1) × (1.0 $\underline{2.1\ 2.2}$ 1.3)	$1^1 \leftarrow 2^2 \rightarrow 1^1$
8	(3.1 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 1.3)	$3^1 \leftarrow 2^2 \rightarrow 1^1$
9	(3.1 2.2 1.3 0.3) × ($\underline{3.0\ 3.1}$ 2.2 1.3)	$3^2 \rightarrow 2^1 \leftrightarrow 1^1$
*	(3.1 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2}$ 1.3)	$1^1 \leftarrow 3^2 \rightarrow 1^1$
*	(3.1 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2}$ 1.3)	$2^1 \leftarrow 3^2 \rightarrow 1^1$
*	(3.2 2.2 1.3 0.1) × (1.0 3.1 $\underline{2.2\ 2.3}$)	$1^1 \leftrightarrow 3^1 \leftarrow 2^2$
*	(3.2 2.2 1.3 0.2) × (2.0 3.1 $\underline{2.2\ 2.3}$)	$2^1 \leftrightarrow 3^1 \leftarrow 2^2$
*	(3.2 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2}$ 2.3)	$1^1 \leftarrow 3^2 \rightarrow 2^1$
*	(3.2 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2}$ 2.3)	$2^1 \leftarrow 3^2 \rightarrow 2^1$

4. Tetradic thematizations:

*	(3.1 2.2 1.3 0.1) × ($\underline{1.0}$ $\underline{3.1}$ $\underline{2.2}$ $\underline{1.3}$)	$1^1 \leftrightarrow 3^1 \leftrightarrow 2^1 \leftrightarrow 1^1$
*	(3.1 2.2 1.3 0.2) × ($\underline{2.0}$ $\underline{3.1}$ $\underline{2.2}$ $\underline{1.3}$)	$2^1 \leftrightarrow 3^1 \leftrightarrow 2^1 \leftrightarrow 1^1$

We can now group these n-adic thematizations to tetratomic n-ads. It turns out that reality thematics, which present dyadic thematization, can be grouped into 3 tetratomic tetrads:

Tetratomic Tetrads of dyadic thematization

2	(3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2\ 1.3}$)	$2^1 \leftarrow 1^3$
3	(3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2\ 1.3}$)	$3^1 \leftarrow 1^3$
4	(3.1 2.1 1.2 0.2) × ($\underline{2.0\ 2.1}$ $\underline{1.2\ 1.3}$)	$2^2 \leftrightarrow 1^2$
6	(3.1 2.1 1.3 0.3) × ($\underline{3.0\ 3.1}$ $\underline{1.2\ 1.3}$)	$3^2 \leftrightarrow 1^2$

7	(3.1 2.2 1.2 0.2) × ($\underline{2.0\ 2.1\ 2.2}$ 1.3)	$2^3 \rightarrow 1^1$
10	(3.1 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2}$ 1.3)	$3^3 \rightarrow 1^1$
*	(3.2 2.2 1.2 0.1) × (1.0 $\underline{2.1\ 2.2\ 2.3}$)	$1^1 \leftarrow 2^3$
12	(3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2\ 2.3}$)	$3^1 \leftarrow 2^3$

13	(3.2 2.2 1.3 0.3) × ($\underline{3.0\ 3.1}$ $\underline{2.2\ 2.3}$)	$3^2 \leftrightarrow 2^2$

14	(3.2 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2}$ 2.3)	$3^3 \rightarrow 2^1$
*	(3.3 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2\ 3.3}$)	$1^1 \leftarrow 3^3$
*	(3.3 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2\ 3.3}$)	$2^1 \leftarrow 3^3$

Reality thematics, which present dyadic thematization, can be grouped into 3 tetratomic tetrads plus the $SR_{4,3}$-equivalent of the dual-identical sign class (3.1 2.2 1.3) in $SR_{3,3}$:

*	(3.1 2.1 1.2 0.1) × (1.0 2.1 $\underline{1.2\ 1.3}$)	$1^1 \leftrightarrow 2^1 \leftarrow 1^2$
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 $\underline{1.2\ 1.3}$)	$3^1 \leftrightarrow 2^1 \leftarrow 1^2$
*	(3.1 2.1 1.3 0.1) × (1.0 3.1 $\underline{1.2\ 1.3}$)	$1^1 \leftrightarrow 3^1 \leftarrow 1^2$
*	(3.1 2.1 1.3 0.2) × (2.0 3.1 $\underline{1.2\ 1.3}$)	$2^1 \leftrightarrow 3^1 \leftarrow 1^2$
*	(3.1 2.2 1.2 0.1) × (1.0 $\underline{2.1\ 2.2}$ 1.3)	$1^1 \leftarrow 2^2 \rightarrow 1^1$
8	(3.1 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 1.3)	$3^1 \leftarrow 2^2 \rightarrow 1^1$
*	(3.1 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2}$ 1.3)	$1^1 \leftarrow 3^2 \rightarrow 1^1$
*	(3.2 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2}$ 2.3)	$2^1 \leftarrow 3^2 \rightarrow 2^1$
*	(3.1 2.3 1.3 0.2) × (2.0 $\underline{3.1\ 3.2}$ 1.3)	$2^1 \leftarrow 3^2 \rightarrow 1^1$
*	(3.2 2.2 1.3 0.1) × (1.0 3.1 $\underline{2.2\ 2.3}$)	$1^1 \leftrightarrow 3^1 \leftarrow 2^2$
*	(3.2 2.2 1.3 0.2) × (2.0 3.1 $\underline{2.2\ 2.3}$)	$2^1 \leftrightarrow 3^1 \leftarrow 2^2$
*	(3.2 2.3 1.3 0.1) × (1.0 $\underline{3.1\ 3.2}$ 2.3)	$1^1 \leftarrow 3^2 \rightarrow 2^1$
9	(3.1 2.2 1.3 0.3) × ($\underline{3.0\ 3.1}$ 2.2 1.3)	$3^2 \rightarrow 2^1 \leftrightarrow 1^1$

Although the tetradic pre-semiotic sign class (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3) is only dual-invariant respecting its triadic part relation (3.1 2.2 1.3), the sign class (3.1 2.2 1.3 0.3) and its reality thematic (3.0 3.1 2.2 1.3) hang together with all other sign classes and reality thematics of this tetratomic tetrad of triadic thematization, respectively, by at least one sub-sign. Thus, the pre-semiotic dual system (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3) shares this type of connectedness with the semiotic dual system (3.1 2.2 1.3) × (3.1 2.2 1.3).

4. In Toth (2008b), we have shown that both the semiotic system SS10 over $SR_{3,3}$ and the pre-semiotic system SS27 over $SR_{3,3}$ with abolishment of the semiotic inclusion order $a \leq b \leq c$ are homeostatic. It thus may astonish that also both SS15 and SS30 over $SR_{4,3}$ are homeostatic, despite their lacking of a (genuine) dual-identical sign class. The reason is the for-mentioned connectedness of the pre-semiotic dual system (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3) by at least one sub-sign to all other pre-semiotic dual systems both from SS15 and from SS30:

1	(3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)
2	(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)
3	(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)
*	(3.1 2.1 1.2 0.1) × (1.0 2.1 1.2 1.3)
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)
*	(3.1 2.1 1.3 0.1) × (1.0 3.1 1.2 1.3)
**	(3.1 2.1 1.3 0.2) × (2.0 3.1 1.2 1.3)
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)
*	(3.1 2.2 1.2 0.1) × (1.0 2.1 2.2 1.3)
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)
*	(3.1 2.2 1.3 0.1) × (1.0 3.1 2.2 1.3)
**	(3.1 2.2 1.3 0.2) × (2.0 3.1 2.2 1.3)
9	**(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)**
*	(3.1 2.3 1.3 0.1) × (1.0 3.1 3.2 1.3)
**	(3.1 2.3 1.3 0.2) × (2.0 3.1 3.2 1.3)
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
*	(3.2 2.2 1.2 0.1) × (1.0 2.1 2.2 2.3)
11	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)
*	(3.2 2.2 1.3 0.1) × (1.0 3.1 2.2 2.3)
**	(3.2 2.2 1.3 0.2) × (2.0 3.1 2.2 2.3)
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)
*	(3.2 2.3 1.3 0.1) × (1.0 3.1 3.2 2.3)
**	(3.2 2.3 1.3 0.2) × (2.0 3.1 3.2 2.3)
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)
*	(3.3 2.3 1.3 0.1) × (1.0 3.1 3.2 3.3)
**	(3.3 2.3 1.3 0.2) × (2.0 3.1 3.2 3.3)
15	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)

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