Prof. Dr. Alfred Toth

Tetradic, triadic, and dyadic sign classes

1. In Toth (2008a, pp. 179 ss.), we have constructed a tetradic-tetratomic semiotics on the basis of the following 4 × 4 matrix:

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3

based on the general tetradic-tetratomic sign relation

SR_4 = R(Q, M, O, I); SR_4 = R(.0., .1., .2., .3.);
SR_4 = (((Q ⇒ M) ⇒ O) ⇒ I); SR_4 = (((.0. ⇒ .1.) ⇒ .2.) ⇒ .3.)

with the tetratomic semiotic inclusion order

(3.a 2.b 1.c 0.d) with a, b, c, d ∈ {.0., .1., .2., .3.} und a ≤ b ≤ c ≤ d

We can then construct the following 35 tetradic-tetratomic sign classes and their dual reality thematics:

1	(3.0	2.0	1.0	0.0)	×	(0.0	0.1	0.2	0.3)
2	(3.0	2.0	1.0	0.1)	×	(1.0	0.1	0.2	0.3)
3	(3.0	2.0	1.0	0.2)	×	(2.0	0.1	0.2	0.3)
4	(3.0	2.0	1.0	0.3)	×	(3.0	0.1	0.2	0.3)
5	(3.0	2.0	1.1	0.1)	×	(1.0	1.1	0.2	0.3)
6	(3.0	2.0	1.1	0.2)	×	(2.0	1.1	0.2	0.3)
7	(3.0	2.0	1.1	0.3)	×	(3.0	1.1	0.2	0.3)
8	(3.0	2.0	1.2	0.2)	×	(2.0	2.1	0.2	0.3)
9	(3.0	2.0	1.2	0.3)	×	(3.0	2.1	0.2	0.3)
10	(3.0	2.0	1.3	0.3)	×	(3.0	3.1	0.2	0.3)
11	(3.0	2.1	1.1	0.1)	×	(1.0	1.1	1.2	0.3)
12	(3.0	2.1	1.1	0.2)	×	(2.0	1.1	1.2	0.3)
13	(3.0	2.1	1.1	0.3)	×	(3.0	1.1	1.2	0.3)
14	(3.0	2.1	1.2	0.2)	×	(2.0	2.1	1.2	0.3)
15	(3.0	2.1	1.2	0.3)	×	(3.0	2.1	1.2	0.3)

16	(3.0	2.1	1.3	0.3)	×	(3.0	3.1	1.2	0.3)
17	(3.0	2.2	1.2	0.2)	×	(2.0	2.1	2.2	0.3)
18	(3.0	2.2	1.2	0.3)	×	(3.0	2.1	2.2	0.3)
19	(3.0	2.2	1.3	0.3)	×	(3.0	3.1	2.2	0.3)
20	(3.0	2.3	1.3	0.3)	×	(3.0	3.1	3.2	0.3)
21	(3.1	2.1	1.1	0.1)	×	(1.0	1.1	1.2	1.3)
22	(3.1	2.1	1.1	0.2)	×	(2.0	1.1	1.2	1.3)
23	(3.1	2.1	1.1	0.3)	×	(3.0	1.1	1.2	1.3)
24	(3.1	2.1	1.2	0.2)	×	(2.0	2.1	1.2	1.3)
25	(3.1	2.1	1.2	0.3)	×	(3.0	2.1	1.2	1.3)
26	(3.1	2.1	1.3	0.3)	×	(3.0	3.1	1.2	1.3)
27	(3.1	2.2	1.2	0.2)	×	(2.0	2.1	2.2	1.3)
28	(3.1	2.2	1.2	0.3)	×	(3.0	2.1	2.2	1.3)
29	(3.1	2.2	1.3	0.3)	×	(3.0	3.1	2.2	1.3)
30	(3.1	2.3	1.3	0.3)	×	(3.0	3.1	3.2	1.3)
31	(3.2	2.2	1.2	0.2)	×	(2.0	2.1	2.2	2.3)
32	(3.2	2.2	1.2	0.3)	×	(3.0	2.1	2.2	2.3)
33	(3.2	2.2	1.3	0.3)	×	(3.0	3.1	2.2	2.3)
34	(3.2	2.3	1.3	0.3)	×	(3.0	3.1	3.2	2.3)
35	(3.3	2.3	1.3	0.3)	×	(3.0	3.1	3.2	3.3)

The 35 representation systems can be ordered into the following system of **4 Tetratomic Tetrads of structural realities with dyadic thematization**:

1	(3.0	2.0	1.0	0.0)	×	(0.0	0.1	0.2	0.3)
2	(3.0	2.0	1.0	0.1)	×	(1.0	0.1	0.2	0.3)
3	(3.0	2.0	1.0	0.2)	×	(2.0	0.1	0.2	0.3)
4	(3.0	2.0	1.0	0.3)	×	(3.0	0.1	0.2	0.3)
11	(3.0	2.1	1.1	0.1)	×	(1.0	1.1	1.2	0.3)
21	(3.1	2.1	1.1	0.1)	×	(1.0	1.1	1.2	1.3)
22	(3.1	2.1	1.1	0.2)	×	(2.0	1.1	1.2	1.3)
23	(3.1	2.1	1.1	0.3)	×	(3.0	1.1	1.2	1.3)
17	(3.0	2.2	1.2	0.2)	×	(2.0	2.1	2.2	0.3)
27	(3.1	2.2	1.2	0.2)	×	(2.0	2.1	2.2	1.3)
31	(3.2	2.2	1.2	0.2)	×	(2.0	2.1	2.2	2.3)
32	(3.2	2.2	1.2	0.3)	×	(3.0	2.1	2.2	2.3)
20	(3.0	2.3	1.3	0.3)	×	(3.0	3.1	3.2	0.3)
30	(3.1	2.3	1.3	0.3)	×	(3.0	3.1	3.2	1.3)

34	(3.2	2.3	1.3	0.3)	×	(3.0	3.1	3.2	2.3)
35	(3.3	2.3	1.3	0.3)	×	(3.0	3.1	3.2	3.3)

Moreover, the 35 representation systems can also be ordered into the following system **of 4 Tetratomic Triads of triadic thematization**:

1	(3.0	2.0	1.0	0.0)	×	(0.0	0.1	0.2	0.3)
6	(3.0	2.0	1.1	0.2)	×	(2.0	1.1	0.2	0.3)
9	(3.0	2.0	1.2	0.3)	×	(3.0	2.1	0.2	0.3)
7	(3.0	2.0	1.1	0.3)	×	(3.0	1.1	0.2	0.3)
12	(3.0	2.1	1.1	0.2)	×	(2.0	1.1	1.2	0.3)
21	(3.1	2.1	1.1	0.1)	×	(1.0	1.1	1.2	1.3)
25	(3.1	2.1	1.2	0.3)	×	(3.0	2.1	1.2	1.3)
13	(3.0	2.1	1.1	0.3)	×	(3.0	1.1	1.2	0.3)
14	(3.0	2.1	1.2	0.2)	×	(2.0	2.1	1.2	0.3)
28	(3.1	2.2	1.2	0.3)	×	(3.0	2.1	2.2	1.3)
31	(3.2	2.2	1.2	0.2)	×	(2.0	2.1	2.2	2.3)
18	(3.0	2.2	1.2	0.3)	×	(3.0	2.1	2.2	0.3)
16	(3.0	2.1	1.3	0.3)	×	(3.0	3.1	1.2	0.3)
29	(3.1	2.2	1.3	0.3)	×	(3.0	3.1	2.2	1.3)
19	(3.0	2.2	1.3	0.3)	×	(3.0	3.1	2.2	0.3)
35	(3.3	2.3	1.3	0.3)	×	(3.0	3.1	3.2	3.3)

2. Triadic-trichotomic semiotics that is constructed by aid of the following 3 × 3 matrix:

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

on the basis of the general triadic-trichotomic sign relation

SR_3 = R(M, O, I); SR_3 = R(.1., .2., .3.);
SR_3 = ((M ⇒ O) ⇒ I); SR_3 = ((.1. ⇒ .2.) ⇒ .3.)

with the trichotomic semiotic inclusion order

(3.a 2.b 1.c) with a, b, c ∈ {.1., .2., .3.} und a ≤ b ≤ c

has the following 10 triadic-trichotomic sign classes and their dual reality thematics:

1	(3.1	2.1	1.1)	×	(1.1	1.2	1.3)
2	(3.1	2.1	1.2)	×	(2.1	1.2	1.3)
3	(3.1	2.1	1.3)	×	(3.1	1.2	1.3)
4	(3.1	2.2	1.2)	×	(2.1	2.2	1.3)
5	(3.1	2.2	1.3)	×	(3.1	2.2	1.3)
6	(3.1	2.3	1.3)	×	(3.1	3.2	1.3)
7	(3.2	2.2	1.2)	×	(2.1	2.2	2.3)
8	(3.2	2.2	1.3)	×	(3.1	2.2	2.3)
9	(3.2	2.3	1.3)	×	(3.1	3.2	2.3)
10	(3.3	2.3	1.3)	×	(3.1	3.2	3.3)

The 10 representation systems can be ordered into the following system of **3 Trichotomic Triads** (Walther 1981, 1982):

1	(3.1	2.1	1.1)	×	(1.1	1.2	1.3)
2	(3.1	2.1	1.2)	×	(2.1	1.2	1.3)
3	(3.1	2.1	1.3)	×	(3.1	1.2	1.3)
4	(3.1	2.2	1.2)	×	(2.1	2.2	1.3)
7	(3.2	2.2	1.2)	×	(2.1	2.2	2.3)
8	(3.2	2.2	1.3)	×	(3.1	2.2	2.3)
6	(3.1	2.3	1.3)	×	(3.1	3.2	1.3)
9	(3.2	2.3	1.3)	×	(3.1	3.2	2.3)
10	(3.3	2.3	1.3)	×	(3.1	3.2	3.3)

Here, the dual-invariant sign class (3.1 2.2 1.3) × (3.1 2.2 1.3), the determinant of the triadic-trichotomic matrix, determines the system of the Trichotomic Triads. In the 2 systems of the 35 tetradic sign classes, the dual-invariant sign class (3.0 2.1 1.2 0.3) × (3.0 2.1 1.2 0.3), the determinant of the tetradic-tetratomic matrix, determines the 2 systems of the Tetratomic Tetrads. While (3.1 2.2 1.3) has the following three types of thematizations and thus structural realities:

(3.1 2.2 1.3) × (3.1 2.1 1.3) → { (3.1, 2.1)-them. (1.3)
(3.1, 1.3)-them. (2.2)
(2.2, 1.3)-them. (3.1),

the sign class (3.0 2.1 1.2 0.3) has 10 types of thematizations and structural realities (thematized realities are underlined):

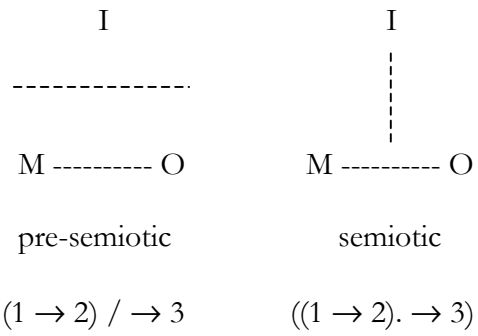
$$(3.0\ 2.1\ 1.2\ 0.3) \times (\underline{3.0}\ \underline{2.1}\ \underline{1.2}\ 0.3) \rightarrow$$

$(\underline{3.0}\ \underline{2.1}\ \underline{1.2}\ 0.3)$
$(\underline{3.0}\ \underline{2.1}\ 1.2\ 0.3)$
$(\underline{3.0}\ 2.1\ 1.2\ 0.3)$
$(3.0\ \underline{2.1}\ \underline{1.2}\ \underline{0.3})$
$(3.0\ 2.1\ \underline{1.2}\ \underline{0.3})$
$(3.0\ 2.1\ 1.2\ \underline{0.3})$
$(\underline{3.0}\ 2.1\ 1.2\ \underline{0.3})$
$(3.0\ \underline{2.1}\ \underline{1.2}\ 0.3)$
$(3.0\ \underline{2.1}\ 1.2\ 0.3)$
$(3.0\ 2.1\ \underline{1.2}\ 0.3)$

Thus, from their structural realities and from their possibilities to be ordered into a system of n-atomic n-ads, SR_3 is **not** a part of SR_4, since SR_4 has quite different n-adic n-atomic and thematization structures than SR_3.

3. Ditterich (1990, pp. 29, 81) has defined the dyadic sign relation of de Saussure, which he calls „pre-semiotic", by aid of the semiotic matrix as a sub-relation of the triadic-trichotomic Peircean sign relation SR_3:

	.1	.2	.3
3.	3.1	3.2	3.3
2.	2.1	2.2	2.3
1.	1.1	1.2	1.3



If we write the dyadic sign relation as SR_2, then we have according to Ditterich:

$SR_2 \subset SR_3$,

However, it is not clear, if this inclusion holds beyond the pure quantitative point of view. In the triadic sign model, the third category, the interpretant or the thirdness, alone guarantees that the triadic sign is a "mediating function between World and Consiousness" (Bense 1975, p. 16; 1976, p. 91; Toth 2008b). Thus, if the interpretant relation falls off, the sign cannot mediate anymore between the dyadic rest-function and the consciousness of the interpreter. Therefore, the interpretant relation which embeds the dyadic relation ($M \Rightarrow O$) into the triadic relation ($(M \Rightarrow O) \Rightarrow I$) crosses the contexture of the denomination function ($M \Rightarrow O$) that belongs to the "world" and adds to it the designation function ($O \Rightarrow I$) that belongs to the "consciousness". Hence, already the triadic sign relation involves two logical contextures, world and consciousness, or object and subject that are bridged in the triadic

sign relation. From that it follows, that Ditterich's inclusion relation does not hold from the qualitative point of view (cf. also Toth 1991), so that we have

$SR_2 \not\subset SR_3$.

4. In Toth (2008c), I have introduced the tetradic-trichotomic pre-semiotic sign relation

PSR = (0., .1., .2., .3.); $SR_{4,3}$ (3.a 2.b 1.c 0.d)

with the corresponding trichotomic inclusion order

(a ≤ b ≤ c),

whose corresponding semiotic structure is thus 4-adic, but 3-ary, since in Z^r_k, the categorial number k ≠ 0 (Bense 1975, p. 65), and therefore the pre-semiotic matrix is "defective" from the viewpoint of a quadratic matrix of Cartesian products over (.0., .1., .2., .3.):

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

From this semiotic matrix, we can construct the following 15 tetradic-trichotomic sign classes and their dual reality thematics:

1 (3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)
2 (3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)
3 (3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)
4 (3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
5 (3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)
6 (3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)
7 (3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
8 (3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)
9 (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)
10 (3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
11 (3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
12 (3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)
13 (3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)
14 (3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)
15 (3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3),

whose number corresponds to the 15 trito-numbers of the polycontextural contexture T_4 (cf. Kronthaler 1986, p. 34), which underlines the fact that these 15 pre-semiotic sign classes are both quantitative and qualitative sign classes, because the integration of the zeroness into the triadic sign relation bridges the polycontextural border between the ontological space of objects and the semiotic space of signs (cf. Bense 1975, p. 65; Toth 2003).

Moreover, we notice that $SR_{4,3}$, unlike the systems SR_3 and SR_4, does not have a dual-identical sign class. On the other side, $SR_{4,3}$ displays, in the system of its dual reality thematics, semiotic structures that do neither occur in SR_3 nor in SR_4. Finally, in $SR_{4,3}$, we do not get any type of n-atomic n-ads, but the following system of **3 tetradic pentatomies** to which the 15 pre-semiotic sign classes can be ordered:

1 (3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)
2 (3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)
4 (3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
7 (3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
5 (3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)

11 (3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
3 (3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)
6 (3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)
10 (3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
9 (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)

14 (3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)
12 (3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)
13 (3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)
14 (3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)
8 (3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)

5. As it was shown in Toth (2008c, d),

$SR_{4,3} \not\subset SR_4$,

since the category of zeroness appears only as tetradic, not as trichotomic semiotic value. Moreover, since zeroness (0.) or qualitiy (Q) localizes SR_3 in the ontological space (Bense 1975, p. 65), we also have

$SR_3 \not\subset SR_{4,3}$,

so that, by transitivity,

$SR_3 \not\subset SR_{4,3} \not\subset SR_4$,

and since we found above that

$SR_2 \not\subset SR_3$,

we finally obtain

$SR_2 \not\subset SR_3 \not\subset SR_{4,3} \not\subset SR_4$,

which means that the dyadic Saussurean sign relation is not a sub-relation of the triadic-trichotomic Peircean sign relation, the Peircean sign relation is not a sub-relation of the tetradic-trichotomic) pre-semiotic sign relation, and the latter is not a sub-relation of the tetradic-tetratomic sign relation, either!

However, it is true, from an exclusively quantitative standpoint, that we can visualize an "inclusion" relation between the four sign relations in the following semiotic matrix:

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3,

but in doing so, we ultimately "monocontexturalize" all higher semiotic relations down to the dyadic Saussurean "sign relation", which is not even a sign relation, but a dyadic sub-relation, namely the denomination relation of the complete triadic sign relation. Since the Saussurean sign relation corresponds exactly to the semiotic status of numbers in mono-contextural mathematics, the following two systems of monocontexturalization of the four sign relations:

(I) $SR_4 \rightarrow SR_3 \rightarrow SR_2$
(II) $SR_{4,3} \rightarrow SR_3 \rightarrow SR_2$

correspond to the reversal of fiberings from the system of Peano numbers into the system of polycontextural numbers (cf. Kronthaler 1986, pp. 93 s.). However, in semiotics, we have two different levels of semiotic monocontexturalization: In (I), the monocontexturalization goes strictly over the abolishment of categories, in $SR_3 \rightarrow SR_2$, the abolisment of the category of thirdness breaks down the "bridge" between world and consciousness or object and subject and turns the triadic sign relation into an "unsaturated" or "partial" sub-sign relation (Bense 1975, p. 44). Such a "sign relation" is thus beneath the recognition of a polycontextural border between sign and object, and this "sign relation" therefore cannot mediate between them. In (II), the monocontexturalization $SR_{4,3} \rightarrow SR_3$ abolishes the quality of zeroness and thus the qualitative embedding of SR_3; with the loss of this strictly qualitative category, the sign relation cannot mediate anymore between the levels of keno-

and morphogrammatics on the one side, and semiotics on the other side, thus the polycontextural border between semiotic and ontological space (Bense 1975, p. 65) is abolished.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976

Ditterich, Joseph, Selbstreferentielle Modellierungen. Klagenfurt 1990

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1986

Kronthaler, Engelbert, Zahl – Zeichen – Begriff. In: Semiosis 65-68, 1992, pp. 282-302

Toth, Alfred, Bemerkungen zum Saussureschen Arbitraritätsgesetz und Zeichenmodell. In: Semiosis 63/64, 1991, pp. 43-62. Reprinted in: Eckhardt, Michael/Engell, Lorenz (eds.), Das Programm des Schönen. Weimar 2002, pp. 71-88

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2008 (2008a)

Toth, Alfred, The sign as a "disjunction between world and consciousness". Ch. 23 (vol. I) (2008b)

Toth, Alfred, Tetradic sign classes from relational and categorial numbers. Ch. 41 (2008c)

Toth, Alfred, Towards a reality theory of pre-semiotics. Ch. 42 (2008d)

Walther, Elisabeth, Vorläufige Bemerkungen zu Trichotomischen Triaden. In: Semiosis 21, 1981, pp. 29-39

Walther, Elisabeth, Nachtrag zu Trichotomischen Triaden. In: Semiosis 27, 1982, pp. 15-20