Prof. Dr. Alfred Toth

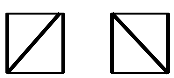
The semiotic Swastika and related shortest symmetric semiotic graphs

1. We will ask the question, which are the shortest symmetric paths through the grids that represent the semiotic matrices of $SR_{2,2}$, $SR_{3,3}$, $SR_{4,3}$, and $SR_{4,4}$ (cf. Toth 2008, c-g).

2. Ditterich (1990, pp. 29, 81) has defined the dyadic sign relation $SR_{2,2}$ of de Saussure, which he calls „pre-semiotic", by aid of the semiotic matrix as a sub-relation of the triadic-trichotomic Peircean sign relation $SR_{3,3}$:

	.1	.2	.3
3.	3.1	3.2	3.3
2.	2.1	2.2	2.3
1.	1.1	1.2	1.3

Obviously, $SR_{2,2}$ corresponds to a 1×1 grid, and the shortest symmetric paths through this grid are:



In the language of semiotic category theory (Toth 2008a), we thus have:

1. ((2.1, 1.1), (1.1, 2.2), (2.2, 1.2)) ≡ [[α°, id1], [α, α], [α°, id2]]
2. ((1.1, 2.1), (2.1, 1.2), (1.2, 2.2)) ≡ [[α, id1], [α°, α], [α, id2]]

Obviously, the "N" graph of $SR_{2,2}$ and its corresponding graph, rotated by 180°, show those connections between "signifiant" (1.1, 1.2) ≡ [id1, α] or (1.2, 1.1) ≡ [id1, α°] and "signifié" (2.1, 2.2) ≡ [id2, α] or (2.2, 2.1) ≡ [id2, α°] which Saussure (1916) compared to the recto and verso side of a sheet of paper.

3. Triadic-trichotomic semiotics is constructed by aid of the following 3×3 matrix:

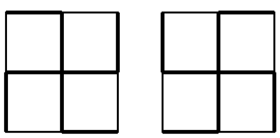
	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

from the triadic-trichotomic sign relation

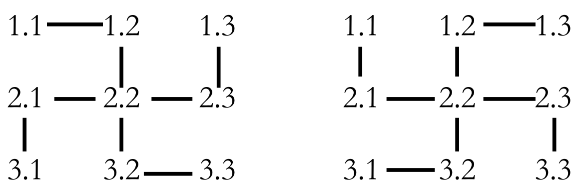
$SR_{3,3}$ = (.1., .2., .3.)
with the trichotomic semiotic inclusion order

(3.a 2.b 1.c) with a, b, c $\in$ {.1., .2., .3.} and a $\leq$ b $\leq$ c.

$SR_{3,3}$ can thus be represented by a 2×2 grid, and the shortest symmetric paths are the following two Swastikas:

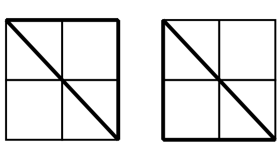


Moreover, the semiotic Swastikas are the smallest graphs to connect all sub-signs of $SR_{3,3}$, and the point of rotational symmetry is (2.2):

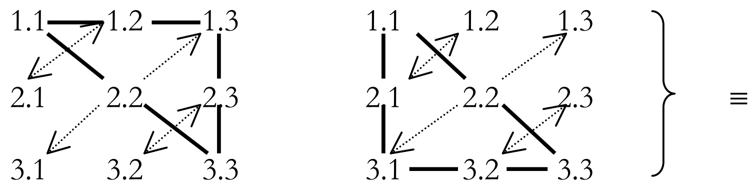


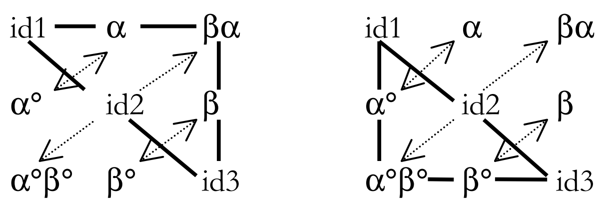
The Swastika is also the graph of the Diophantine function $x^4 - y^4 = 2xy$ (Haskell 1951; Gardner 1984, § 20.6). Swastikas with even path lengths can only be drawn into $n \times n$ grids with even n and thus n-adic m-otomic sign relations with odd n and m, and n = m, so the next higher semiotic Swastikas are the ones for $SR_{5,5}$, $SR_{7,7}$, $SR_{9,9}$, ...

If we further assume, given the law of semiotic antisymmetry (cf. Toth 1996), according to which for each sub-sign (a.b) also the sub-sign (b.a) must be in a semiotic matrix, we get the following two smallest graphs:



both of which contain the main diagonal of the Genuine Category Class (cf. Bense 1992). In the first graph, (2.1), (3.1) and (3.2) are gained by retrosemiosis or as dual morphisms from (1.2), (1.3) and (2.3); in the second graph, (1.2), (1.3) and (2.3) are gained in the same way from (2.1), (3.1) and (3.2):





4. The tetradic-trichotomic pre-semiotic sign relation (Toth 2008b)

$SR_{4,3}$ (0., .1., .2., .3.)

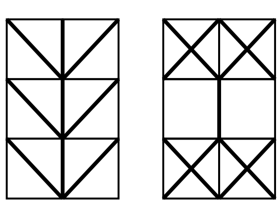
has the trichotomic inclusion order

(3.a 2.b 1.c 0.d) with a, b, c, d $\in$ {.1., .2., .3.} and a $\geq$ b $\geq$ c $\geq$ d

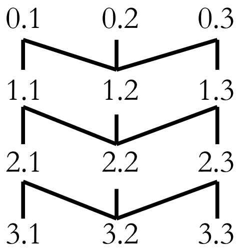
and the following non-quadratic semiotic matrix

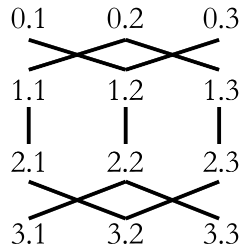
	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

Thus, $SR_{4,3}$ is represented by a 4 $\times$ 3 grid, and two of the smallest symmetric graphs are:



These two graphs connect the following sub-signs from the matrix of $SR_{4,3}$:





5. In Toth (2008a, pp. 179 ss.), I had constructed a tetradic-tetratomic semiotics on the basis of the following 4×4 matrix:

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3

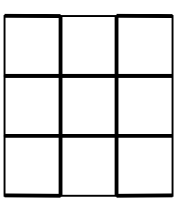
from the general sign relation

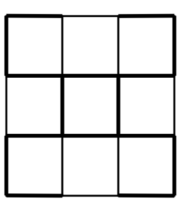
$SR_{4,4}$ = (.0., .1., .2., .3.);

with the tetratomic semiotic inclusion order

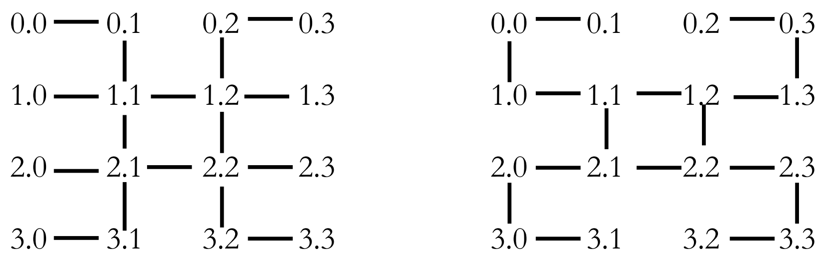
(3.a 2.b 1.c 0.d) with a, b, c, d $\in$ {.0., .1., .2., .3.} und a $\leq$ b $\leq$ c $\leq$ d.

The only two minimal symmetric graphs seem to be:





which encompass the following sub-signs of $SR_{4,4}$:



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