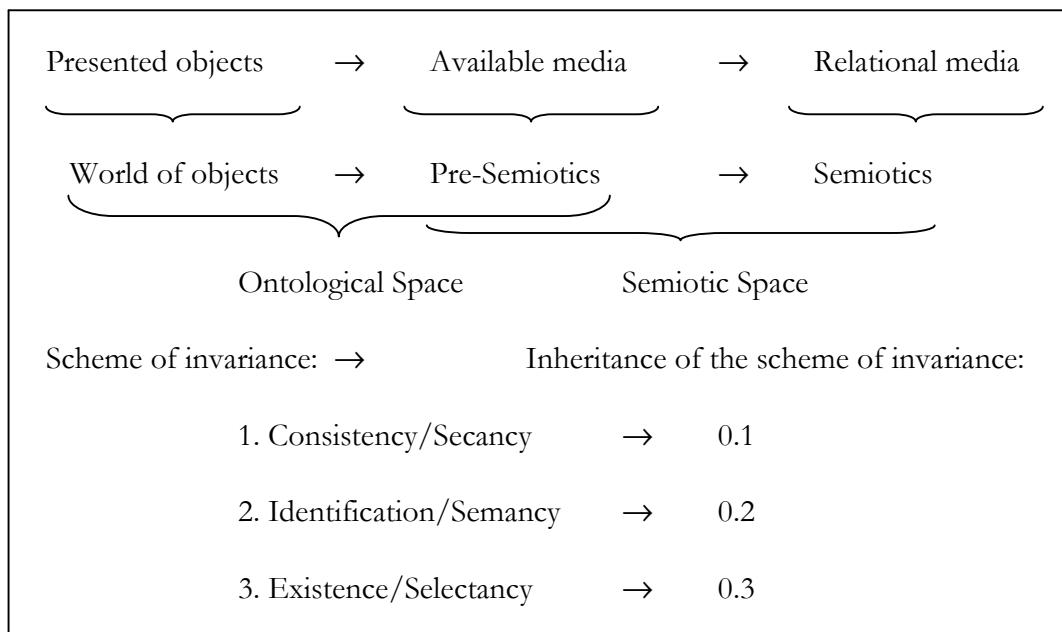


Prof. Dr. Alfred Toth

Semiotic-pre-semiotic sign connections

1. In Toth (2008b), I presented the fundamentals of a general sign grammar, to which I added in Toth (2008c) the basics of a general sign grammar for pre-semiotics. In the present study, I will show the basic sign connections between semiotic and pre-semiotic signs. I therefore investigate the intersection of the pre-semiotic signs of this fuzzy never-land between semiotic and ontological space (Bense 1975, p. 65), or between the realm of signs and the realm of objects. We are thus in the area where the meaning- and sense-full sign relations get thinner and thinner and the form- and matter-less kenogram relations get stronger and stronger, and we are mostly interested in the network places where they meet. In order to show where we are standing, I reproduce an illustration from Toth (2008d):



Therefore, in this never-land, form and matter, sign and object, eternally separated in monocontextural sciences, come together. We are in the reign of contexture-borders. We find here the last residues of sign relations before they get polycontexturalized and loose these basic dichotomies. Thus, since pre-semiotics is a part of semiotics, we could say **that all that is subject to semiotics, which is based on the fundamental dichotomy between form and matter.**

In a still other way, we are dealing in the present study with what Bense called the operation of "Mitführung" (carry-on) of the presentamen, i.e. the presented object, in the representamen, i.e. the sign (Bense 1979, p. 43). Since signs cannot change the objects to which they refer, it follows that objects must persist in some way represented in their signs.

2. The pre-semiotic sign is a tetradic relation consisting of the four part-relations

$$(0), (0 \Rightarrow 1), ((0 \Rightarrow 1) \Rightarrow 2), (0 \Rightarrow 1 \Rightarrow 2 \Rightarrow 3)$$

i.e., it is a relation over a monadic, a dyadic, a triadic, and a tetradic relation:

$$SR = (a, (a \Rightarrow b), ((a \Rightarrow b) \Rightarrow c), (a \Rightarrow b \Rightarrow c \Rightarrow d))$$

The possible sign values for a, b, and c, or 1, 2, and 3 are obtained by Cartesian multiplication of the four possible pre-semiotic prime-signs (0., 1., 2., 3.) in the rows and the three possible pre-semiotic prime-signs (.1, .2, .3) in the columns, as displayed in the pre-semiotic matrix:

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

In doing so, one gets the following sets of values for the four part-relations:

$$a = \{0.1, 0.2, 0.3\}$$

$$b = \{1.1, 1.2, 1.3\}$$

$$c = \{2.1, 2.2, 2.3\}$$

$$d = \{3.1, 3.2, 3.3\}$$

However, the pre-semiotic sign model as an extension of the Peircean sign model requires that a semiotic value be selected out of each of the four sets of values a, b, c, d and that the sign relation SR be ordered according to the following scheme of tetradicity:

$$SR = <3.w, 2.x, 1.y, 0.z> \text{ with } w, x, y, z \in \{1, 2, 3\}$$

with special respect to the pre-semiotic inclusion order

$$w \leq x \leq y \leq z$$

By aid of these two constraints, the $4^9 = 262'144$ possible sign relations are reduced to the following 15 pre-semiotic sign classes:

1	(3.1 2.1 1.1 0.1)	9	(3.1 2.2 1.3 0.3)
2	(3.1 2.1 1.1 0.2)	10	(3.1 2.3 1.3 0.3)
3	(3.1 2.1 1.1 0.3)	11	(3.2 2.2 1.2 0.2)
4	(3.1 2.1 1.2 0.2)	12	(3.2 2.2 1.2 0.3)

5	(3.1 2.1 1.2 0.3)	13	(3.2 2.2 1.3 0.3)
6	(3.1 2.1 1.3 0.3)	14	(3.2 2.3 1.3 0.3)
7	(3.1 2.2 1.2 0.2)	15	(3.3 2.3 1.3 0.3)
8	(3.1 2.2 1.2 0.3)		

Thus, the abstract sign scheme underlying these 15 pre-semiotic sign classes can be noted as follows:

$$SR = (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square)$$

By aid of the sign scheme, the 15 pre-semiotic sign classes can be displayed as follows (cf. Toth 2008c):

1	(3.1 2.1 1.1 0.1) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
2	(3.1 2.1 1.1 0.2) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
3	(3.1 2.1 1.1 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
4	(3.1 2.1 1.2 0.2) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
5	(3.1 2.1 1.2 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
6	(3.1 2.1 1.3 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
7	(3.1 2.2 1.2 0.2) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
8	(3.1 2.2 1.2 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
9	(3.1 2.2 1.3 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
10	(3.1 2.3 1.3 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
11	(3.2 2.2 1.2 0.2) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
12	(3.2 2.2 1.2 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
13	(3.2 2.2 1.3 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
14	(3.2 2.3 1.3 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)
15	(3.3 2.3 1.3 0.3) = ($\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square$)

However, because of the asymmetry between tetrads and trichromies in sign classes, and triads and tetratomies in reality thematics (cf. the pre-semiotic matrix), we need a special new reality scheme in order to show a dualized sign class. The reason is that (1.0), (2.0), and (3.0) are not defined in sign classes, and that (0.1), (0.2), (0.3) are not defined in reality thematics, due to the non-quadratic matrix of $SR_{4,3}$. In order to construct a reality scheme, we proceed in the same way as we did in sign schemes, i.e. we order the variables for sub-signs in decreasing order.

Example: $(3.1 2.1 1.1 0.1) \times (1.0 1.1 1.2 1.3)$
 $(\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \square\square\square\square \ \square\square\square\square)$

1	$(3.1 2.1 1.1 0.1) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \square\square\square\square \ \square\square\square\square) \equiv$ $(1.0 1.1 1.2 1.3)$
2	$(3.1 2.1 1.1 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \square\square\square\square \ \square\square\square\square) \equiv$ $(2.0 1.1 1.2 1.3)$
3	$(3.1 2.1 1.1 0.3) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \square\square\square\square \ \square\square\square\square) \equiv$ $(3.0 1.1 1.2 1.3)$
4	$(3.1 2.1 1.2 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \square\square\square\square \ \square\square\square\square) \equiv$

- 5 (2.0 2.1 1.2 1.3)
 (3.1 2.1 1.2 0.3) \equiv (□□■ □□■ □■□ ■□□) \times (□□□■ □□■□ ■■□□ □□□) \equiv
 (3.0 2.1 1.2 1.3)

6 (3.1 2.1 1.3 0.3) \equiv (□□■ □□■ □■□ ■□□) \times (□□■■ □□□□ ■■□□ □□□) \equiv
 (3.0 3.1 1.2 1.3)

7 (3.1 2.2 1.2 0.2) \equiv (□□■ □■□ □■□ □■□) \times (□□□□ □■■■■ ■□□□ □□□) \equiv
 (2.0 2.1 2.2 1.3)

8 (3.1 2.2 1.2 0.3) \equiv (□□■ □■□ □■□ ■□□) \times (□□□■ □■■■□ ■□□□ □□□) \equiv
 (3.0 2.1 2.2 1.3)

9 (3.1 2.2 1.3 0.3) \equiv (□□■ □■□ ■□□ ■□□) \times (□□■■ □■■□□ ■□□□ □□□) \equiv
 (3.0 3.1 2.2 1.3)

10 (3.1 2.3 1.3 0.3) \equiv (□□■ ■□□ ■□□ ■□□) \times (□■■■■ □□□□ ■□□□ □□□) \equiv
 (3.0 3.1 3.2 1.3)

11 (3.2 2.2 1.2 0.2) \equiv (□■□ □■□ □■□ □■□) \times (□□□□ ■■■■■ □□□□ □□□) \equiv
 (2.0 2.1 2.2 2.3)

12 (3.2 2.2 1.2 0.3) \equiv (□■□ □■□ □■□ ■□□) \times (□□□■ ■■■■□ □□□□ □□□) \equiv
 (3.0 2.1 2.2 2.3)

13 (3.2 2.2 1.3 0.3) \equiv (□■□ □■□ ■□□ ■□□) \times (□□■■ ■■■■□ □□□□ □□□) \equiv
 (3.0 3.1 2.2 2.3)

14 (3.2 2.3 1.3 0.3) \equiv (□■□ ■□□ ■□□ ■□□) \times (□■■■■ ■□□□ □□□□ □□□) \equiv
 (3.0 3.1 3.2 2.3)

15 (3.3 2.3 1.3 0.3) \equiv (■□□ ■□□ ■□□ ■□□) \times (■■■■■ □□□□ □□□□ □□□) \equiv
 (3.0 3.1 3.2 3.3)

3. In this place, we shall have a look at some semiotic and pre-semiotic operations (Toth 2008b, c) that also apply for connections between semiotic and pre-semiotic sign relations.

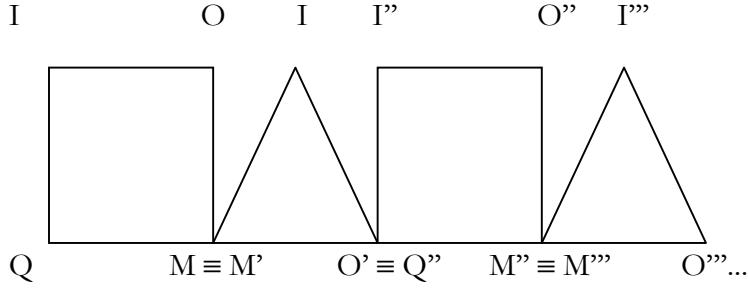
3.1. A first semiotic-pre-semiotic operator is the adjuncor (\cup). “Adjunction is a sign operation with serial, concatenating character” (Bense and Walther 1973, p. 11).

Semiotic example: $(3.1 \ 2.1 \ 1.1) \cup (3.1 \ 2.1 \ 1.2) \cup \dots$
 $(\square\square\blacksquare \ \square\square\blacksquare \ \square\square\blacksquare) \cup (\square\square\blacksquare \ \square\square\blacksquare \ \square\square\blacksquare) \cup \dots$

Pre-semiotic example: $(3.1 \ 2.1 \ 1.1 \ 0.1) \cup (3.1 \ 2.1 \ 1.2 \ 0.2) \cup \dots$
 $(\square\square\blacksquare \ \square\square\blacksquare \ \square\square\blacksquare \ \square\square\blacksquare) \cup (\square\square\blacksquare \ \square\square\blacksquare \ \square\square\square \ \square\square\square) \cup \dots$

Semiotic-pre-semiotic example: $(3.1\ 2.1\ 1.1\ 0.1) \cup (3.1\ 2.1\ 1.2) \cup \dots$
 $(\square\square\blacksquare\ \square\square\blacksquare\ \square\square\blacksquare\ \square\square\blacksquare) \cup (\square\square\blacksquare\ \square\square\blacksquare\ \square\square\blacksquare) \cup \dots$

Using the tetradic-trichotomic pre-semiotic square sign model and the triadic-trichotomic semiotic triangle model, we can display semiotic-pre-semiotic adjunction as follows:



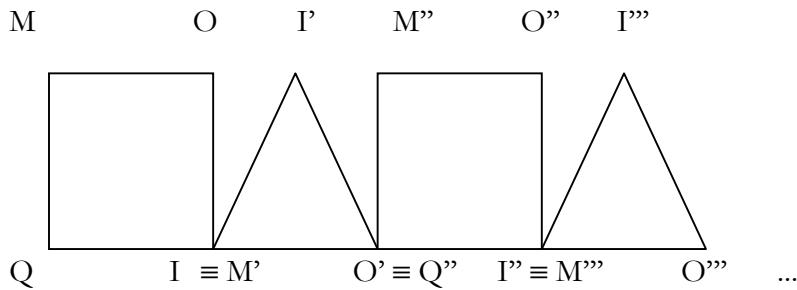
3.2. A second example is the superizer (\cap): “Superization is a sign process in the sense of the comprising wholeness formation of a set of single signs to a gestalt, a structure, or a configuration” (Bense and Walther 1973, p. 106).

Semiotic example: $(3.1 \ 2.1 \ 1.1) \cap (3.1 \ 2.1 \ 1.2) \cap \dots$
 $(\square\square\square \ \square\square\square \ \square\square\square) \cap (\square\square\square \ \square\square\square \ \square\square\square) \cap \dots$

Pre-semiotic example: $(3.1 \ 2.1 \ 1.1 \ 0.1) \cap (3.1 \ 2.1 \ 1.2 \ 0.2) \cap \dots$
 $(\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \cap (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \cap \dots$

Semiotic-pre-semiotic example: $(3.1 \ 2.1 \ 1.1) \cap (3.1 \ 2.1 \ 1.2 \ 0.2) \cap \dots$
 $(\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \cap (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \cap \dots$

Using the combination of tetradic-trichotomic pre-semiotic square and triadic-trichotomic triangle sign models, we can display semiotic-pre-semiotic superization as follows:



3.3. A third semiotic-pre-semiotic operator is the iterator (\circ): “Iteration is an operation, which reaches all subsets of the sign repertory and which can be displayed as power function” (Bense and Walther 1973, p. 46).

Semiotic example: $(3.1 \ 2.2 \ 1.3), (3.1 \ 2.2 \ 1.3)', (3.1 \ 2.2 \ 1.3)'', \dots$
 $(\square\square\square \ \square\square\square \ \square\square\square), (\square\square\square \ \square\square\square \ \square\square\square)', (\square\square\square \ \square\square\square \ \square\square\square)'', \dots$

Pre-semiotic example: $(3.1 \ 2.2 \ 1.3 \ 0.3), (3.1 \ 2.2 \ 1.3 \ 0.3)', (3.1 \ 2.2 \ 1.3 \ 0.3)'', \dots$

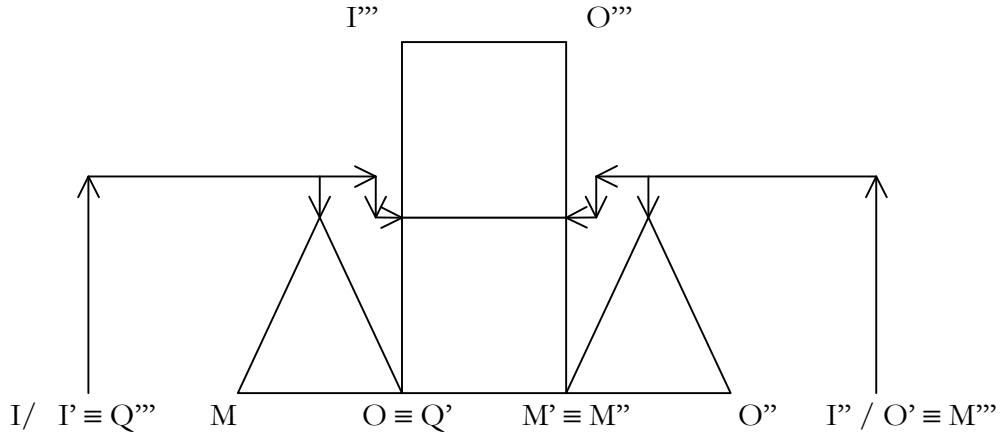
$$(\square\square\square\square\square\square\square\square), (\square\square\square\square\square\square\square\square)', \\ (\square\square\square\square\square\square\square\square)'', \dots$$

Semiotic-pre-semiotic example: (3.1 2.2 1.3 0.3), (3.1 2.2 1.3)', (3.1 2.2 1.3 0.3)'', ...

$$(\square\square\square\square\square\square\square\square), (\square\square\square\square\square\square\square\square)', (\square\square\square\square\square\square\square\square)'', \dots$$

In this example, a pre-semiotic sign class is iterated, but the result of this iteration is the semiotic eigen-real sign class (3.1 2.2 1.3). After, the eigen-real sign class is iterated and the result of this iteration is the pre-semiotic pseudo-eigen-real tetradic sign class (3.1 2.2 1.3 0.3), whose triadic part-relation (3.1 2.2 1.3) is. Such an iteration with first a categorial reduction and then a categorial restitution is only possible due to the application of the semiotic-pre-semiotic carry-on operation (Mitführung; cf. Bense 1979, p. 43).

Using the tetradic-trichotomic pre-semiotic square sign model in connection with the triadic-trichotomic semiotic triangle model, we can display pre-semiotic iteration as follows:



3.4. A fourth example for a semiotic-pre-semiotic operator is the reflexion of a triadic semiotic sign relation inside of a tetradic pre-semiotic sign relation. We show first an example for such a part-reflexion:

Symbol: $R_{\dots\dots\dots}$: Part-reflexion of all positions, marked by i

Example: $R_{\dots\dots\dots} (3.1 2.2 1.3 0.3) = * (3.1 2.3 1.2 0.3)$ (irregular)

$$R_{\dots\dots\dots} (\square\square\square\square\square\square\square\square) = * (\square\square\square\square\square\square\square\square)$$

In the following example, the semiotic triadic sign class (3.1 2.2 1.3) is reflected inside of the pre-semiotic tetradic sign class (3.1 2.2 1.3 0.3):

$$R_{\dots\dots\dots} ((\square\square\square\square\square\square\square\square) \square\square\square) = ((\square\square\square\square\square\square\square\square) \square\square\square)$$

Another form of reflexion, which we shall call mirroring, we get, if we do not start with the numerical form of the sign classes, but directly with their corresponding sign schemes. We shall mark the mirroring operator by “—“:

- 1 $(3.1 \ 2.1 \ 1.1 \ 0.1) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) — (\blacksquare\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \equiv$
 $(3.3 \ 3.0 \ 2.1 \ 2.0 \ 1.2)$
- 2 $(3.1 \ 2.1 \ 1.1 \ 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) — (\square\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \equiv$
 $(3.2 \ 3.0 \ 2.1 \ 1.2)$
- 3 $(3.1 \ 2.1 \ 1.1 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \equiv$
 $(3.1 \ 3.0 \ 2.1 \ 1.2)$
- 4 $(3.1 \ 2.1 \ 1.2 \ 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) — (\square\square\square\square \ \blacksquare\square\square \ \square\square\square \ \square\square\square) \equiv$
 $(3.2 \ 2.3 \ 2.1 \ 1.2)$
- 5 $(3.1 \ 2.1 \ 1.2 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \blacksquare\square\square \ \square\square\square \ \square\square\square) \equiv$
 $(3.1 \ 2.3 \ 2.1 \ 1.2)$
- 6 $(3.1 \ 2.1 \ 1.3 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$
 $(3.1 \ 2.2 \ 2.1 \ 1.2)$
- 7 $(3.1 \ 2.2 \ 1.2 \ 0.2) \equiv (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square) — (\square\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$
 $(3.2 \ 2.3 \ 2.0 \ 1.2)$
- 8 $(3.1 \ 2.2 \ 1.2 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$
 $(3.1 \ 2.3 \ 2.0 \ 1.2)$
- 9 $(3.1 \ 2.2 \ 1.3 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \equiv$
 $(3.1 \ 2.2 \ 2.0 \ 1.2)$
- 10 $(3.1 \ 2.3 \ 1.3 \ 0.3) \equiv (\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \equiv$
 $(3.1 \ 2.2 \ 1.3 \ 1.2)$
- 11 $(3.2 \ 2.2 \ 1.2 \ 0.2) \equiv (\blacksquare\square\square \ \square\square\square \ \square\square\square \ \square\square\square) — (\square\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$
 $(3.2 \ 2.3 \ 2.0 \ 1.1)$
- 12 $(3.2 \ 2.2 \ 1.2 \ 0.3) \equiv (\blacksquare\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$
 $(3.1 \ 2.3 \ 2.0 \ 1.1)$
- 13 $(3.2 \ 2.2 \ 1.3 \ 0.3) \equiv (\blacksquare\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \equiv$
 $(3.1 \ 2.2 \ 2.0 \ 1.1)$
- 14 $(3.2 \ 2.3 \ 1.3 \ 0.3) \equiv (\blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \equiv$
 $(3.1 \ 2.2 \ 1.3 \ 1.1)$
- 15 $(3.3 \ 2.3 \ 1.3 \ 0.3) \equiv (\blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \equiv$
 $(3.1 \ 2.2 \ 1.3 \ 1.0)$

Due to the fact already observed in connection with dualization, the pre-semiotic sign structures for mirrored sign classes is different from the original pre-semiotic sign structures. As one recognizes, mirrored sign classes are not only irregular without exception, but in mirrored pre-semiotic sign classes, the three sub-signs of the triad of zeroness is never assigned a semiotic value. Hence it seems that by mirroring pre-semiotic sign classes one reaches zeroness only as trichotomic values of the categories of the semiotic (and not pre-semiotic) triads thirdness, secondness, and firstness.

3.5. As a fifth example of operations, which apply to the connections of semiotic and pre-semiotic sign classes, we handle addition and subtraction.

3.5.1. Symbol for addition: +

Semiotic example: $(3.1 \ 2.2 \ 1.3) + (3.2 \ 2.2 \ 1.3) = (3.2 \ 2.2 \ 1.3)$
 $(\square\square\square \ \square\square\square \ \blacksquare\square\square) + (\square\square\square \ \square\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \blacksquare\square\square)$

Pre-semiotic example: $(3.1 \ 2.2 \ 1.3 \ 0.3) + (3.2 \ 2.2 \ 1.3 \ 0.3) = (3.2 \ 2.2 \ 1.3 \ 0.3)$
 $(\square\square\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square) + (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square) =$
 $(\square\square\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square)$

Thus, addition is identical with lattice-theoretic union (cf. Toth 2007, pp. 71 ss.).

3.5.2. Symbol for subtraction: -

Semiotic example: $(3.2 \ 2.3 \ 1.3) - (3.2 \ 2.2 \ 1.3) = (3.1 \ 2.2 \ 1.3)$
 $(\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) - (\square\square\square \ \square\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \blacksquare\square\square)$

Pre-semiotic example: $(3.2 \ 2.3 \ 1.3 \ 0.3) - (3.2 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.2 \ 1.3 \ 0.3)$
 $(\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) - (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square) =$
 $(\square\square\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square)$

Thus, subtraction is identical with lattice-theoretical intersection (cf. Toth 2007, pp. 71 ss.).

3.6. A pre-semiotic tetradic sign class can be split into a triadic semiotic sign class and a monadic sign relation:

Symbol: $Z_{i,j} = Z(\cap_i \cap_j)$: Splitting in two part of lengths i and j; $i + j = m$

Example: $Z_{6,2}(3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.2 \ 1.3); (0.3)$

It can also be split in all other possible part-relations, e.g.

$Z_{2,4}(3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1); (2.2 \ 1.3 \ 0.3)$
 $Z_{2,4}(\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square); (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square)$

Here, the second split part-relation $(2.2 \ 1.3 \ 0.3)$ can further be transformed by the normal-form operator: $N(2.2 \ 1.3 \ 0.3) = (3.2 \ 2.3 \ 0.3)$, cf. 3.7.

3.7. Normal-form Operator: By aid of normal-form operators (N_i), irregular sign classes can be transformed into regular ones. Since a pre-semiotic sign class is regular, if $(3.a \leq 2.b \leq 1.c \leq 0.d)$ where $a, b, c \in \{1, 2, 3\}$, normal-form operators are mostly ambiguous.

Examples: $N * (3.2 \ 2.1 \ 1.3 \ 0.3) = (3.1 \ 2.1 \ 1.3 \ 0.3), (3.2 \ 2.2 \ 1.2 \ 0.3), (3.2 \ 2.2 \ 1.3 \ 0.3)$ or $(3.2 \ 2.3 \ 1.3 \ 0.3)$;
but cf. $N * (3.3 \ 2.1 \ 1.1 \ 0.3) = N * (3.3 \ 2.1 \ 1.2 \ 0.3) = \dots = N(3.2 \ 2.2 \ 1.3 \ 0.3) = \dots$
 $= N * (3.3 \ 2.3 \ 1.2 \ 0.3) = (3.3 \ 2.3 \ 1.3 \ 0.3)$
 $N * (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square), (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square), (\square\square\square \ \blacksquare\square\square \ \square\square\square \ \blacksquare\square\square)$ or $(\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square)$;

but cf. $N^*(\blacksquare\square\square\square\square\square\square\square) = N^*(\square\blacksquare\square\square\square\square\square\square) = \dots = N(\square\square\square\square\square\square\square\square)$
 $\square\square\square\square\square\square\square\square) = \dots = N^*(\blacksquare\square\square\square\square\square\square\square) = (\blacksquare\square\square\square\square\square\square\square)$

4. In this chapter, we want to have a look at those places, where pre-semiotic and semiotic sign connections hold. Between a triadic-trichotomic semiotic triangle model and a tetradic-trichotomic pre-semiotic square model, the following the sign connection are possible:

4.1. Monadic semiotic-pre-semiotic sign connections. Here, the connections connect two vertices of the two sign models:

$$Q \equiv Q; Q \equiv M; Q \equiv O; Q \equiv I$$

4.2. Dyadic semiotic-pre-semiotic sign connections. Here, the connections connect

4.2.1 one vertex and one edge:

$$Q \equiv Q-Q; Q \equiv Q-M; Q \equiv Q-O; Q \equiv Q-I; Q \equiv M-M; Q \equiv M-O; Q \equiv M-I; Q \equiv O-O; Q \equiv O-I; Q \equiv M-I \quad \text{or}$$

4.2.2. two edges:

$$Q-Q \equiv Q-Q; Q-Q \equiv Q-M; Q-Q \equiv Q-O; Q-Q \equiv Q-I; Q-Q \equiv M-M; Q-Q \equiv O-O; Q-Q \equiv I-I; Q-Q \equiv M-O; Q-Q \equiv O-I; Q-Q \equiv M-I; Q-M \equiv Q-M, \dots, Q-O \equiv Q-O, \dots, Q-I \equiv Q-I.$$

4.3. We than get micro-(pre-)semiotic connections by replacing the symbols for the semiotic and pre-semiotic categories by their tetradic-trichotomic values, thus

$$Q = \{0.1, 0.2, 0.3\}$$

$$M = \{1.1, 1.2, 1.3\}$$

$$O = \{2.1, 2.2, 2.3\}$$

$$I = \{3.1, 3.2, 3.3\}$$

The construction of the macro-(pre-semiotic) connections follows the respective lists in Toth (2008b, pp. 51 ss.; 2008c); e.g. (the examples to the right are the chiral equivalents):

$Q \equiv Q'$		$Q' \equiv Q$
$0.1 \equiv 0.1'$	\Leftrightarrow	$[id0, id1]$
$0.2 \equiv 0.1'$	\Leftrightarrow	$[id0, \alpha^\circ]$
$0.3 \equiv 0.1'$	\Leftrightarrow	$[id0, \alpha^\circ\beta^\circ]$
$0.2 \equiv 0.2'$	\Leftrightarrow	$[id0, id2]$
$0.3 \equiv 0.2'$	\Leftrightarrow	$[id0, \beta^\circ]$
$0.3 \equiv 0.3'$	\Leftrightarrow	$[id0, id3]$

$Q \equiv Q'-M'$		$Q'-M' \equiv Q$
$0.1 \equiv 0.1'-1.1'$	\equiv	$[[\gamma], [\gamma, id1]]$
$0.1 \equiv 0.2'-1.1'$	\equiv	$[[\gamma], [\gamma, \alpha^\circ]]$

$$Q-M \equiv Q'-M'$$

$0.1-1.1 \equiv 0.1'-1.1'$	$[[\gamma, id1], [\gamma, id1]]$
$0.1-1.1 \equiv 0.2'-1.1'$	$[[\gamma, id1], [\gamma, \alpha^\circ]]$
$0.1-1.1 \equiv 0.3'-1.1'$	$[[\gamma, id1], [\gamma, \alpha^\circ\beta^\circ]]$
$0.1-1.1 \equiv 0.2'-1.2'$	$[[\gamma, id1], [\gamma, id2]]$
$0.1-1.1 \equiv 0.3'-1.2'$	$[[\gamma, id1], [\gamma, \beta^\circ]]$
$0.1-1.1 \equiv 0.3'-1.3'$	$[[\gamma, id1], [\gamma, id3]]$

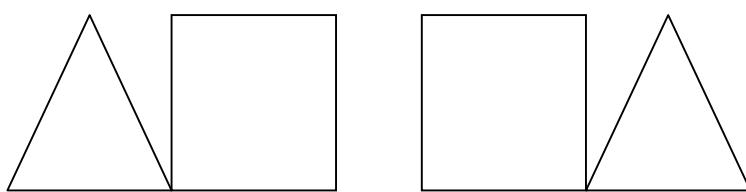
$$Q'-M' \equiv Q-M$$

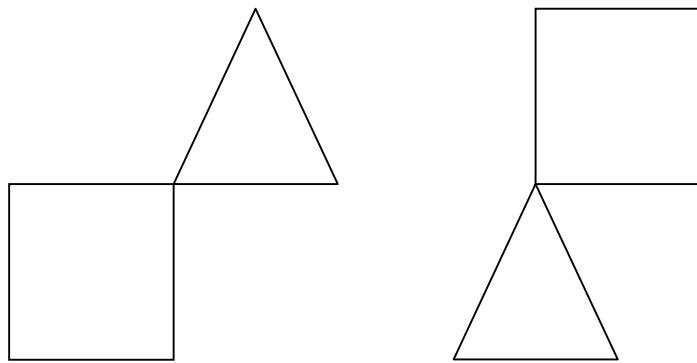
$0.1' - 1.1' \equiv 0.1 - 1.1$	$[[\gamma, \text{id}1], [\gamma, \text{id}1]]$
$0.2' - 1.1' \equiv 0.1 - 1.1$	$[[\gamma, \alpha^\circ], [\gamma, \text{id}1]]$
$0.3' - 1.1' \equiv 0.1 - 1.1$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \text{id}1]]$
$0.2' - 1.2' \equiv 0.1 - 1.1$	$[[\gamma, \text{id}2], [\gamma, \text{id}1]]$
$0.3' - 1.2' \equiv 0.1 - 1.1$	$[[\gamma, \beta^\circ], [\gamma, \text{id}1]]$
$0.3' - 1.3' \equiv 0.1 - 1.1$	$[[\gamma, \text{id}3], [\gamma, \text{id}1]]$

$0.2 \cdot 1 \cdot 1 \equiv 0.1' \cdot 1 \cdot 1'$	$[[\gamma, \alpha^\circ], [\gamma, \text{id}1]]$	$0.1' \cdot 1 \cdot 1' \equiv 0.2 \cdot 1 \cdot 1$	$[[\gamma, \text{id}1], [\gamma, \alpha^\circ]]$
$0.2 \cdot 1 \cdot 1 \equiv 0.2' \cdot 1 \cdot 1'$	$[[\gamma, \alpha^\circ], [\gamma, \alpha^\circ]]$	$0.2' \cdot 1 \cdot 1' \equiv 0.2 \cdot 1 \cdot 1$	$[[\gamma, \alpha^\circ], [\gamma, \alpha^\circ]]$
$0.2 \cdot 1 \cdot 1 \equiv 0.3' \cdot 1 \cdot 1'$	$[[\gamma, \alpha^\circ], [\gamma, \alpha^\circ \beta^\circ]]$	$0.3' \cdot 1 \cdot 1' \equiv 0.2 \cdot 1 \cdot 1$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \alpha^\circ]]$
$0.2 \cdot 1 \cdot 1 \equiv 0.2' \cdot 1 \cdot 2'$	$[[\gamma, \alpha^\circ], [\gamma, \text{id}2]]$	$0.2' \cdot 1 \cdot 2' \equiv 0.2 \cdot 1 \cdot 1$	$[[\gamma, \text{id}2], [\gamma, \alpha^\circ]]$
$0.2 \cdot 1 \cdot 1 \equiv 0.3' \cdot 1 \cdot 2'$	$[[\gamma, \alpha^\circ], [\gamma, \beta^\circ]]$	$0.3' \cdot 1 \cdot 2' \equiv 0.2 \cdot 1 \cdot 1$	$[[\gamma, \beta^\circ], [\gamma, \alpha^\circ]]$
$0.2 \cdot 1 \cdot 1 \equiv 0.3' \cdot 1 \cdot 3'$	$[[\gamma, \alpha^\circ], [\gamma, \text{id}3]]$	$0.3' \cdot 1 \cdot 3' \equiv 0.2 \cdot 1 \cdot 1$	$[[\gamma, \text{id}3], [\gamma, \alpha^\circ]]$
$0.3 \cdot 1 \cdot 1 \equiv 0.1' \cdot 1 \cdot 1'$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \text{id}1]]$	$0.1' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 1$	$[[\gamma, \text{id}1], [\gamma, \alpha^\circ \beta^\circ]]$
$0.3 \cdot 1 \cdot 1 \equiv 0.2' \cdot 1 \cdot 1'$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \alpha^\circ]]$	$0.2' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 1$	$[[\gamma, \alpha^\circ], [\gamma, \alpha^\circ \beta^\circ]]$
$0.3 \cdot 1 \cdot 1 \equiv 0.3' \cdot 1 \cdot 1'$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \alpha^\circ \beta^\circ]]$	$0.3' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 1$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \alpha^\circ \beta^\circ]]$
$0.3 \cdot 1 \cdot 1 \equiv 0.2' \cdot 1 \cdot 2'$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \text{id}2]]$	$0.2' \cdot 1 \cdot 2' \equiv 0.3 \cdot 1 \cdot 1$	$[[\gamma, \text{id}2], [\gamma, \alpha^\circ \beta^\circ]]$
$0.3 \cdot 1 \cdot 1 \equiv 0.3' \cdot 1 \cdot 2'$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \beta^\circ]]$	$0.3' \cdot 1 \cdot 2' \equiv 0.3 \cdot 1 \cdot 1$	$[[\gamma, \beta^\circ], [\gamma, \alpha^\circ \beta^\circ]]$
$0.3 \cdot 1 \cdot 1 \equiv 0.3' \cdot 1 \cdot 3'$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \text{id}3]]$	$0.3' \cdot 1 \cdot 3' \equiv 0.3 \cdot 1 \cdot 1$	$[[\gamma, \text{id}3], [\gamma, \alpha^\circ \beta^\circ]]$
$0.2 \cdot 1 \cdot 2 \equiv 0.1' \cdot 1 \cdot 1'$	$[[\gamma, \text{id}2], [\gamma, \text{id}1]]$	$0.1' \cdot 1 \cdot 1' \equiv 0.2 \cdot 1 \cdot 2$	$[[\gamma, \text{id}1], [\gamma, \text{id}2]]$
$0.2 \cdot 1 \cdot 2 \equiv 0.2' \cdot 1 \cdot 1'$	$[[\gamma, \text{id}2], [\gamma, \alpha^\circ]]$	$0.2' \cdot 1 \cdot 1' \equiv 0.2 \cdot 1 \cdot 2$	$[[\gamma, \alpha^\circ], [\gamma, \text{id}2]]$
$0.2 \cdot 1 \cdot 2 \equiv 0.3' \cdot 1 \cdot 1'$	$[[\gamma, \text{id}2], [\gamma, \alpha^\circ \beta^\circ]]$	$0.3' \cdot 1 \cdot 1' \equiv 0.2 \cdot 1 \cdot 2$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \text{id}2]]$
$0.2 \cdot 1 \cdot 2 \equiv 0.2' \cdot 1 \cdot 2'$	$[[\gamma, \text{id}2], [\gamma, \text{id}2]]$	$0.2' \cdot 1 \cdot 2' \equiv 0.2 \cdot 1 \cdot 2$	$[[\gamma, \text{id}2], [\gamma, \text{id}2]]$
$0.2 \cdot 1 \cdot 2 \equiv 0.3' \cdot 1 \cdot 2'$	$[[\gamma, \text{id}2], [\gamma, \beta^\circ]]$	$0.3' \cdot 1 \cdot 2' \equiv 0.2 \cdot 1 \cdot 2$	$[[\gamma, \beta^\circ], [\gamma, \text{id}2]]$
$0.2 \cdot 1 \cdot 2 \equiv 0.3' \cdot 1 \cdot 3'$	$[[\gamma, \text{id}2], [\gamma, \text{id}3]]$	$0.3' \cdot 1 \cdot 3' \equiv 0.2 \cdot 1 \cdot 2$	$[[\gamma, \text{id}3], [\gamma, \text{id}2]]$
$0.3 \cdot 1 \cdot 2 \equiv 0.1' \cdot 1 \cdot 1'$	$[[\gamma, \beta^\circ], [\gamma, \text{id}1]]$	$0.1' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 2$	$[[\gamma, \text{id}1], [\gamma, \beta^\circ]]$
$0.3 \cdot 1 \cdot 2 \equiv 0.2' \cdot 1 \cdot 1'$	$[[\gamma, \beta^\circ], [\gamma, \alpha^\circ]]$	$0.2' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 2$	$[[\gamma, \alpha^\circ], [\gamma, \beta^\circ]]$
$0.3 \cdot 1 \cdot 2 \equiv 0.3' \cdot 1 \cdot 1'$	$[[\gamma, \beta^\circ], [\gamma, \alpha^\circ \beta^\circ]]$	$0.3' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 2$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \beta^\circ]]$
$0.3 \cdot 1 \cdot 2 \equiv 0.2' \cdot 1 \cdot 2'$	$[[\gamma, \beta^\circ], [\gamma, \text{id}2]]$	$0.2' \cdot 1 \cdot 2' \equiv 0.3 \cdot 1 \cdot 2$	$[[\gamma, \text{id}2], [\gamma, \beta^\circ]]$
$0.3 \cdot 1 \cdot 2 \equiv 0.3' \cdot 1 \cdot 2'$	$[[\gamma, \beta^\circ], [\gamma, \beta^\circ]]$	$0.3' \cdot 1 \cdot 2' \equiv 0.3 \cdot 1 \cdot 2$	$[[\gamma, \beta^\circ], [\gamma, \beta^\circ]]$
$0.3 \cdot 1 \cdot 2 \equiv 0.3' \cdot 1 \cdot 3'$	$[[\gamma, \beta^\circ], [\gamma, \text{id}3]]$	$0.3' \cdot 1 \cdot 3' \equiv 0.3 \cdot 1 \cdot 2$	$[[\gamma, \text{id}3], [\gamma, \beta^\circ]]$
$0.3 \cdot 1 \cdot 3 \equiv 0.1' \cdot 1 \cdot 1'$	$[[\gamma, \text{id}3], [\gamma, \text{id}1]]$	$0.1' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 3$	$[[\gamma, \text{id}1], [\gamma, \text{id}3]]$
$0.3 \cdot 1 \cdot 3 \equiv 0.2' \cdot 1 \cdot 1'$	$[[\gamma, \text{id}3], [\gamma, \alpha^\circ]]$	$0.2' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 3$	$[[\gamma, \alpha^\circ], [\gamma, \text{id}3]]$
$0.3 \cdot 1 \cdot 3 \equiv 0.3' \cdot 1 \cdot 1'$	$[[\gamma, \text{id}3], [\gamma, \alpha^\circ \beta^\circ]]$	$0.3' \cdot 1 \cdot 1' \equiv 0.3 \cdot 1 \cdot 3$	$[[\gamma, \alpha^\circ \beta^\circ], [\gamma, \text{id}3]]$
$0.3 \cdot 1 \cdot 3 \equiv 0.2' \cdot 1 \cdot 2'$	$[[\gamma, \text{id}3], [\gamma, \text{id}2]]$	$0.2' \cdot 1 \cdot 2' \equiv 0.3 \cdot 1 \cdot 3$	$[[\gamma, \text{id}2], [\gamma, \text{id}3]]$
$0.3 \cdot 1 \cdot 3 \equiv 0.3' \cdot 1 \cdot 2'$	$[[\gamma, \text{id}3], [\gamma, \beta^\circ]]$	$0.3' \cdot 1 \cdot 2' \equiv 0.3 \cdot 1 \cdot 3$	$[[\gamma, \beta^\circ], [\gamma, \text{id}3]]$
$0.3 \cdot 1 \cdot 3 \equiv 0.3' \cdot 1 \cdot 3'$	$[[\gamma, \text{id}3], [\gamma, \text{id}3]]$	$0.3' \cdot 1 \cdot 3' \equiv 0.3 \cdot 1 \cdot 3$	$[[\gamma, \text{id}3], [\gamma, \text{id}3]]$

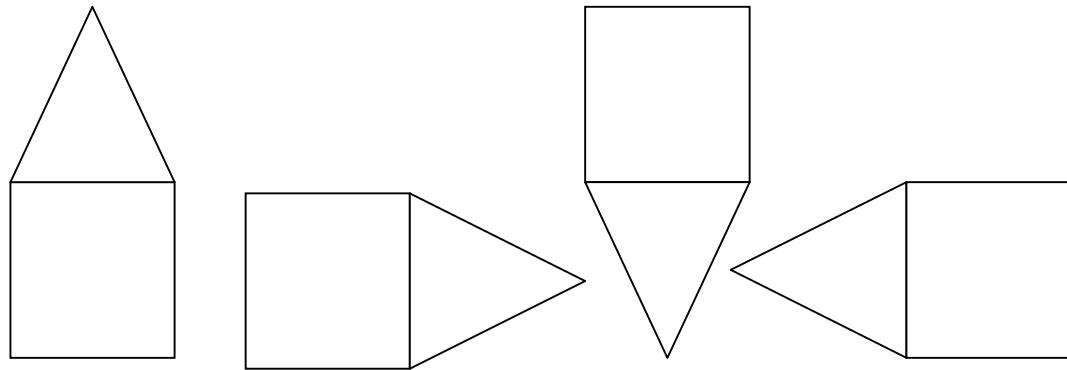
6. In order to conclude, we show here a few basic semiotic-pre-semiotic sign-configurations, which are to be compared to the semiotic sign-configurations in Toth (2008b, pp. 62 ss.) and to the pre-semiotic sign-configurations in Toth (2008c):

6.1. Type 1: Monadic sign connections, i.e. semiotic-pre-semiotic sign connections that hang together by 1 vertex:

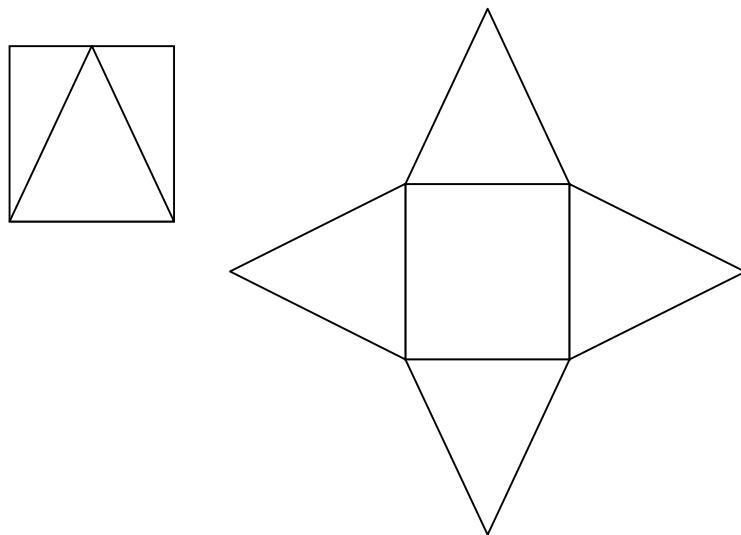


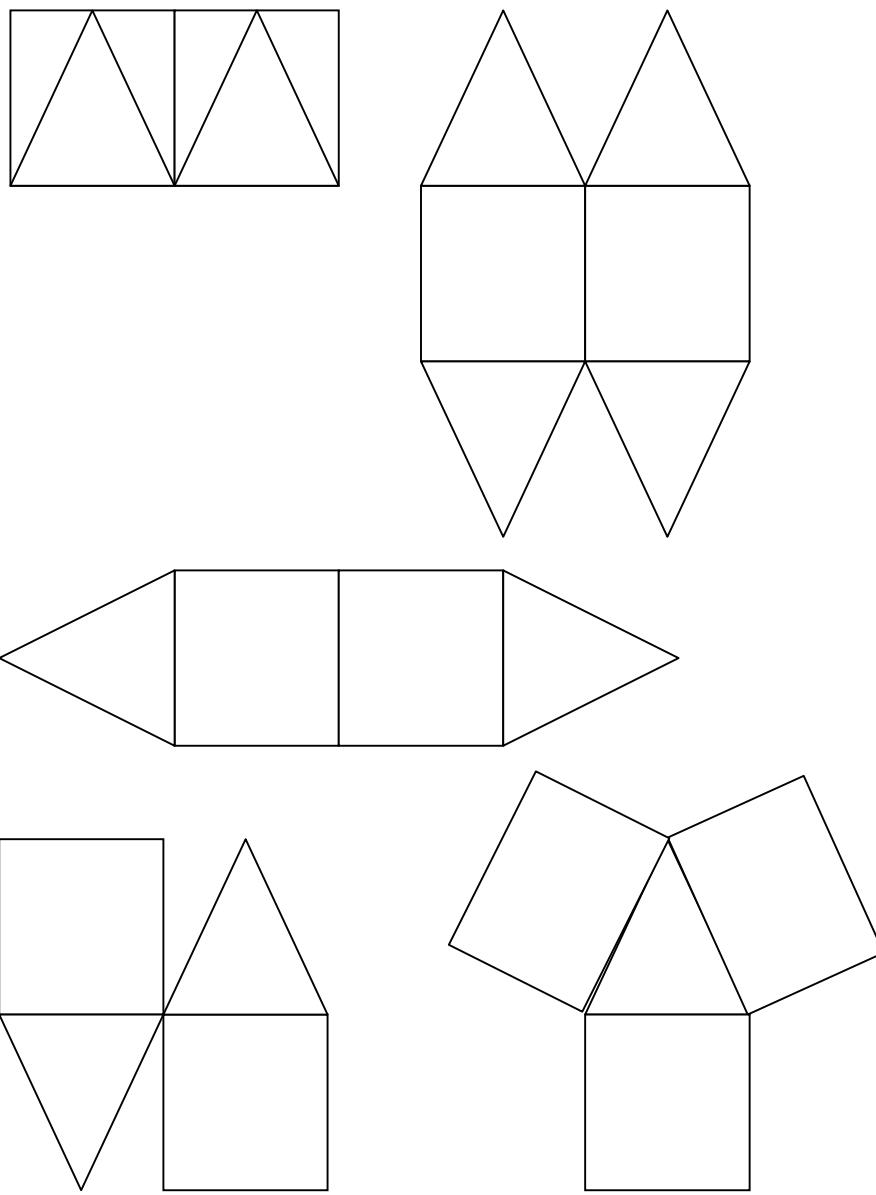


6.2. Type 2: Dyadic sign connections, i.e. semiotic-pre-semiotic sign connections that hang together by 2 edges:

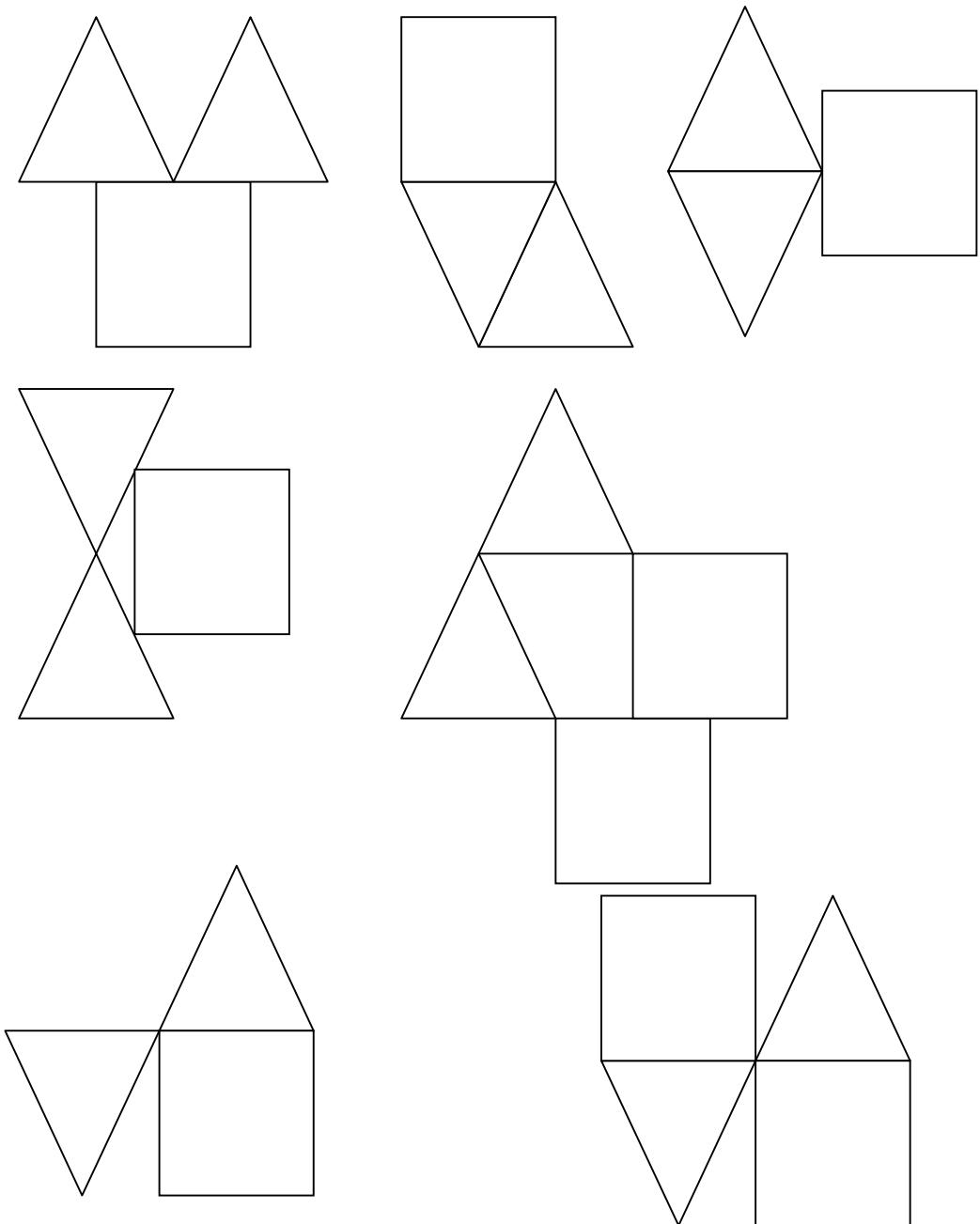


6.3. “Fancy” pre-semiotic-semiotic sign connections (selection):





A special type has to be mentioned here: These are pre-semiotic-semiotic sign connections, in which either the semiotic sign model or the pre-semiotic sign model intersects only partly or even tangentially with the edge of either a pre-semiotic or semiotic sign model. These are typical polycontextural connections; cf. Günther 1976, pp. 336 ss.; Toth 2008a, pp. 61 ss.):



Finally, I want to point out that all these connections between “triangles” and “squares” have not been put together by mathematical, but by pure semiotic and pre-semiotic reasoning. Thus, semiotics and pre-semiotics display an independent source of patterns and tilings.

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