

## **Prof. Dr. Alfred Toth**

### **Fundamentals for a general sign grammar of pre-semiotics**

1. The present study continues my book “Entwurf einer allgemeinen Zeichengrammatik/Outlines of a General Sign Grammar” (Toth 2008b), which is based on the previous works of Schnelle (1962), Bense (1975, pp. 78 ss.), and Stiebing (1978). As suggested in the title, at this place, we are interested in establishing a general framework of sign grammar for pre-semiotics, introduced in Toth (2008c, d, e) and other works. Especially, we shall focus on the interconnections between semiotic and ontological space (Bense 1975, p. 65) and its modeling in a semiotic-pre-semiotic sign grammar.

2. The pre-semiotic sign is a tetradic relation consisting of the four part-relations

$$(0), (0 \Rightarrow 1), ((0 \Rightarrow 1) \Rightarrow 2), (0 \Rightarrow 1 \Rightarrow 2 \Rightarrow 3)$$

i.e., it is a relation over a monadic, a dyadic, a triadic, and a tetradic relation; generally:

$$SR = (a, (a \Rightarrow b), ((a \Rightarrow b) \Rightarrow c), (a \Rightarrow b \Rightarrow c \Rightarrow d))$$

The possible sign values for a, b, and c, or 1, 2, and 3 are obtained by Cartesian multiplication of the four possible pre-semiotic prime-signs (0., 1., 2., 3.) in the rows and the three possible pre-semiotic prime-signs (.1, .2, .3) in the columns, as displayed in the pre-semiotic matrix:

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

In doing so, one gets the following sets of values for the four part-relations:

$$a = \{0.1, 0.2, 0.3\}$$

$$b = \{1.1, 1.2, 1.3\}$$

$$c = \{2.1, 2.2, 2.3\}$$

$$d = \{3.1, 3.2, 3.3\}$$

However, the pre-semiotic sign model as an extension of the Peircean sign model requires that a semiotic value be selected out of each of the four sets of values a, b, c, d and that the sign relation SR be ordered according to the following scheme of tetradicity:

$SR = \langle 3.w, 2.x, 1.y, 0.z \rangle$  with  $w, x, y, z \in \{1, 2, 3\}$

with special respect to the pre-semiotic inclusion order

$$w \leq x \leq y \leq z$$

By aid of these two constraints, the  $4^9 = 262'144$  possible sign relations are reduced to the following 15 pre-semiotic sign classes:

1	(3.1 2.1 1.1 0.1)	9	(3.1 2.2 1.3 0.3)
2	(3.1 2.1 1.1 0.2)	10	(3.1 2.3 1.3 0.3)
3	(3.1 2.1 1.1 0.3)	11	(3.2 2.2 1.2 0.2)
4	(3.1 2.1 1.2 0.2)	12	(3.2 2.2 1.2 0.3)
5	(3.1 2.1 1.2 0.3)	13	(3.2 2.2 1.3 0.3)
6	(3.1 2.1 1.3 0.3)	14	(3.2 2.3 1.3 0.3)
7	(3.1 2.2 1.2 0.2)	15	(3.3 2.3 1.3 0.3)
8	(3.1 2.2 1.2 0.3)		

Thus, the abstract sign scheme underlying these 15 pre-semiotic sign classes can be noted as follows:

$$SR = (\square\square\square \square\square\square \square\square\square \square\square\square)$$

The four empty patterns of three variables are to be ordered according to decreasing indicated sign values (3.3, 3.2, 3.1; 2.3, 2.2, 2.1; 1.3, 1.2, 1.1; 0.3, 0.2, 0.1). Thus, according to the tetradicity principle, in the first 3-variables-pattern, a sign value from the set  $c = (3.1, 3.2, 3.3)$ , in the second 3-variables-pattern, a sign value from the set  $b = (2.1, 2.2, 2.3)$ , in the third 3-variables-pattern, a sign value from the set  $c = (1.1, 1.2, 1.3)$ , and in the fourth 3-variables-pattern, a sign value from the set  $d = (0.1, 0.2, 0.3)$  has to be chosen. Note that the choice of the sign value from the set  $d$  depends on the choices for the sign values from the sets  $c$ ,  $b$ , and  $a$ ; the choice for  $c$  depends on  $b$ , and  $a$ , and the choice for  $b$  depends on  $a$ . In the abstract scheme, we therefore must and are allowed to assign four empty places by ( $\blacksquare$ ). In doing so, by aid of the sign scheme, the 15 pre-semiotic sign classes can be displayed as follows:

1	(3.1 2.1 1.1 0.1) = ( $\square\square\blacksquare \square\square\blacksquare \square\square\blacksquare \square\square\blacksquare$ )
2	(3.1 2.1 1.1 0.2) = ( $\square\square\blacksquare \square\square\blacksquare \square\square\blacksquare \square\blacksquare\square$ )
3	(3.1 2.1 1.1 0.3) = ( $\square\square\blacksquare \square\square\blacksquare \square\square\blacksquare \blacksquare\square\square$ )
4	(3.1 2.1 1.2 0.2) = ( $\square\square\blacksquare \square\square\blacksquare \square\blacksquare\square \square\blacksquare\square$ )
5	(3.1 2.1 1.2 0.3) = ( $\square\square\blacksquare \square\square\blacksquare \square\blacksquare\square \blacksquare\square\square$ )
6	(3.1 2.1 1.3 0.3) = ( $\square\square\blacksquare \square\square\blacksquare \blacksquare\square\square \blacksquare\square\square$ )
7	(3.1 2.2 1.2 0.2) = ( $\square\square\blacksquare \square\blacksquare\square \square\blacksquare\square \square\blacksquare\square$ )
8	(3.1 2.2 1.2 0.3) = ( $\square\square\blacksquare \square\blacksquare\square \square\blacksquare\square \blacksquare\square\square$ )
9	(3.1 2.2 1.3 0.3) = ( $\square\square\blacksquare \square\blacksquare\square \blacksquare\square\square \blacksquare\square\square$ )
10	(3.1 2.3 1.3 0.3) = ( $\square\square\blacksquare \blacksquare\square\square \blacksquare\square\square \blacksquare\square\square$ )
11	(3.2 2.2 1.2 0.2) = ( $\blacksquare\square\square \square\blacksquare\square \square\blacksquare\square \square\blacksquare\square$ )
12	(3.2 2.2 1.2 0.3) = ( $\blacksquare\square\square \square\blacksquare\square \square\blacksquare\square \blacksquare\square\square$ )

- 13  $(3.2 \ 2.2 \ 1.3 \ 0.3) = (\square \blacksquare \square \blacksquare \square \blacksquare \square \blacksquare)$   
 14  $(3.2 \ 2.3 \ 1.3 \ 0.3) = (\square \blacksquare \square \blacksquare \square \blacksquare \square \blacksquare)$   
 15  $(3.3 \ 2.3 \ 1.3 \ 0.3) = (\blacksquare \square \blacksquare \square \blacksquare \square \blacksquare)$

In the following, we will use sign schemes – abstract one as well as assigned ones – in order to show how the semiotic operators work.

3.1. Bense (1971, S. 34) defined the following semiotic operators

$$o := (M \Rightarrow O) \text{ and}$$

$$i := (O \Rightarrow I)$$

In addition to these two operators, a third one was introduced later: “A clear distinction between the designation function and the determination function, thus  $(M \Rightarrow O)$  and  $(O \Rightarrow I)$ , allows to introduce the relation  $(I \Rightarrow M)$  as application function (a)” (Walther 1979, pp. 72s.):

$$a := (I \Rightarrow M)$$

In pre-semiotics, however, we have to introduce the following operator, which we will call “qualification”, and abbreviate it by m:

$$m: (Q \Rightarrow M)$$

Moreover, besides  $a := (I \Rightarrow M)$ , there is a “contexturalization function” c:

$$c := (I \Rightarrow Q).$$

Unlike the semiotic functions o, i, and a, the functions m and c are bridging functions between pre-signs and signs, or between semiotic and ontological spaces.

Besides these semiotic and pre-semiotic-semiotic operators, which are usually called “functions”, there are, according to Walther (1979, pp. 116 ss.) 9 more operators which apply both to semiotics and pre-semiotics.

### 3.2. Substitutor (/)

Example:  $(3.1 \ 2.1 \ 1.1 \ 0.1) / (0.1/0.3) \equiv$   
 $(\square \blacksquare \square \blacksquare \square \blacksquare \square \blacksquare) / (0.1/0.3) = (\square \blacksquare \square \blacksquare \square \blacksquare \square \blacksquare \blacksquare)$

### 3.3. Selector (>)

Example:  $(3.1 \ 2.1 \ 1.1 \ 0.3), (1.1) > (1.2) \equiv$   
 $(\square \blacksquare \square \blacksquare \square \blacksquare \blacksquare), (1.1) > (1.2) (\square \blacksquare \square \blacksquare \square \blacksquare \blacksquare)$

Bense (1981, p. 108) still differentiated between separative (/), abstractive (>) and associative (X) selection. The first kind of selection is restricted to the medium relation, the second to the object relation, and the third to the interpretant relation of the triadic sign relation. In addition, we may introduce the “differentiating” selection operator, which works on the level of pre-semiotic quality. Note that all four operators apply only on trichotomies.

### 3.4. Coordinator ( $| \rightarrow$ )

Example:  $(2.1) | \rightarrow (1.1)$

$$(\square\square\square \square\square\square \square\square\square \blacksquare \square\square\square), | \rightarrow (2.1, 1.1) = (\square\square\square \square\square\square \square\square\square \blacksquare \square\square\square)$$

Bense (1983, p. 57) further differentiates between founding ( $| \rightarrow$ ), reflexive ( $\leftrightarrow$ ), and analogue ( $> \rightarrow$ ) coordinator. In addition, we may introduce the “availability” coordinator, which works on the qualitative pre-semiotic level and coordinates between zeroness and firstness.

### 3.5. Creator (realizator) ( $>>$ )

3.1

Example:  $\wedge > 1.2$

0.2

$$>> (0.2, 3.1) = (1.2) \equiv$$

$$>> ((\square\square\square \square\square\square \square\square\square \square\square\square \blacksquare \square\square\square), (\square\square\square \square\square\square \square\square\square \square\square\square)) = (\square\square\square \square\square\square \square\square\square \blacksquare \square\square\square),$$

which means that an interpreting consciousness (3.1) selects from the available pre-semiotic qualities (1.2) in order to create or realize a medium (1.2).

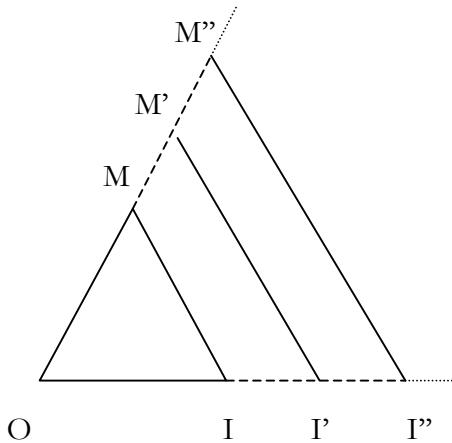
### 3.6. Adjuncter ( $\cup$ )

Example:  $(3.1 2.1 1.1 0.1) \cup (3.1 2.1 1.2 0.2) \cup \dots$

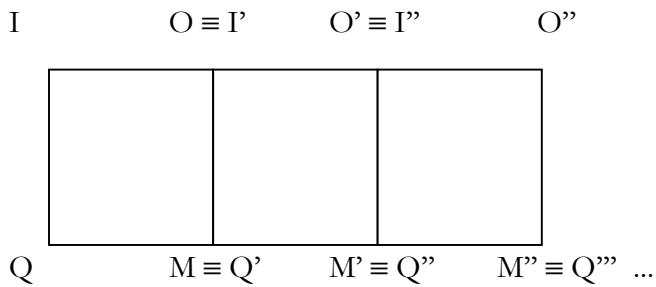
$$(\blacksquare \square\square\square \square\square\square \square\square\square \blacksquare \square\square\square) \cup (\blacksquare \square\square\square \square\square\square \square\square\square \blacksquare \square\square\square) \cup \dots$$

“Adjunction is a sign operation with serial, concatenating character” (Bense and Walther 1973, p. 11).

Display of an adjunction after Bense (1971, p. 53):



Using the tetradic-trichotomic pre-semiotic square sign model, we can display pre-semiotic adjunction as follows:

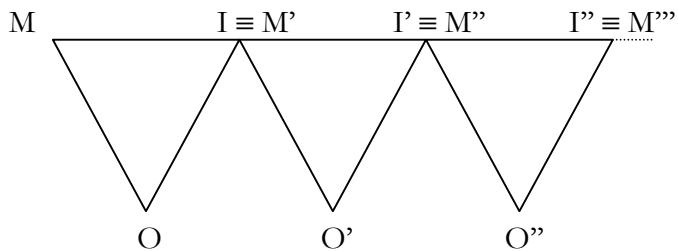


### 3.7. Superizator ( $\cap$ )

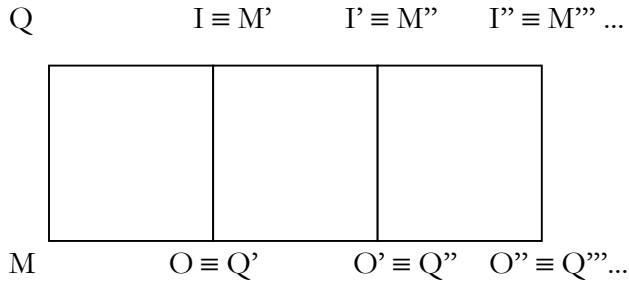
Example:  $(3.1 \ 2.1 \ 1.1 \ 0.1) \cap (3.1 \ 2.1 \ 1.2 \ 0.2) \cap \dots$   
 $(\square \square \blacksquare \ \square \square \blacksquare \ \square \square \blacksquare \ \square \square \blacksquare) \cap (\square \square \blacksquare \ \square \square \blacksquare \ \square \blacksquare \square \ \square \blacksquare \square) \cap \dots$

“Superization is a sign process in the sense of the comprising wholeness formation of a set of single signs to a gestalt, a structure, or a configuration” (Bense and Walther 1973, p. 106).

Display of a superization after Bense (1971, p. 54):



Using the tetradic-trichotomic pre-semiotic square sign model, we can display pre-semiotic superization as follows:



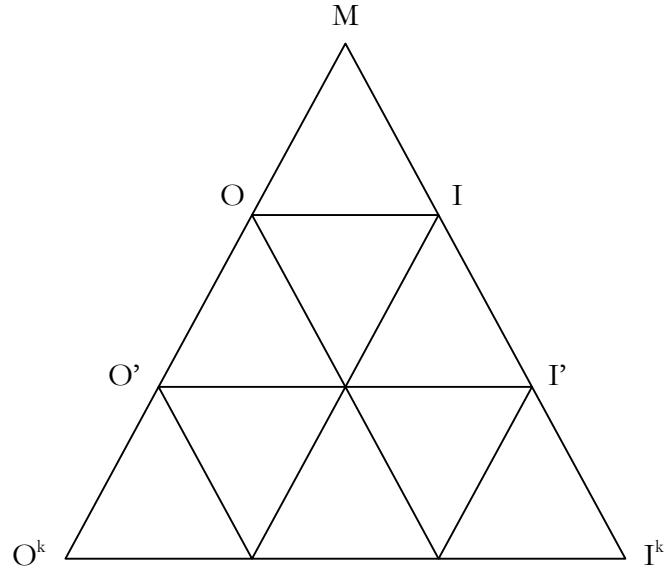
### 3.8. Iterator (')

Example:  $(2.1), (2.1)', (2.1)'' \dots$

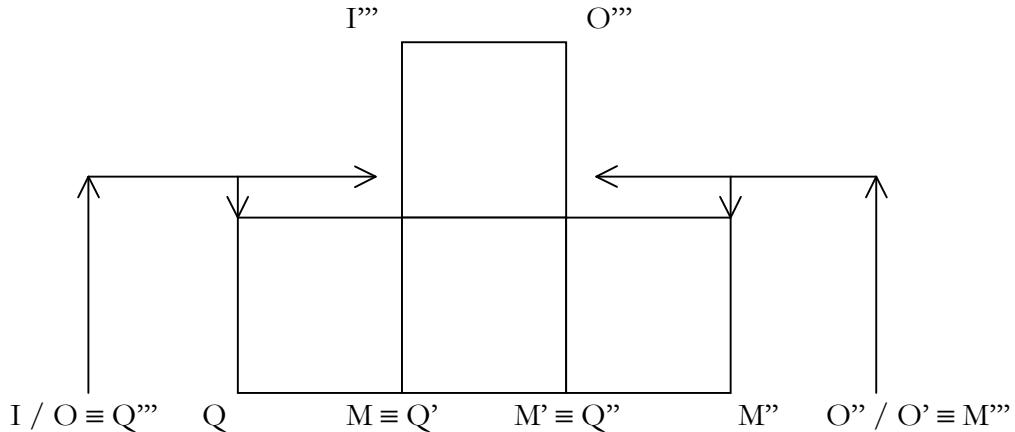
$(\square\square\square \square\square\square \square\square\square \square\square\square), (\square\square\square \square\square\square \square\square\square \square\square\square)', (\square\square\square \square\square\square \square\square\square \square\square\square)'', \dots$

“Iteration is an operation, which reaches all subsets of the sign repertory and which can be displayed as power function” (Bense and Walther 1973, p. 46).

Display of an iteration after Bense (1971, p. 55):



Using the tetradic-trichotomic pre-semiotic square sign model, we can display pre-semiotic iteration as follows:



### 3.9 Thetic introduction ( $\vdash$ )

Note that only a sign relation with categorial number  $> 0$  can be thetically introduced (cf. Bense 1975, p. 65; Toth 2008c). Thus, only the triadic part-relation of the pre-semiotic sign relation PSR = (3.a 2.b 1.c 0.d) is thetically introduced.

Example:  $\vdash (2.1)$   
 $\vdash (2.1) (\square\square\square \square\square\square \square\square\square \square\square\square) = (\square\square\square \square\square\square \square\square\square \blacksquare \square\square\square)$

### 3.10 Autoreproductor ( $\sqcap$ )

Example:  $(2.3) \sqcap (2.3)$   
 $(2.3) \sqcap (2.3) (\square\square\square \blacksquare\square\square \square\square\square \square\square\square) = (\square\square\square \blacksquare\square\square \square\square\square \square\square\square)$

Bense does not mention the dualizor, which Bense (1976, pp. 53 ss.) had introduced and which maps a sign class onto a reality thematic, amongst the semiotic operators.

### 3.11 Dualizor ( $\times$ )

Because of the asymmetry between tetrads and trichotomies in sign classes, and triads and tetratomies in reality themetics, we need a special new reality scheme in order to show a dualized sign class. The reason is that (1.0), (2.0), and (3.0) are not defined in sign classes, and that (0.1), (0.2), (0.3) are not defined in reality themetics, due to the non-quadratic matrix of SR<sub>4,3</sub>. In order to construct a reality scheme, we proceed in the same way as we did for sign schemes, i.e. we order the variables for sub-signs in decreasing order.

Example:  $(3.1 \ 2.1 \ 1.1 \ 0.1) \times (1.0 \ 1.1 \ 1.2 \ 1.3)$   
 $(\square\square\blacksquare \ \square\blacksquare\square \ \square\blacksquare\square \ \square\square\blacksquare) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\blacksquare\blacksquare \ \square\square\square)$

- 1  $(3.1 \ 2.1 \ 1.1 \ 0.1) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\blacksquare \ \square\square\square) \equiv$   
 $(1.0 \ 1.1 \ 1.2 \ 1.3)$
- 2  $(3.1 \ 2.1 \ 1.1 \ 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\blacksquare\square \ \square\square\square) \equiv$   
 $(2.0 \ 1.1 \ 1.2 \ 1.3)$
- 3  $(3.1 \ 2.1 \ 1.1 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\blacksquare\square \ \square\square\square) \equiv$   
 $(3.0 \ 1.1 \ 1.2 \ 1.3)$
- 4  $(3.1 \ 2.1 \ 1.2 \ 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\blacksquare\square \ \square\square\square) \equiv$   
 $(2.0 \ 2.1 \ 1.2 \ 1.3)$
- 5  $(3.1 \ 2.1 \ 1.2 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 2.1 \ 1.2 \ 1.3)$
- 6  $(3.1 \ 2.1 \ 1.3 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 3.1 \ 1.2 \ 1.3)$
- 7  $(3.1 \ 2.2 \ 1.2 \ 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\blacksquare\square \ \square\square\square) \equiv$   
 $(2.0 \ 2.1 \ 2.2 \ 1.3)$
- 8  $(3.1 \ 2.2 \ 1.2 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 2.1 \ 2.2 \ 1.3)$
- 9  $(3.1 \ 2.2 \ 1.3 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 3.1 \ 2.2 \ 1.3)$
- 10  $(3.1 \ 2.3 \ 1.3 \ 0.3) \equiv (\square\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \square\square\square\square \ \blacksquare\blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 3.1 \ 3.2 \ 1.3)$
- 11  $(3.2 \ 2.2 \ 1.2 \ 0.2) \equiv (\blacksquare\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \times (\square\square\square\square \ \blacksquare\blacksquare\blacksquare \ \square\square\square\square \ \square\square\square) \equiv$   
 $(2.0 \ 2.1 \ 2.2 \ 2.3)$
- 12  $(3.2 \ 2.2 \ 1.2 \ 0.3) \equiv (\blacksquare\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \blacksquare\blacksquare\blacksquare \ \square\square\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 2.1 \ 2.2 \ 2.3)$
- 13  $(3.2 \ 2.2 \ 1.3 \ 0.3) \equiv (\blacksquare\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \blacksquare\blacksquare\blacksquare \ \square\square\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 3.1 \ 2.2 \ 2.3)$
- 14  $(3.2 \ 2.3 \ 1.3 \ 0.3) \equiv (\blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \times (\square\square\square\square \ \blacksquare\blacksquare\blacksquare \ \square\square\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 3.1 \ 3.2 \ 2.3)$
- 15  $(3.3 \ 2.3 \ 1.3 \ 0.3) \equiv (\blacksquare\blacksquare\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) \times (\blacksquare\blacksquare\blacksquare \ \square\square\square\square \ \square\square\square\square \ \square\square\square) \equiv$   
 $(3.0 \ 3.1 \ 3.2 \ 3.3)$

### 3.12 Carry-on (Mitführung)

“Mitführung (carry-on) means that the ‘presentamen’ remains present gradually or partly in the ‘representamen’ (Bense 1979, p. 43). Thus, this operation of the pre-semiotic neverland between kenogrammatics and semiotics refers to the “thinning” of the world of objects on the one side and to the poly-affinity of sign classes and reality thematics on the other side (cf. Toth 2008a, pp. 166 ss.).

### 3.13. Additive Association

“Starting with the two configurations of the fundamental categorial three-digit order relations:

$$\begin{array}{ccc} 3. & 2. & 1. \\ .1 & .2 & .3 \end{array}$$

one gains by additive association the order of the sub-signs of the diagonal dual-invariant sign class-reality theamics (3.1 2.2 1.3)” (Bense 1981, p. 204). Displayed by aid of a structurel sign scheme:

$$((\square \blacksquare \blacksquare \blacksquare), (\square \blacksquare \blacksquare \blacksquare)) = (\blacksquare \blacksquare \blacksquare \blacksquare) \approx (\square \square \blacksquare \square \square \blacksquare \square \square \square)$$

In the following, we introduce some more semiotic and pre-semiotic operators, which had been used for polycontextural semiotics (cf. Toth 2003, pp. 36 ss.):

### 3.14. Abolishment

Symbol:  $L_i$ : Abolishment of position i

$$\begin{aligned} \text{Example: } L_1(3.1 2.2 1.3 0.3) &= (\emptyset 1 2.2 1.3 0.3) \\ L_1(\blacksquare \blacksquare \blacksquare \blacksquare \blacksquare) &= (\square \blacksquare \blacksquare \blacksquare \blacksquare) \end{aligned}$$

### 3.15. Assignment

Symbol:  $B_{ik}$ : Assignment of position i with value k

$$\begin{aligned} \text{Example: } B_{22}(3.\emptyset 2.2 1.3 0.3) &= (3.2 2.2 1.3 0.3) \\ B_{22}(\square \blacksquare \blacksquare \blacksquare \blacksquare) &= (\blacksquare \blacksquare \blacksquare \blacksquare \blacksquare) \end{aligned}$$

### 3.16. Nulling

Symbol:  $N_i$ : Nulling of position i

$$\begin{aligned} \text{Example: } N_5(3.1 2.2 1.3 0.3) &= (3.1 2.2 \emptyset 3 0.3) \\ N_5(\blacksquare \blacksquare \blacksquare \blacksquare) &= (\blacksquare \blacksquare \blacksquare \square \blacksquare) \end{aligned}$$

### 3.17. Maximization

Symbol:  $\text{Max}_i$ : Maximizing of position i

$$\begin{aligned} \text{Example: } \text{Max}_4(3.1 2.2 1.3 0.3) &= (3.1 2.3 1.3 0.3) \\ \text{Max}_4(\square \blacksquare \blacksquare \blacksquare \blacksquare) &= (\square \blacksquare \blacksquare \blacksquare \square \blacksquare) \end{aligned}$$

### 3.18. Minimization

Symbol:  $\text{Min}_i$ : Minimizing of position i

$$\begin{aligned} \text{Example: } \text{Min}_4(3.1 2.2 1.3 0.3) &= (3.1 2.1 1.3 0.3) \\ \text{Min}_4(\square \blacksquare \blacksquare \blacksquare \blacksquare) &= (\square \blacksquare \blacksquare \square \blacksquare \blacksquare) \end{aligned}$$

### 3.19. Assignment changing

Symbol:  $w_{ik}$ : Assignment changing  $w_i \rightarrow k$

Example:  $w_{22} (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.2 \ 2.2 \ 1.3 \ 0.3)$

$$W_{22} (\square \square \blacksquare \quad \square \blacksquare \square \quad \blacksquare \square \square \quad \blacksquare \square \square) = (\square \blacksquare \square \quad \square \blacksquare \square \quad \blacksquare \square \square \quad \blacksquare \square \square)$$

### 3.20. Transposition

Symbol:  $T_{ik}$ : Transposition of  $w_i$  and  $w_k$

Example:  $w_{23} (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.2 \ 1.2 \ 1.3 \ 0.3)$

$$W_{23} (\square \square \blacksquare \quad \square \blacksquare \square \quad \blacksquare \square \square \quad \blacksquare \square \square) = (\square \blacksquare \square \quad \square \square \square \quad \blacksquare \blacksquare \square \quad \blacksquare \square \square)$$

Permutation is an m-digit transposition:

Example:  $w_{312111}(3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.1 \ 1.1 \ 0.3)$

$$W_{312111} (\square \square \blacksquare \ \square \blacksquare \square \ \blacksquare \square \square \ \blacksquare \square \square) = (\square \square \blacksquare \ \square \square \blacksquare \ \square \square \blacksquare \ \blacksquare \square \square)$$

### 3.21. Reflexion

Symbol:  $R_{\dots\dots}$ : Part-reflexion of all positions, marked by i

Example:  $R_{\dots\dots} (3.1 \ 2.2 \ 1.3 \ 0.3) = * (3.1 \ 2.3 \ 1.2 \ 0.3)$  (irregular)

$$R_{\square\square\square\dots}(\square\square\square\square\square\square\square\square\square) = *(\square\square\square\square\square\square\square\square\square)$$

An m-digit reflexion  $R_m$  is a total reflexion:

Examples:  $R_6(3.1\ 2.2\ 1.3\ 0.3) = (3.1\ 2.2\ 1.3\ 03)$ ;  $R_6(3.1\ 2.1\ 1.1\ 0.3) = (1.1\ 1.2\ 1.3\ 0.3)$ .

$$R_6(\square\square\blacksquare \square\blacksquare\square \blacksquare\square\square \blacksquare\square\square) = (\square\square\blacksquare \square\blacksquare\square \blacksquare\square\square \blacksquare\square\square);$$

$$R_6 (\square \square \blacksquare \quad \square \square \blacksquare \quad \square \square \blacksquare \quad \blacksquare \square \square) = (\square \square \square \quad \square \square \square \quad \blacksquare \blacksquare \blacksquare \quad \blacksquare \square \square)$$

Thus, the total reflector is identical with the dualizer introduced in 3.11. Hence, only the dual-identical triadic part-relation of the pre-semiotic sign class (3.1 2.2 1.3 0.3) is mapped onto itself by  $R_m$ .

Another form of reflexion, which we shall call mirroring, we get, if we do not start with the numerical form of the sign classes, but with their corresponding sign schemes. We shall mark the mirroring operator by “—“:

- 1 (3.1 2.1 1.1 0.1) ≡ (□□■ □□■ □□■ □□■) — (■□□■ □□■□ □■□□ □□□) ≡  
(3.3 3.0 2.1 2.0 1.2)
  - 2 (3.1 2.1 1.1 0.2) ≡ (□□■ □□■ □□■ □■□) — (□■□■ □□■□ □■□□ □□□) ≡  
(3.2 3.0 2.1 1.2)
  - 3 (3.1 2.1 1.1 0.3) ≡ (□□■ □□■ □□■ ■□□) — (□□■■ □□■□ □■□□ □□□) ≡  
(3.1 3.0 2.1 1.2)
  - 4 (3.1 2.1 1.2 0.2) ≡ (□□■ □□■ □■□ □■□) — (□■□□ ■□■□ □■□□ □□□) ≡  
(3.2 2.3 2.1 1.2)
  - 5 (3.1 2.1 1.2 0.3) ≡ (□□■ □□■ □■□ ■□□) — (□□■■ ■□■□ □■□□ □□□) ≡  
(3.1 2.3 2.1 1.2)

- 6  $(3.1 \ 2.1 \ 1.3 \ 0.3) \equiv (\square\square\blacksquare \ \square\square\blacksquare \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\blacksquare\square \ \square\square\blacksquare\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.2 \ 2.1 \ 1.2)$
- 7  $(3.1 \ 2.2 \ 1.2 \ 0.2) \equiv (\square\square\blacksquare \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.2 \ 2.3 \ 2.0 \ 1.2)$
- 8  $(3.1 \ 2.2 \ 1.2 \ 0.3) \equiv (\square\square\blacksquare \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\blacksquare\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.3 \ 2.0 \ 1.2)$
- 9  $(3.1 \ 2.2 \ 1.3 \ 0.3) \equiv (\square\square\blacksquare \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\blacksquare\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.2 \ 2.0 \ 1.2)$
- 10  $(3.1 \ 2.3 \ 1.3 \ 0.3) \equiv (\square\square\blacksquare \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\blacksquare\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.2 \ 1.3 \ 1.2)$
- 11  $(3.2 \ 2.2 \ 1.2 \ 0.2) \equiv (\blacksquare\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.2 \ 2.3 \ 2.0 \ 1.1)$
- 12  $(3.2 \ 2.2 \ 1.2 \ 0.3) \equiv (\blacksquare\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\blacksquare\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.3 \ 2.0 \ 1.1)$
- 13  $(3.2 \ 2.2 \ 1.3 \ 0.3) \equiv (\blacksquare\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\blacksquare\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.2 \ 2.0 \ 1.1)$
- 14  $(3.2 \ 2.3 \ 1.3 \ 0.3) \equiv (\blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\blacksquare\square \ \square\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.2 \ 1.3 \ 1.1)$
- 15  $(3.3 \ 2.3 \ 1.3 \ 0.3) \equiv (\blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) — (\square\square\blacksquare\square \ \blacksquare\square\square \ \blacksquare\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.2 \ 1.3 \ 1.0)$

Therefore, by mirror regular pre-semiotic sign classes, we get exclusively irregular ones, while mirroring regular semiotic classes leads to exclusively regular ones; cf. Toth 2008b, p. 18). Since in the latter system (SS10), mirroring operation is identical with symplexosis (cf. Toth 2007, p. 45), it follows, that in pre-semiotics, mirroring is not identical with any group theoretic binary operation.

### 3.22. Addition

Symbol: +

Example:  $(3.1 \ 2.2 \ 1.3 \ 0.3) + (3.2 \ 2.2 \ 1.3 \ 0.3) = (3.2 \ 2.2 \ 1.3 \ 0.3)$   
 $(\square\square\blacksquare \ \square\square\blacksquare \ \blacksquare\square\square \ \blacksquare\square\square) + (\blacksquare\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = (\blacksquare\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square)$

Thus, addition is identical with lattice-theoretic union (cf. Toth 2007, pp. 71 ss.).

### 3.23. Subtraction

Symbol: -

Example:  $(3.2 \ 2.3 \ 1.3 \ 0.3) - (3.2 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.2 \ 1.3 \ 0.3)$   
 $(\blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square \ \blacksquare\square\square) - (\blacksquare\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = (\square\square\blacksquare \ \square\square\blacksquare \ \blacksquare\square\square \ \blacksquare\square\square)$

Thus, subtraction is identical with lattice-theoretical intersection (cf. Toth 2007, pp. 71 ss.).

### 3.24. Splitting

Symbol:  $Z_{mi,j} = Z(\cap_i \cap_j)$ : Splitting in two part of lengths  $i$  and  $j$ ;  $i + j = m$

Example:  $Z_{2,4}(3.1\ 2.2\ 1.3\ 0.3) = (3.1); (2.2\ 1.3\ 0.3)$

$$Z_{2,4}(\square\square\square\ \square\square\square\ \square\square\square\ \square\square\square) = (\square\square\square\ \square\square\square\ \square\square\square\ \square\square\square); (\square\square\square\ \square\square\square\ \square\square\square\ \square\square\square)$$

$Z_m$  is the splitting in merely single parts of length 1.

Example:  $Z_6(3.1\ 2.2\ 1.3\ 0.3) = 3; 1; 2; 2; 1; 3; 0; 3$

$$Z_6(\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare) = (\blacksquare_3); (\blacksquare_1); (\blacksquare_2); (\blacksquare_2); (\blacksquare_1); (\blacksquare_3); (\blacksquare_0); (\blacksquare_3)$$

Thus, total splitting is the operation which is the basis of the semiotic catastrophe, introduced by Arin (1981, pp. 328 ss.).

### 3.25. Normal-form Operator

By aid of normal-form operators ( $N_i$ ), irregular sign classes can be transformed into regular ones. Since a pre-semiotic sign class is regular, if  $(3.a \leq 2.b \leq 1.c \leq 0.d)$  where  $a, b, c \in \{1, 2, 3\}$  normal-form operators are mostly ambiguous.

Examples:  $N^*(3.2\ 2.1\ 1.3\ 0.3) = (3.1\ 2.1\ 1.3\ 0.3), (3.2\ 2.2\ 1.2\ 0.3), (3.2\ 2.2\ 1.3\ 0.3)$  or  $(3.2\ 2.3\ 1.3\ 0.3)$ ;

$$\text{but cf. } N^*(3.3\ 2.1\ 1.1\ 0.3) = N^*(3.3\ 2.1\ 1.2\ 0.3) = \dots = N(3.2\ 2.2\ 1.3\ 0.3) = \dots = N^*(3.3\ 2.3\ 1.2\ 0.3) = (3.3\ 2.3\ 1.3\ 0.3)$$

$$N^*(\square\square\square\ \square\square\square\ \square\square\square\ \square\square\square) = (\square\square\square\ \square\square\square\ \square\square\square\ \square\square\square), (\square\square\square\ \square\square\square\ \square\square\square\ \square\square\square), (\square\square\square\ \square\square\square\ \square\square\square\ \square\square\square) \text{ or } (\square\square\square\ \square\square\square\ \square\square\square\ \square\square\square);$$

$$\text{but cf. } N^*(\blacksquare\square\square\ \square\square\square\ \square\square\square\ \square\square\square) = N^*(\blacksquare\square\square\ \square\square\square\ \square\square\square\ \square\square\square) = \dots = N(\blacksquare\square\square\ \square\square\square\ \square\square\square\ \square\square\square) = \dots = N^*(\blacksquare\square\square\ \square\square\square\ \square\square\square\ \square\square\square) = (\blacksquare\square\square\ \square\square\square\ \square\square\square\ \square\square\square)$$

4. In this chapter, we want to have a look at the pre-semiotic and semiotic sign connections achieved by the operators introduced in chapter 3. First, we shall show the monadic pre-semiotic sign connections:

#### $Q \equiv Q'$

$$0.1 \equiv 0.1' \Leftrightarrow$$

$$[\text{id0}, \text{id1}]$$

$$0.2 \equiv 0.1' \Leftrightarrow$$

$$[\text{id0}, \alpha^\circ]$$

$$0.3 \equiv 0.1' \Leftrightarrow$$

$$[\text{id0}, \alpha^\circ \beta^\circ]$$

$$0.2 \equiv 0.2' \Leftrightarrow$$

$$[\text{id0}, \text{id2}]$$

$$0.3 \equiv 0.2' \Leftrightarrow$$

$$[\text{id0}, \beta^\circ]$$

$$0.3 \equiv 0.3' \Leftrightarrow$$

$$[\text{id0}, \text{id3}]$$

#### $Q' \equiv Q$

$$0.1' \equiv 0.1 \Leftrightarrow$$

$$[\text{id0}, \text{id1}]$$

$$0.1' \equiv 0.2 \Leftrightarrow$$

$$[\text{id0}, \alpha]$$

$$0.1' \equiv 0.3 \Leftrightarrow$$

$$[\text{id0}, \beta \alpha]$$

$$0.2' \equiv 0.2 \Leftrightarrow$$

$$[\text{id0}, \text{id2}]$$

$$0.2' \equiv 0.3 \Leftrightarrow$$

$$[\text{id0}, \beta]$$

$$0.3' \equiv 0.3 \Leftrightarrow$$

$$[\text{id0}, \text{id3}]$$

#### $Q \equiv M'$

$$0.1 \equiv 1.1' \Leftrightarrow$$

$$[\gamma, \text{id1}]$$

$$0.2 \equiv 1.1' \Leftrightarrow$$

$$[\gamma, \alpha^\circ]$$

$$0.3 \equiv 1.1' \Leftrightarrow$$

$$[\gamma, \alpha^\circ \beta^\circ]$$

$$0.1 \equiv 1.2' \Leftrightarrow$$

$$[\gamma, \alpha]$$

#### $M' \equiv Q$

$$1.1' \equiv 0.1 \Leftrightarrow$$

$$[\gamma^\circ, \text{id1}]$$

$$1.1' \equiv 0.2 \Leftrightarrow$$

$$[\gamma^\circ, \alpha]$$

$$1.1' \equiv 0.3 \Leftrightarrow$$

$$[\gamma^\circ, \beta \alpha]$$

$$1.2' \equiv 0.1 \Leftrightarrow$$

$$[\gamma^\circ, \alpha^\circ]$$

$0.2 \equiv 1.2'$	$\Leftrightarrow$	$[\gamma, \text{id2}]$	$1.2' \equiv 0.2$	$\Leftrightarrow$	$[\gamma^\circ, \text{id2}]$
$0.3 \equiv 1.2'$	$\Leftrightarrow$	$[\gamma, \beta^\circ]$	$1.2' \equiv 0.3$	$\Leftrightarrow$	$[\gamma^\circ, \beta]$
$0.1 \equiv 1.3'$	$\Leftrightarrow$	$[\gamma, \beta\alpha]$	$1.3' \equiv 0.1$	$\Leftrightarrow$	$[\gamma^\circ, \alpha^\circ\beta^\circ]$
$0.2 \equiv 1.3'$	$\Leftrightarrow$	$[\gamma, \beta]$	$1.3' \equiv 0.2$	$\Leftrightarrow$	$[\gamma^\circ, \beta^\circ]$
$0.3 \equiv 1.3'$	$\Leftrightarrow$	$[\gamma, \text{id3}]$	$1.3' \equiv 0.3$	$\Leftrightarrow$	$[\gamma^\circ, \text{id3}]$

<b><math>Q \equiv O'</math></b>	<b><math>O' \equiv Q</math></b>
$0.1 \equiv 2.1'$	$\Leftrightarrow$ $[\delta, \text{id1}]$
$0.2 \equiv 2.1'$	$\Leftrightarrow$ $[\delta, \alpha^\circ]$
$0.3 \equiv 2.1'$	$\Leftrightarrow$ $[\delta, \alpha^\circ\beta^\circ]$
$0.1 \equiv 2.2'$	$\Leftrightarrow$ $[\delta, \alpha]$
$0.2 \equiv 2.2'$	$\Leftrightarrow$ $[\delta, \text{id2}]$
$0.3 \equiv 2.2'$	$\Leftrightarrow$ $[\delta, \beta^\circ]$
$0.1 \equiv 2.3'$	$\Leftrightarrow$ $[\delta, \beta\alpha]$
$0.2 \equiv 2.3'$	$\Leftrightarrow$ $[\delta, \beta]$
$0.3 \equiv 2.3'$	$\Leftrightarrow$ $[\delta, \text{id3}]$

<b><math>Q \equiv I'</math></b>	<b><math>I' \equiv Q</math></b>
$0.1 \equiv 3.1'$	$\Leftrightarrow$ $[\delta\gamma, \text{id1}]$
$0.2 \equiv 3.1'$	$\Leftrightarrow$ $[\delta\gamma, \alpha^\circ]$
$0.3 \equiv 3.1'$	$\Leftrightarrow$ $[\delta\gamma, \alpha^\circ\beta^\circ]$
$0.1 \equiv 3.2'$	$\Leftrightarrow$ $[\delta\gamma, \alpha]$
$0.2 \equiv 3.2'$	$\Leftrightarrow$ $[\delta\gamma, \text{id2}]$
$0.3 \equiv 3.2'$	$\Leftrightarrow$ $[\delta\gamma, \beta^\circ]$
$0.1 \equiv 3.3'$	$\Leftrightarrow$ $[\delta\gamma, \beta\alpha]$
$0.2 \equiv 3.3'$	$\Leftrightarrow$ $[\delta\gamma, \beta]$
$0.3 \equiv 3.3'$	$\Leftrightarrow$ $[\delta\gamma, \text{id3}]$

<b><math>M \equiv M'</math></b>	<b><math>M' \equiv M</math></b>
$1.1 \equiv 1.1'$	$\Leftrightarrow$ $[\text{id1}, \text{id1}]$
$1.2 \equiv 1.1'$	$\Leftrightarrow$ $[\text{id1}, \alpha^\circ]$
$1.3 \equiv 1.1'$	$\Leftrightarrow$ $[\text{id1}, \alpha^\circ\beta^\circ]$
$1.2 \equiv 1.2'$	$\Leftrightarrow$ $[\text{id1}, \text{id2}]$
$1.3 \equiv 1.2'$	$\Leftrightarrow$ $[\text{id1}, \beta^\circ]$
$1.3 \equiv 1.3'$	$\Leftrightarrow$ $[\text{id1}, \text{id3}]$

<b><math>M \equiv O'</math></b>	<b><math>O' \equiv M</math></b>
$1.1 \equiv 2.1'$	$\Leftrightarrow$ $[\alpha, \text{id1}]$
$1.2 \equiv 2.1'$	$\Leftrightarrow$ $[\alpha, \alpha^\circ]$
$1.3 \equiv 2.1'$	$\Leftrightarrow$ $[\alpha, \alpha^\circ\beta^\circ]$
$1.1 \equiv 2.2'$	$\Leftrightarrow$ $[\alpha, \alpha]$
$1.2 \equiv 2.2'$	$\Leftrightarrow$ $[\alpha, \text{id2}]$
$1.3 \equiv 2.2'$	$\Leftrightarrow$ $[\alpha, \beta^\circ]$

$1.1 \equiv 2.3'$	$\Leftrightarrow$	$[\alpha, \beta\alpha]$	$2.3' \equiv 1.1$	$\Leftrightarrow$	$[\alpha^\circ, \alpha^\circ\beta^\circ]$
$1.2 \equiv 2.3'$	$\Leftrightarrow$	$[\alpha, \beta]$	$2.3' \equiv 1.2$	$\Leftrightarrow$	$[\alpha^\circ, \beta^\circ]$
$1.3 \equiv 2.3'$	$\Leftrightarrow$	$[\alpha, \text{id}3]$	$2.3' \equiv 1.3$	$\Leftrightarrow$	$[\alpha^\circ, \text{id}3]$

<b>M ≡ M'</b>		<b>I' ≡ M</b>
$1.1 \equiv 3.1'$	$\Leftrightarrow$	$[\beta\alpha, \text{id}1]$
$1.2 \equiv 3.1'$	$\Leftrightarrow$	$[\beta\alpha, \alpha^\circ]$
$1.3 \equiv 3.1'$	$\Leftrightarrow$	$[\beta\alpha, \alpha^\circ\beta^\circ]$
$1.1 \equiv 3.2'$	$\Leftrightarrow$	$[\beta\alpha, \alpha]$
$1.2 \equiv 3.2'$	$\Leftrightarrow$	$[\beta\alpha, \text{id}2]$
$1.3 \equiv 3.2'$	$\Leftrightarrow$	$[\beta\alpha, \beta^\circ]$
$1.1 \equiv 3.3'$	$\Leftrightarrow$	$[\beta\alpha, \beta\alpha]$
$1.2 \equiv 3.3'$	$\Leftrightarrow$	$[\beta\alpha, \beta]$
$1.3 \equiv 3.3'$	$\Leftrightarrow$	$[\beta\alpha, \text{id}3]$
		$3.1' \equiv 1.1$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \text{id}1]$
		$3.1' \equiv 1.2$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \alpha]$
		$3.1' \equiv 1.3$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \beta\alpha]$
		$3.2' \equiv 1.1$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \alpha^\circ]$
		$3.2' \equiv 1.2$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \text{id}2]$
		$3.2' \equiv 1.3$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \beta]$
		$3.3' \equiv 1.1$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \alpha^\circ\beta^\circ]$
		$3.3' \equiv 1.2$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \beta^\circ]$
		$3.3' \equiv 1.3$
		$\Leftrightarrow$
		$[\alpha^\circ\beta^\circ, \text{id}3]$

<b>O ≡ O'</b>		<b>M' ≡ O</b>
$2.1 \equiv 1.1'$	$\Leftrightarrow$	$[\alpha^\circ, \text{id}1]$
$2.2 \equiv 1.1'$	$\Leftrightarrow$	$[\alpha^\circ, \alpha^\circ]$
$2.3 \equiv 1.1'$	$\Leftrightarrow$	$[\alpha^\circ, \alpha^\circ\beta^\circ]$
$2.1 \equiv 1.2'$	$\Leftrightarrow$	$[\alpha^\circ, \alpha]$
$2.2 \equiv 1.2'$	$\Leftrightarrow$	$[\alpha^\circ, \text{id}2]$
$2.3 \equiv 1.2'$	$\Leftrightarrow$	$[\alpha^\circ, \beta^\circ]$
$2.1 \equiv 1.3'$	$\Leftrightarrow$	$[\alpha^\circ, \beta\alpha]$
$2.2 \equiv 1.3'$	$\Leftrightarrow$	$[\alpha^\circ, \beta]$
$2.3 \equiv 1.3'$	$\Leftrightarrow$	$[\alpha^\circ, \text{id}3]$
		$1.1' \equiv 2.1$
		$\Leftrightarrow$
		$[\alpha, \text{id}1]$
		$1.1' \equiv 2.2$
		$\Leftrightarrow$
		$[\alpha, \alpha]$
		$1.1' \equiv 2.3$
		$\Leftrightarrow$
		$[\alpha, \beta\alpha]$
		$1.2' \equiv 2.1$
		$\Leftrightarrow$
		$[\alpha, \alpha^\circ]$
		$1.2' \equiv 2.2$
		$\Leftrightarrow$
		$[\alpha, \text{id}2]$
		$1.2' \equiv 2.3$
		$\Leftrightarrow$
		$[\alpha, \beta]$
		$1.3' \equiv 2.1$
		$\Leftrightarrow$
		$[\alpha, \alpha^\circ\beta^\circ]$
		$1.3' \equiv 2.2$
		$\Leftrightarrow$
		$[\alpha, \beta^\circ]$
		$1.3' \equiv 2.3$
		$\Leftrightarrow$
		$[\alpha, \text{id}3]$

<b>O ≡ O'</b>		<b>O' ≡ O</b>
$2.1 \equiv 2.1'$	$\Leftrightarrow$	$[\text{id}2, \text{id}1]$
$2.2 \equiv 2.1'$	$\Leftrightarrow$	$[\text{id}2, \alpha^\circ]$
$2.3 \equiv 2.1'$	$\Leftrightarrow$	$[\text{id}2, \alpha^\circ\beta^\circ]$
$2.2 \equiv 2.2'$	$\Leftrightarrow$	$[\text{id}2, \text{id}2]$
$2.3 \equiv 2.2'$	$\Leftrightarrow$	$[\text{id}2, \beta^\circ]$
$2.3 \equiv 2.3'$	$\Leftrightarrow$	$[\text{id}2, \text{id}3]$
		$2.1' \equiv 2.1$
		$\Leftrightarrow$
		$[\text{id}2, \text{id}1]$
		$2.1' \equiv 2.2$
		$\Leftrightarrow$
		$[\text{id}2, \alpha]$
		$2.1' \equiv 2.3$
		$\Leftrightarrow$
		$[\text{id}2, \beta\alpha]$
		$2.2' \equiv 2.2$
		$\Leftrightarrow$
		$[\text{id}2, \text{id}2]$
		$2.2' \equiv 2.3$
		$\Leftrightarrow$
		$[\text{id}2, \beta]$
		$2.3' \equiv 2.3$
		$\Leftrightarrow$
		$[\text{id}2, \text{id}3]$

<b>O ≡ I'</b>		<b>I' ≡ O</b>
$2.1 \equiv 3.1'$	$\Leftrightarrow$	$[\beta, \text{id}1]$
$2.2 \equiv 3.1'$	$\Leftrightarrow$	$[\beta, \alpha^\circ]$
$2.3 \equiv 3.1'$	$\Leftrightarrow$	$[\beta, \alpha^\circ\beta^\circ]$
$2.1 \equiv 3.2'$	$\Leftrightarrow$	$[\beta, \alpha]$
$2.2 \equiv 3.2'$	$\Leftrightarrow$	$[\beta, \text{id}2]$
$2.3 \equiv 3.2'$	$\Leftrightarrow$	$[\beta, \beta^\circ]$
$2.1 \equiv 3.3'$	$\Leftrightarrow$	$[\beta, \beta\alpha]$
$2.2 \equiv 3.3'$	$\Leftrightarrow$	$[\beta, \beta]$
		$3.1' \equiv 2.1$
		$\Leftrightarrow$
		$[\beta^\circ, \text{id}1]$
		$3.1' \equiv 2.2$
		$\Leftrightarrow$
		$[\beta^\circ, \alpha]$
		$3.1' \equiv 2.3$
		$\Leftrightarrow$
		$[\beta^\circ, \beta\alpha]$
		$3.2' \equiv 2.1$
		$\Leftrightarrow$
		$[\beta^\circ, \alpha^\circ]$
		$3.2' \equiv 2.2$
		$\Leftrightarrow$
		$[\beta^\circ, \text{id}2]$
		$3.2' \equiv 2.3$
		$\Leftrightarrow$
		$[\beta^\circ, \beta]$
		$3.3' \equiv 2.1$
		$\Leftrightarrow$
		$[\beta^\circ, \alpha^\circ\beta^\circ]$
		$3.3' \equiv 2.2$
		$\Leftrightarrow$
		$[\beta^\circ, \beta^\circ]$

$$2.3 \equiv 3.3' \Leftrightarrow [\beta, \text{id3}] \quad 3.3' \equiv 2.3 \Leftrightarrow [\beta^\circ, \text{id3}]$$

**I ≡ M'**

$$\begin{array}{lll} 3.1 \equiv 1.1' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \text{id1}] \\ 3.2 \equiv 1.1' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \alpha^\circ] \\ 3.3 \equiv 1.1' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \alpha^\circ \beta^\circ] \\ 3.1 \equiv 1.2' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \alpha] \\ 3.2 \equiv 1.2' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \text{id2}] \\ 3.3 \equiv 1.2' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \beta^\circ] \\ 3.1 \equiv 1.3' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \beta \alpha] \\ 3.2 \equiv 1.3' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \beta] \\ 3.3 \equiv 1.3' & \Leftrightarrow & [\alpha^\circ \beta^\circ, \text{id3}] \end{array} \quad \begin{array}{lll} 1.1' \equiv 3.1 & \Leftrightarrow & [\beta \alpha, \text{id1}] \\ 1.1' \equiv 3.2 & \Leftrightarrow & [\beta \alpha, \alpha] \\ 1.1' \equiv 3.3 & \Leftrightarrow & [\beta \alpha, \beta \alpha] \\ 1.2' \equiv 3.1 & \Leftrightarrow & [\beta \alpha, \alpha^\circ] \\ 1.2' \equiv 3.2 & \Leftrightarrow & [\beta \alpha, \text{id2}] \\ 1.2' \equiv 3.3 & \Leftrightarrow & [\beta \alpha, \beta] \\ 1.3' \equiv 3.1 & \Leftrightarrow & [\beta \alpha, \alpha^\circ \beta^\circ] \\ 1.3' \equiv 3.2 & \Leftrightarrow & [\beta \alpha, \beta^\circ] \\ 1.3' \equiv 3.3 & \Leftrightarrow & [\beta \alpha, \text{id3}] \end{array}$$

**I ≡ O'**

$$\begin{array}{lll} 3.1 \equiv 2.1' & \Leftrightarrow & [\beta^\circ, \text{id1}] \\ 3.2 \equiv 2.1' & \Leftrightarrow & [\beta^\circ, \alpha^\circ] \\ 3.3 \equiv 2.1' & \Leftrightarrow & [\beta^\circ, \alpha^\circ \beta^\circ] \\ 3.1 \equiv 2.2' & \Leftrightarrow & [\beta^\circ, \alpha] \\ 3.2 \equiv 2.2' & \Leftrightarrow & [\beta^\circ, \text{id2}] \\ 3.3 \equiv 2.2' & \Leftrightarrow & [\beta^\circ, \beta^\circ] \\ 3.1 \equiv 2.3' & \Leftrightarrow & [\beta^\circ, \beta \alpha] \\ 3.2 \equiv 2.3' & \Leftrightarrow & [\beta^\circ, \beta] \\ 3.3 \equiv 2.3' & \Leftrightarrow & [\beta^\circ, \text{id3}] \end{array} \quad \begin{array}{lll} 2.1' \equiv 3.1 & \Leftrightarrow & [\beta, \text{id1}] \\ 2.1' \equiv 3.2 & \Leftrightarrow & [\beta, \alpha] \\ 2.1' \equiv 3.3 & \Leftrightarrow & [\beta, \beta \alpha] \\ 2.2' \equiv 3.1 & \Leftrightarrow & [\beta, \alpha^\circ] \\ 2.2' \equiv 3.2 & \Leftrightarrow & [\beta, \text{id2}] \\ 2.2' \equiv 3.3 & \Leftrightarrow & [\beta, \beta] \\ 2.3' \equiv 3.1 & \Leftrightarrow & [\beta, \alpha^\circ \beta^\circ] \\ 2.3' \equiv 3.2 & \Leftrightarrow & [\beta, \beta^\circ] \\ 2.3' \equiv 3.3 & \Leftrightarrow & [\beta, \text{id3}] \end{array}$$

**I ≡ I'**

$$\begin{array}{lll} 3.1 \equiv 3.1' & \Leftrightarrow & [\text{id3}, \text{id1}] \\ 3.2 \equiv 3.1' & \Leftrightarrow & [\text{id3}, \alpha^\circ] \\ 3.3 \equiv 3.1' & \Leftrightarrow & [\text{id3}, \alpha^\circ \beta^\circ] \\ 3.2 \equiv 3.2' & \Leftrightarrow & [\text{id3}, \text{id2}] \\ 3.3 \equiv 3.2' & \Leftrightarrow & [\text{id3}, \beta^\circ] \\ 3.3 \equiv 3.3' & \Leftrightarrow & [\text{id3}, \text{id3}] \end{array} \quad \begin{array}{lll} 3.1' \equiv 3.1 & \Leftrightarrow & [\text{id3}, \text{id1}] \\ 3.1' \equiv 3.2 & \Leftrightarrow & [\text{id3}, \alpha] \\ 3.1' \equiv 3.3 & \Leftrightarrow & [\text{id3}, \beta \alpha] \\ 3.2' \equiv 3.2 & \Leftrightarrow & [\text{id3}, \text{id2}] \\ 3.2' \equiv 3.3 & \Leftrightarrow & [\text{id3}, \beta] \\ 3.3' \equiv 3.3 & \Leftrightarrow & [\text{id3}, \text{id3}] \end{array}$$

5. Second, we shall present the dyadic pre-semiotic sign connections. For the sake of clearness, we first deal with the pre-semiotic connections separately.

**Q/M ≡ Q'/M'**

$$\begin{array}{lll} 0.1-1.1 \equiv 0.1'-1.1' & \Leftrightarrow & [[\gamma, \text{id1}], [\gamma, \text{id1}]] \\ 0.1-1.1 \equiv 0.2'-1.1' & \Leftrightarrow & [[\gamma, \text{id1}], [\gamma, \alpha^\circ]] \\ 0.1-1.1 \equiv 0.3'-1.1' & \Leftrightarrow & [[\gamma, \text{id1}], [\gamma, \alpha^\circ \beta^\circ]] \\ 0.1-1.1 \equiv 0.2'-1.2' & \Leftrightarrow & [[\gamma, \text{id1}], [\gamma, \text{id2}]] \\ 0.1-1.1 \equiv 0.3'-1.2' & \Leftrightarrow & [[\gamma, \text{id1}], [\gamma, \beta^\circ]] \\ 0.1-1.1 \equiv 0.3'-1.3' & \Leftrightarrow & [[\gamma, \text{id1}], [\gamma, \text{id3}]] \\ 0.2-1.1 \equiv 0.1'-1.1' & \Leftrightarrow & [[\gamma, \alpha^\circ], [\gamma, \text{id1}]] \end{array} \quad \begin{array}{lll} 0.1'-1.1' \equiv 0.1-1.1 & \Leftrightarrow & [[\gamma, \text{id1}], [\gamma, \text{id1}]] \\ 0.2'-1.1' \equiv 0.1-1.1 & \Leftrightarrow & [[\gamma, \alpha^\circ], [\gamma, \text{id1}]] \\ 0.3'-1.1' \equiv 0.1-1.1 & \Leftrightarrow & [[\gamma, \alpha^\circ \beta^\circ], [\gamma, \text{id1}]] \\ 0.2'-1.2' \equiv 0.1-1.1 & \Leftrightarrow & [[\gamma, \text{id2}], [\gamma, \text{id1}]] \\ 0.3'-1.2' \equiv 0.1-1.1 & \Leftrightarrow & [[\gamma, \beta^\circ], [\gamma, \text{id1}]] \\ 0.3'-1.3' \equiv 0.1-1.1 & \Leftrightarrow & [[\gamma, \text{id3}], [\gamma, \text{id1}]] \\ 0.1'-1.1' \equiv 0.2-1.1 & \Leftrightarrow & [[\gamma, \text{id1}], [\gamma, \alpha^\circ]] \end{array}$$

0.2-1.1 ≡ 0.2'-1.1'	[[γ, α°], [γ, α°]]	0.2'-1.1' ≡ 0.2-1.1	[[γ, α°], [γ, α°]]
0.2-1.1 ≡ 0.3'-1.1'	[[γ, α°], [γ, α°β°]]	0.3'-1.1' ≡ 0.2-1.1	[[γ, α°β°], [γ, α°]]
0.2-1.1 ≡ 0.2'-1.2'	[[γ, α°], [γ, id2]]	0.2'-1.2' ≡ 0.2-1.1	[[γ, id2], [γ, α°]]
0.2-1.1 ≡ 0.3'-1.2'	[[γ, α°], [γ, β°]]	0.3'-1.2' ≡ 0.2-1.1	[[γ, β°], [γ, α°]]
0.2-1.1 ≡ 0.3'-1.3'	[[γ, α°], [γ, id3]]	0.3'-1.3' ≡ 0.2-1.1	[[γ, id3], [γ, α°]]
0.3-1.1 ≡ 0.1'-1.1'	[[γ, α°β°], [γ, id1]]	0.1'-1.1' ≡ 0.3-1.1	[[γ, id1], [γ, α°β°]]
0.3-1.1 ≡ 0.2'-1.1'	[[γ, α°β°], [γ, α°]]	0.2'-1.1' ≡ 0.3-1.1	[[γ, α°], [γ, α°β°]]
0.3-1.1 ≡ 0.3'-1.1'	[[γ, α°β°], [γ, α°β°]]	0.3'-1.1' ≡ 0.3-1.1	[[γ, α°β°], [γ, α°β°]]
0.3-1.1 ≡ 0.2'-1.2'	[[γ, α°β°], [γ, id2]]	0.2'-1.2' ≡ 0.3-1.1	[[γ, id2], [γ, α°β°]]
0.3-1.1 ≡ 0.3'-1.2'	[[γ, α°β°], [γ, β°]]	0.3'-1.2' ≡ 0.3-1.1	[[γ, β°], [γ, α°β°]]
0.3-1.1 ≡ 0.3'-1.3'	[[γ, α°β°], [γ, id3]]	0.3'-1.3' ≡ 0.3-1.1	[[γ, id3], [γ, α°β°]]
0.2-1.2 ≡ 0.1'-1.1'	[[γ, id2], [γ, id1]]	0.1'-1.1' ≡ 0.2-1.2	[[γ, id1], [γ, id2]]
0.2-1.2 ≡ 0.2'-1.1'	[[γ, id2], [γ, α°]]	0.2'-1.1' ≡ 0.2-1.2	[[γ, α°], [γ, id2]]
0.2-1.2 ≡ 0.3'-1.1'	[[γ, id2], [γ, α°β°]]	0.3'-1.1' ≡ 0.2-1.2	[[γ, α°β°], [γ, id2]]
0.2-1.2 ≡ 0.2'-1.2'	[[γ, id2], [γ, id2]]	0.2'-1.2' ≡ 0.2-1.2	[[γ, id2], [γ, id2]]
0.2-1.2 ≡ 0.3'-1.2'	[[γ, id2], [γ, β°]]	0.3'-1.2' ≡ 0.2-1.2	[[γ, β°], [γ, id2]]
0.2-1.2 ≡ 0.3'-1.3'	[[γ, id2], [γ, id3]]	0.3'-1.3' ≡ 0.2-1.2	[[γ, id3], [γ, id2]]
0.3-1.2 ≡ 0.1'-1.1'	[[γ, β°], [γ, id1]]	0.1'-1.1' ≡ 0.3-1.2	[[γ, id1], [γ, β°]]
0.3-1.2 ≡ 0.2'-1.1'	[[γ, β°], [γ, α°]]	0.2'-1.1' ≡ 0.3-1.2	[[γ, α°], [γ, β°]]
0.3-1.2 ≡ 0.3'-1.1'	[[γ, β°], [γ, α°β°]]	0.3'-1.1' ≡ 0.3-1.2	[[γ, α°β°], [γ, β°]]
0.3-1.2 ≡ 0.2'-1.2'	[[γ, β°], [γ, id2]]	0.2'-1.2' ≡ 0.3-1.2	[[γ, id2], [γ, β°]]
0.3-1.2 ≡ 0.3'-1.2'	[[γ, β°], [γ, β°]]	0.3'-1.2' ≡ 0.3-1.2	[[γ, β°], [γ, β°]]
0.3-1.2 ≡ 0.3'-1.3'	[[γ, β°], [γ, id3]]	0.3'-1.3' ≡ 0.3-1.2	[[γ, id3], [γ, β°]]
0.3-1.3 ≡ 0.1'-1.1'	[[γ, id3], [γ, id1]]	0.1'-1.1' ≡ 0.3-1.3	[[γ, id1], [γ, id3]]
0.3-1.3 ≡ 0.2'-1.1'	[[γ, id3], [γ, α°]]	0.2'-1.1' ≡ 0.3-1.3	[[γ, α°], [γ, id3]]
0.3-1.3 ≡ 0.3'-1.1'	[[γ, id3], [γ, α°β°]]	0.3'-1.1' ≡ 0.3-1.3	[[γ, α°β°], [γ, id3]]
0.3-1.3 ≡ 0.2'-1.2'	[[γ, id3], [γ, id2]]	0.2'-1.2' ≡ 0.3-1.3	[[γ, id2], [γ, id3]]
0.3-1.3 ≡ 0.3'-1.2'	[[γ, id3], [γ, β°]]	0.3'-1.2' ≡ 0.3-1.3	[[γ, β°], [γ, id3]]
0.3-1.3 ≡ 0.3'-1.3'	[[γ, id3], [γ, id3]]	0.3'-1.3' ≡ 0.3-1.3	[[γ, id3], [γ, id3]]

### Q/O ≡ Q'/O'

0.1-2.1 ≡ 0.1'-2.1'	[[δ, id1], [δ, id1]]
0.1-2.1 ≡ 0.2'-2.1'	[[δ, id1], [δ, α°]]
0.1-2.1 ≡ 0.3'-2.1'	[[δ, id1], [δ, α°β°]]
0.1-2.1 ≡ 0.2'-2.2'	[[δ, id1], [δ, id2]]
0.1-2.1 ≡ 0.3'-2.2'	[[δ, id1], [δ, β°]]
0.1-2.1 ≡ 0.3'-2.3'	[[δ, id1], [δ, id3]]
0.2-2.1 ≡ 0.1'-2.1'	[[δ, α°], [δ, id1]]
0.2-2.1 ≡ 0.2'-2.1'	[[δ, α°], [δ, α°]]
0.2-2.1 ≡ 0.3'-2.1'	[[δ, α°], [δ, α°β°]]
0.2-2.1 ≡ 0.2'-2.2'	[[δ, α°], [δ, id2]]
0.2-2.1 ≡ 0.3'-2.2'	[[δ, α°], [δ, β°]]

### Q'/O' ≡ Q/O

0.1'-2.1' ≡ 0.1-2.1	[[δ, id1], [δ, id1]]
0.2'-2.1' ≡ 0.1-2.1	[[δ, α°], [δ, id1]]
0.3'-2.1' ≡ 0.1-2.1	[[δ, α°β°], [δ, id1]]
0.2'-2.2' ≡ 0.1-2.1	[[δ, id2], [δ, id1]]
0.3'-2.2' ≡ 0.1-2.1	[[δ, β°], [δ, id1]]
0.3'-2.3' ≡ 0.1-2.1	[[δ, id3], [δ, id1]]
0.1'-2.1' ≡ 0.2-2.1	[[δ, id1], [δ, α°]]
0.2'-2.1' ≡ 0.2-2.1	[[δ, α°], [δ, α°]]
0.3'-2.1' ≡ 0.2-2.1	[[δ, α°β°], [δ, α°]]
0.2'-2.2' ≡ 0.2-2.1	[[δ, id2], [δ, α°]]
0.3'-2.2' ≡ 0.2-2.1	[[δ, β°], [δ, α°]]

$0.2-2.1 \equiv 0.3'-2.3'$	$[[\delta, \alpha^\circ], [\delta, \text{id}3]]$	$0.3'-2.3' \equiv 0.2-2.1$	$[[\delta, \text{id}3], [\delta, \alpha^\circ]]$
$0.3-2.1 \equiv 0.1'-2.1'$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \text{id}1]]$	$0.1'-2.1' \equiv 0.3-2.1$	$[[\delta, \text{id}1], [\delta, \alpha^\circ\beta^\circ]]$
$0.3-2.1 \equiv 0.2'-2.1'$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \alpha^\circ]]$	$0.2'-2.1' \equiv 0.3-2.1$	$[[\delta, \alpha^\circ], [\delta, \alpha^\circ\beta^\circ]]$
$0.3-2.1 \equiv 0.3'-2.1'$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \alpha^\circ\beta^\circ]]$	$0.3'-2.1' \equiv 0.3-2.1$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \alpha^\circ\beta^\circ]]$
$0.3-2.1 \equiv 0.2'-2.2'$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \text{id}2]]$	$0.2'-2.2' \equiv 0.3-2.1$	$[[\delta, \text{id}2], [\delta, \alpha^\circ\beta^\circ]]$
$0.3-2.1 \equiv 0.3'-2.2'$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \beta^\circ]]$	$0.3'-2.2' \equiv 0.3-2.1$	$[[\delta, \beta^\circ], [\delta, \alpha^\circ\beta^\circ]]$
$0.3-2.1 \equiv 0.3'-2.3'$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \text{id}3]]$	$0.3'-2.3' \equiv 0.3-2.1$	$[[\delta, \text{id}3], [\delta, \alpha^\circ\beta^\circ]]$
$0.2-2.2 \equiv 0.1'-2.1'$	$[[\delta, \text{id}2], [\delta, \text{id}1]]$	$0.1'-2.1' \equiv 0.2-2.2$	$[[\delta, \text{id}1], [\delta, \text{id}2]]$
$0.2-2.2 \equiv 0.2'-2.1'$	$[[\delta, \text{id}2], [\delta, \alpha^\circ]]$	$0.2'-2.1' \equiv 0.2-2.2$	$[[\delta, \alpha^\circ], [\delta, \text{id}2]]$
$0.2-2.2 \equiv 0.3'-2.1'$	$[[\delta, \text{id}2], [\delta, \alpha^\circ\beta^\circ]]$	$0.3'-2.1' \equiv 0.2-2.2$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \text{id}2]]$
$0.2-2.2 \equiv 0.2'-2.2'$	$[[\delta, \text{id}2], [\delta, \text{id}2]]$	$0.2'-2.2' \equiv 0.2-2.2$	$[[\delta, \text{id}2], [\delta, \text{id}2]]$
$0.2-2.2 \equiv 0.3'-2.2'$	$[[\delta, \text{id}2], [\delta, \beta^\circ]]$	$0.3'-2.2' \equiv 0.2-2.2$	$[[\delta, \beta^\circ], [\delta, \text{id}2]]$
$0.2-2.2 \equiv 0.3'-2.3'$	$[[\delta, \text{id}2], [\delta, \text{id}3]]$	$0.3'-2.3' \equiv 0.2-2.2$	$[[\delta, \text{id}3], [\delta, \text{id}2]]$
$0.3-2.2 \equiv 0.1'-2.1'$	$[[\delta, \beta^\circ], [\delta, \text{id}1]]$	$0.1'-2.1' \equiv 0.3-2.2$	$[[\delta, \text{id}1], [\delta, \beta^\circ]]$
$0.3-2.2 \equiv 0.2'-2.1'$	$[[\delta, \beta^\circ], [\delta, \alpha^\circ]]$	$0.2'-2.1' \equiv 0.3-2.2$	$[[\delta, \alpha^\circ], [\delta, \beta^\circ]]$
$0.3-2.2 \equiv 0.3'-2.1'$	$[[\delta, \beta^\circ], [\delta, \alpha^\circ\beta^\circ]]$	$0.3'-2.1' \equiv 0.3-2.2$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \beta^\circ]]$
$0.3-2.2 \equiv 0.2'-2.2'$	$[[\delta, \beta^\circ], [\delta, \text{id}2]]$	$0.2'-2.2' \equiv 0.3-2.2$	$[[\delta, \text{id}2], [\delta, \beta^\circ]]$
$0.3-2.2 \equiv 0.3'-2.2'$	$[[\delta, \beta^\circ], [\delta, \beta^\circ]]$	$0.3'-2.2' \equiv 0.3-2.2$	$[[\delta, \beta^\circ], [\delta, \beta^\circ]]$
$0.3-2.2 \equiv 0.3'-2.3'$	$[[\delta, \beta^\circ], [\delta, \text{id}3]]$	$0.3'-2.3' \equiv 0.3-2.2$	$[[\delta, \text{id}3], [\delta, \beta^\circ]]$
$0.3-2.3 \equiv 0.1'-2.1'$	$[[\delta, \text{id}3], [\delta, \text{id}1]]$	$0.1'-2.1' \equiv 0.3-2.3$	$[[\delta, \text{id}1], [\delta, \text{id}3]]$
$0.3-2.3 \equiv 0.2'-2.1'$	$[[\delta, \text{id}3], [\delta, \alpha^\circ]]$	$0.2'-2.1' \equiv 0.3-2.3$	$[[\delta, \alpha^\circ], [\delta, \text{id}3]]$
$0.3-2.3 \equiv 0.3'-2.1'$	$[[\delta, \text{id}3], [\delta, \alpha^\circ\beta^\circ]]$	$0.3'-2.1' \equiv 0.3-2.3$	$[[\delta, \alpha^\circ\beta^\circ], [\delta, \text{id}3]]$
$0.3-2.3 \equiv 0.2'-2.2'$	$[[\delta, \text{id}3], [\delta, \text{id}2]]$	$0.2'-2.2' \equiv 0.3-2.3$	$[[\delta, \text{id}2], [\delta, \text{id}3]]$
$0.3-2.3 \equiv 0.3'-2.2'$	$[[\delta, \text{id}3], [\delta, \beta^\circ]]$	$0.3'-2.2' \equiv 0.3-2.3$	$[[\delta, \beta^\circ], [\delta, \text{id}3]]$
$0.3-2.3 \equiv 0.3'-2.3'$	$[[\delta, \text{id}3], [\delta, \text{id}3]]$	$0.3'-2.3' \equiv 0.3-2.3$	$[[\delta, \text{id}3], [\delta, \text{id}3]]$

<b>Q/I ≡ Q'/I'</b>		<b>Q'/I' ≡ Q/I</b>
0.1-3.1 ≡ 0.1'-3.1'	[[δγ, id1], [δγ, id1]]	0.1'-3.1' ≡ 0.1-3.1
0.1-3.1 ≡ 0.2'-3.1'	[[δγ, id1], [δγ, α°]]	0.2'-3.1' ≡ 0.1-3.1
0.1-3.1 ≡ 0.3'-3.1'	[[δγ, id1], [δγ, α°β°]]	0.3'-3.1' ≡ 0.1-3.1
0.1-3.1 ≡ 0.2'-3.2'	[[δγ, id1], [δγ, id2]]	0.2'-3.2' ≡ 0.1-3.1
0.1-3.1 ≡ 0.3'-3.2'	[[δγ, id1], [δγ, β°]]	0.3'-3.2' ≡ 0.1-3.1
0.1-3.1 ≡ 0.3'-3.3'	[[δγ, id1], [δγ, id3]]	0.3'-3.3' ≡ 0.1-3.1
0.2-3.1 ≡ 0.1'-3.1'	[[δγ, α°], [δγ, id1]]	0.1'-3.1' ≡ 0.2-3.1
0.2-3.1 ≡ 0.2'-3.1'	[[δγ, α°], [δγ, α°]]	0.2'-3.1' ≡ 0.2-3.1
0.2-3.1 ≡ 0.3'-3.1'	[[δγ, α°], [δγ, α°β°]]	0.3'-3.1' ≡ 0.2-3.1
0.2-3.1 ≡ 0.2'-3.2'	[[δγ, α°], [δγ, id2]]	0.2'-3.2' ≡ 0.2-3.1
0.2-3.1 ≡ 0.3'-3.2'	[[δγ, α°], [δγ, β°]]	0.3'-3.2' ≡ 0.2-3.1
0.2-3.1 ≡ 0.3'-3.3'	[[δγ, α°], [δγ, id3]]	0.3'-3.3' ≡ 0.2-3.1
0.3-3.1 ≡ 0.1'-3.1'	[[δγ, α°β°], [δγ, id1]]	0.1'-3.1' ≡ 0.3-3.1
0.3-3.1 ≡ 0.2'-3.1'	[[δγ, α°β°], [δγ, α°]]	0.2'-3.1' ≡ 0.3-3.1
0.3-3.1 ≡ 0.3'-3.1'	[[δγ, α°β°], [δγ, α°β°]]	0.3'-3.1' ≡ 0.3-3.1

0.3-3.1 ≡ 0.2'-3.2'	[[δγ, α°β°], [δγ, id2]]	0.2'-3.2' ≡ 0.3-3.1	[[δγ, id2], [δγ, α°β°]]
0.3-3.1 ≡ 0.3'-3.2'	[[δγ, α°β°], [δγ, β°]]	0.3'-3.2' ≡ 0.3-3.1	[[δγ, β°], [δγ, α°β°]]
0.3-3.1 ≡ 0.3'-3.3'	[[δγ, α°β°], [δγ, id3]]	0.3'-3.3' ≡ 0.3-3.1	[[δγ, id3], [δγ, α°β°]]
0.2-3.2 ≡ 0.1'-3.1'	[[δγ, id2], [δγ, id1]]	0.1'-3.1' ≡ 0.2-3.2	[[δγ, id1], [δγ, id2]]
0.2-3.2 ≡ 0.2'-3.1'	[[δγ, id2], [δγ, α°]]	0.2'-3.1' ≡ 0.2-3.2	[[δγ, α°], [δγ, id2]]
0.2-3.2 ≡ 0.3'-3.1'	[[δγ, id2], [δγ, α°β°]]	0.3'-3.1' ≡ 0.2-3.2	[[δγ, α°β°], [δγ, id2]]
0.2-3.2 ≡ 0.2'-3.2'	[[δγ, id2], [δγ, id2]]	0.2'-3.2' ≡ 0.2-3.2	[[δγ, id2], [δγ, id2]]
0.2-3.2 ≡ 0.3'-3.2'	[[δγ, id2], [δγ, β°]]	0.3'-3.2' ≡ 0.2-3.2	[[δγ, β°], [δγ, id2]]
0.2-3.2 ≡ 0.3'-3.3'	[[δγ, id2], [δγ, id3]]	0.3'-3.3' ≡ 0.2-3.2	[[δγ, id3], [δγ, id2]]
0.3-3.2 ≡ 0.1'-3.1'	[[δγ, β°], [δγ, id1]]	0.1'-3.1' ≡ 0.3-3.2	[[δγ, id1], [δγ, β°]]
0.3-3.2 ≡ 0.2'-3.1'	[[δγ, β°], [δγ, α°]]	0.2'-3.1' ≡ 0.3-3.2	[[δγ, α°], [δγ, β°]]
0.3-3.2 ≡ 0.3'-3.1'	[[δγ, β°], [δγ, α°β°]]	0.3'-3.1' ≡ 0.3-3.2	[[δγ, α°β°], [δγ, β°]]
0.3-3.2 ≡ 0.2'-3.2'	[[δγ, β°], [δγ, id2]]	0.2'-3.2' ≡ 0.3-3.2	[[δγ, id2], [δγ, β°]]
0.3-3.2 ≡ 0.3'-3.2'	[[δγ, β°], [δγ, β°]]	0.3'-3.2' ≡ 0.3-3.2	[[δγ, β°], [δγ, β°]]
0.3-3.2 ≡ 0.3'-3.3'	[[δγ, β°], [δγ, id3]]	0.3'-3.3' ≡ 0.3-3.2	[[δγ, id3], [δγ, β°]]
0.3-3.3 ≡ 0.1'-3.1'	[[δγ, id3], [δγ, id1]]	0.1'-3.1' ≡ 0.3-3.3	[[δγ, id1], [δγ, id3]]
0.3-3.3 ≡ 0.2'-3.1'	[[δγ, id3], [δγ, α°]]	0.2'-3.1' ≡ 0.3-3.3	[[δγ, α°], [δγ, id3]]
0.3-3.3 ≡ 0.3'-3.1'	[[δγ, id3], [δγ, α°β°]]	0.3'-3.1' ≡ 0.3-3.3	[[δγ, α°β°], [δγ, id3]]
0.3-3.3 ≡ 0.2'-3.2'	[[δγ, id3], [δγ, id2]]	0.2'-3.2' ≡ 0.3-3.3	[[δγ, id2], [δγ, id3]]
0.3-3.3 ≡ 0.3'-3.2'	[[δγ, id3], [δγ, β°]]	0.3'-3.2' ≡ 0.3-3.3	[[δγ, β°], [δγ, id3]]
0.3-3.3 ≡ 0.3'-3.3'	[[δγ, id3], [δγ, id3]]	0.3'-3.3' ≡ 0.3-3.3	[[δγ, id3], [δγ, id3]]

### Q/M ≡ M'/O'

$$0.1-1.1 \equiv 1.1'-2.1'$$

$$[[\gamma, id1], [\alpha, id1]]$$

### M'/O' ≡ Q/M

$$[[\alpha, id1], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.2'-2.1'$$

$$[[\gamma, id1], [\alpha, \alpha°]]$$

$$1.2'-2.1' \equiv 0.1-1.1$$

$$[[\alpha, \alpha°], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.3'-2.1'$$

$$[[\gamma, id1], [\alpha, \alpha°\beta°]]$$

$$1.3'-2.1' \equiv 0.1-1.1$$

$$[[\alpha, \alpha°\beta°], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.2'-2.2'$$

$$[[\gamma, id1], [\alpha, id2]]$$

$$1.2'-2.2' \equiv 0.1-1.1$$

$$[[\alpha, id2], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.3'-2.2'$$

$$[[\gamma, id1], [\alpha, \beta°]]$$

$$1.3'-2.2' \equiv 0.1-1.1$$

$$[[\alpha, \beta°], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.3'-2.3'$$

$$[[\gamma, id1], [\alpha, id3]]$$

$$1.3'-2.3' \equiv 0.1-1.1$$

$$[[\alpha, id3], [\gamma, id1]]$$

$$0.2-1.1 \equiv 1.1'-2.1'$$

$$[[\gamma, \alpha°], [\alpha, id1]]$$

$$1.1'-2.1' \equiv 0.2-1.1$$

$$[[\alpha, id1], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.2'-2.1'$$

$$[[\gamma, \alpha°], [\alpha, \alpha°]]$$

$$1.2'-2.1' \equiv 0.2-1.1$$

$$[[\alpha, \alpha°], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.3'-2.1'$$

$$[[\gamma, \alpha°], [\alpha, \alpha°\beta°]]$$

$$1.3'-2.1' \equiv 0.2-1.1$$

$$[[\alpha, \alpha°\beta°], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.2'-2.2'$$

$$[[\gamma, \alpha°], [\alpha, id2]]$$

$$1.2'-2.2' \equiv 0.2-1.1$$

$$[[\alpha, id2], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.3'-2.2'$$

$$[[\gamma, \alpha°], [\alpha, \beta°]]$$

$$1.3'-2.2' \equiv 0.2-1.1$$

$$[[\alpha, \beta°], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.3'-2.3'$$

$$[[\gamma, \alpha°], [\alpha, id3]]$$

$$1.3'-2.3' \equiv 0.2-1.1$$

$$[[\alpha, id3], [\gamma, \alpha°]]$$

$$0.3-1.1 \equiv 1.1'-2.1'$$

$$[[\gamma, \alpha°\beta°], [\alpha, id1]]$$

$$1.1'-2.1' \equiv 0.3-1.1$$

$$[[\alpha, id1], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.2'-2.1'$$

$$[[\gamma, \alpha°\beta°], [\alpha, \alpha°]]$$

$$1.2'-2.1' \equiv 0.3-1.1$$

$$[[\alpha, \alpha°], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.3'-2.1'$$

$$[[\gamma, \alpha°\beta°], [\alpha, \alpha°\beta°]]$$

$$1.3'-2.1' \equiv 0.3-1.1$$

$$[[\alpha, \alpha°\beta°], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.2'-2.2'$$

$$[[\gamma, \alpha°\beta°], [\alpha, id2]]$$

$$1.2'-2.2' \equiv 0.3-1.1$$

$$[[\alpha, id2], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.3'-2.2'$$

$$[[\gamma, \alpha°\beta°], [\alpha, \beta°]]$$

$$1.3'-2.2' \equiv 0.3-1.1$$

$$[[\alpha, \beta°], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.3'-2.3'$$

$$[[\gamma, \alpha°\beta°], [\alpha, id3]]$$

$$1.3'-2.3' \equiv 0.3-1.1$$

$$[[\alpha, id3], [\gamma, \alpha°\beta°]]$$

$$0.2-1.2 \equiv 1.1'-2.1'$$

$$[[\gamma, id2], [\alpha, id1]]$$

$$1.1'-2.1' \equiv 0.2-1.2$$

$$[[\alpha, id1], [\gamma, id2]]$$

0.2-1.2 ≡ 1.2'-2.1'	[[γ, id2], [α, α°]]	1.2'-2.1' ≡ 0.2-1.2	[[α, α°], [γ, id2]]
0.2-1.2 ≡ 1.3'-2.1'	[[γ, id2], [α, α°β°]]	1.3'-2.1' ≡ 0.2-1.2	[[α, α°β°], [γ, id2]]
0.2-1.2 ≡ 1.2'-2.2'	[[γ, id2], [α, id2]]	1.2'-2.2' ≡ 0.2-1.2	[[α, id2], [γ, id2]]
0.2-1.2 ≡ 1.3'-2.2'	[[γ, id2], [α, β°]]	1.3'-2.2' ≡ 0.2-1.2	[[α, β°], [γ, id2]]
0.2-1.2 ≡ 1.3'-2.3'	[[γ, id2], [α, id3]]	1.3'-2.3' ≡ 0.2-1.2	[[α, id3], [γ, id2]]
0.3-1.2 ≡ 1.1'-2.1'	[[γ, β°], [α, id1]]	1.1'-2.1' ≡ 0.3-1.2	[[α, id1], [γ, β°]]
0.3-1.2 ≡ 1.2'-2.1'	[[γ, β°], [α, α°]]	1.2'-2.1' ≡ 0.3-1.2	[[α, α°], [γ, β°]]
0.3-1.2 ≡ 1.3'-2.1'	[[γ, β°], [α, α°β°]]	1.3'-2.1' ≡ 0.3-1.2	[[α, α°β°], [γ, β°]]
0.3-1.2 ≡ 1.2'-2.2'	[[γ, β°], [α, id2]]	1.2'-2.2' ≡ 0.3-1.2	[[α, id2], [γ, β°]]
0.3-1.2 ≡ 1.3'-2.2'	[[γ, β°], [α, β°]]	1.3'-2.2' ≡ 0.3-1.2	[[α, β°], [γ, β°]]
0.3-1.2 ≡ 1.3'-2.3'	[[γ, β°], [α, id3]]	1.3'-2.3' ≡ 0.3-1.2	[[α, id3], [γ, β°]]
0.3-1.3 ≡ 1.1'-2.1'	[[γ, id3], [α, id1]]	1.1'-2.1' ≡ 0.3-1.3	[[α, id1], [γ, id3]]
0.3-1.3 ≡ 1.2'-2.1'	[[γ, id3], [α, α°]]	1.2'-2.1' ≡ 0.3-1.3	[[α, α°], [γ, id3]]
0.3-1.3 ≡ 1.3'-2.1'	[[γ, id3], [α, α°β°]]	1.3'-2.1' ≡ 0.3-1.3	[[α, α°β°], [γ, id3]]
0.3-1.3 ≡ 1.2'-2.2'	[[γ, id3], [α, id2]]	1.2'-2.2' ≡ 0.3-1.3	[[α, id2], [γ, id3]]
0.3-1.3 ≡ 1.3'-2.2'	[[γ, id3], [α, β°]]	1.3'-2.2' ≡ 0.3-1.3	[[α, β°], [γ, id3]]
0.3-1.3 ≡ 1.3'-2.3'	[[γ, id3], [α, id3]]	1.3'-2.3' ≡ 0.3-1.3	[[α, id3], [γ, id3]]

### **Q/M ≡ O'/I'**

0.1-1.1 ≡ 2.1'-3.1'

[[γ, id1], [β, id1]]

### **O'/I' ≡ Q/M**

2.1'-3.1' ≡ 0.1-1.1

[[β, id1], [γ, id1]]

0.1-1.1 ≡ 2.2'-3.1'

[[γ, id1], [β, α°]]

2.2'-3.1' ≡ 0.1-1.1

[[β, α°], [γ, id1]]

0.1-1.1 ≡ 2.3'-3.1'

[[γ, id1], [β, α°β°]]

2.3'-3.1' ≡ 0.1-1.1

[[β, α°β°], [γ, id1]]

0.1-1.1 ≡ 2.2'-3.2'

[[γ, id1], [β, id2]]

2.2'-3.2' ≡ 0.1-1.1

[[β, id2], [γ, id1]]

0.1-1.1 ≡ 2.3'-3.2'

[[γ, id1], [β, β°]]

2.3'-3.2' ≡ 0.1-1.1

[[β, β°], [γ, id1]]

0.1-1.1 ≡ 2.3'-3.3'

[[γ, id1], [β, id3]]

2.3'-3.3' ≡ 0.1-1.1

[[β, id3], [γ, id1]]

0.2-1.1 ≡ 2.1'-3.1'

[[γ, α°], [β, id1]]

2.1'-3.1' ≡ 0.2-1.1

[[β, id1], [γ, α°]]

0.2-1.1 ≡ 2.2'-3.1'

[[γ, α°], [β, α°]]

2.2'-3.1' ≡ 0.2-1.1

[[β, α°], [γ, α°]]

0.2-1.1 ≡ 2.3'-3.2'

[[γ, α°], [β, id2]]

2.2'-3.2' ≡ 0.2-1.1

[[β, id2], [γ, α°]]

0.2-1.1 ≡ 2.3'-3.2'

[[γ, α°], [β, β°]]

2.3'-3.2' ≡ 0.2-1.1

[[β, β°], [γ, α°]]

0.2-1.1 ≡ 2.3'-3.3'

[[γ, α°], [β, id3]]

2.3'-3.3' ≡ 0.2-1.1

[[β, id3], [γ, α°]]

0.3-1.1 ≡ 2.1'-3.1'

[[γ, α°β°], [β, id1]]

2.1'-3.1' ≡ 0.3-1.1

[[β, id1], [γ, α°β°]]

0.3-1.1 ≡ 2.2'-3.1'

[[γ, α°β°], [β, α°]]

2.2'-3.1' ≡ 0.3-1.1

[[β, α°], [γ, α°β°]]

0.3-1.1 ≡ 2.3'-3.1'

[[γ, α°β°], [β, α°β°]]

2.3'-3.1' ≡ 0.3-1.1

[[β, α°β°], [γ, α°β°]]

0.3-1.1 ≡ 2.2'-3.2'

[[γ, α°β°], [β, id2]]

2.2'-3.2' ≡ 0.3-1.1

[[β, id2], [γ, α°β°]]

0.3-1.1 ≡ 2.3'-3.2'

[[γ, α°β°], [β, β°]]

2.3'-3.2' ≡ 0.3-1.1

[[β, β°], [γ, α°β°]]

0.3-1.1 ≡ 2.3'-3.3'

[[γ, α°β°], [β, id3]]

2.3'-3.3' ≡ 0.3-1.1

[[β, id3], [γ, α°β°]]

0.2-1.2 ≡ 2.1'-3.1'

[[γ, id2], [β, id1]]

2.1'-3.1' ≡ 0.2-1.2

[[β, id1], [γ, id2]]

0.2-1.2 ≡ 2.2'-3.1'

[[γ, id2], [β, α°]]

2.2'-3.1' ≡ 0.2-1.2

[[β, α°], [γ, id2]]

0.2-1.2 ≡ 2.3'-3.1'

[[γ, id2], [β, α°β°]]

2.3'-3.1' ≡ 0.2-1.2

[[β, α°β°], [γ, id2]]

0.2-1.2 ≡ 2.2'-3.2'

[[γ, id2], [β, id2]]

2.2'-3.2' ≡ 0.2-1.2

[[β, id2], [γ, id2]]

0.2-1.2 ≡ 2.3'-3.2'

[[γ, id2], [β, β°]]

2.3'-3.2' ≡ 0.2-1.2

[[β, β°], [γ, id2]]

0.2-1.2 ≡ 2.3'-3.2'

[[γ, id2], [β, id3]]

2.3'-3.2' ≡ 0.2-1.2

[[β, id3], [γ, id2]]

0.2-1.2 ≡ 2.3'-3.2'

[[γ, id2], [β, β°]]

2.3'-3.2' ≡ 0.2-1.2

[[β, β°], [γ, id2]]

0.2-1.2 ≡ 2.3'-3.3'	[[γ, id2], [β, id3]]	2.3'-3.3' ≡ 0.2-1.2	[[β, id3], [γ, id2]]
0.3-1.2 ≡ 2.1'-3.1'	[[γ, β°], [β, id1]]	2.1'-3.1' ≡ 0.3-1.2	[[β, id1], [γ, β°]]
0.3-1.2 ≡ 2.2'-3.1'	[[γ, β°], [β, α°]]	2.2'-3.1' ≡ 0.3-1.2	[[β, α°], [γ, β°]]
0.3-1.2 ≡ 2.3'-3.1'	[[γ, β°], [β, α°β°]]	2.3'-3.1' ≡ 0.3-1.2	[[β, α°β°], [γ, β°]]
0.3-1.2 ≡ 2.2'-3.2'	[[γ, β°], [β, id2]]	2.2'-3.2' ≡ 0.3-1.2	[[β, id2], [γ, β°]]
0.3-1.2 ≡ 2.3'-3.2'	[[γ, β°], [β, β°]]	2.3'-3.2' ≡ 0.3-1.2	[[β, β°], [γ, β°]]
0.3-1.2 ≡ 2.3'-3.3'	[[γ, β°], [β, id3]]	2.3'-3.3' ≡ 0.3-1.2	[[β, id3], [γ, β°]]
0.3-1.3 ≡ 2.1'-3.1'	[[γ, id3], [β, id1]]	2.1'-3.1' ≡ 0.3-1.3	[[β, id1], [γ, id3]]
0.3-1.3 ≡ 2.2'-3.1'	[[γ, id3], [β, α°]]	2.2'-3.1' ≡ 0.3-1.3	[[β, α°], [γ, id3]]
0.3-1.3 ≡ 2.3'-3.1'	[[γ, id3], [β, α°β°]]	2.3'-3.1' ≡ 0.3-1.3	[[β, α°β°], [γ, id3]]
0.3-1.3 ≡ 2.2'-3.2'	[[γ, id3], [β, id2]]	2.2'-3.2' ≡ 0.3-1.3	[[β, id2], [γ, id3]]
0.3-1.3 ≡ 2.3'-3.2'	[[γ, id3], [β, β°]]	2.3'-3.2' ≡ 0.3-1.3	[[β, β°], [γ, id3]]
0.3-1.3 ≡ 2.3'-3.3'	[[γ, id3], [β, id3]]	2.3'-3.3' ≡ 0.3-1.3	[[β, id3], [γ, id3]]

### **Q/M ≡ M'/I'**

$$0.1-1.1 \equiv 1.1'-3.1'$$

$$[[\gamma, id1], [\beta\alpha, id1]]$$

### **M'/I' ≡ Q/M**

$$1.1'-3.1' \equiv 0.1-1.1$$

$$[[\beta\alpha, id1], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.2'-3.1'$$

$$[[\gamma, id1], [\beta\alpha, \alpha°]]$$

$$1.2'-3.1' \equiv 0.1-1.1$$

$$[[\beta\alpha, \alpha°], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.3'-3.1'$$

$$[[\gamma, id1], [\beta\alpha, \alpha°\beta°]]$$

$$1.3'-3.1' \equiv 0.1-1.1$$

$$[[\beta\alpha, \alpha°\beta°], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.2'-3.2'$$

$$[[\gamma, id1], [\beta\alpha, id2]]$$

$$1.2'-3.2' \equiv 0.1-1.1$$

$$[[\beta\alpha, id2], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.3'-3.2'$$

$$[[\gamma, id1], [\beta\alpha, \beta°]]$$

$$1.3'-3.2' \equiv 0.1-1.1$$

$$[[\beta\alpha, \beta°], [\gamma, id1]]$$

$$0.1-1.1 \equiv 1.3'-3.3'$$

$$[[\gamma, id1], [\beta\alpha, id3]]$$

$$1.3'-3.3' \equiv 0.1-1.1$$

$$[[\beta\alpha, id3], [\gamma, id1]]$$

$$0.2-1.1 \equiv 1.1'-3.1'$$

$$[[\gamma, \alpha°], [\beta\alpha, id1]]$$

$$1.1'-3.1' \equiv 0.2-1.1$$

$$[[\beta\alpha, id1], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.2'-3.1'$$

$$[[\gamma, \alpha°], [\beta\alpha, \alpha°]]$$

$$1.2'-3.1' \equiv 0.2-1.1$$

$$[[\beta\alpha, \alpha°], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.3'-3.1'$$

$$[[\gamma, \alpha°], [\beta\alpha, \alpha°\beta°]]$$

$$1.3'-3.1' \equiv 0.2-1.1$$

$$[[\beta\alpha, \alpha°\beta°], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.3'-3.2'$$

$$[[\gamma, \alpha°], [\beta\alpha, id2]]$$

$$1.2'-3.2' \equiv 0.2-1.1$$

$$[[\beta\alpha, id2], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.3'-3.2'$$

$$[[\gamma, \alpha°], [\beta\alpha, \beta°]]$$

$$1.3'-3.2' \equiv 0.2-1.1$$

$$[[\beta\alpha, \beta°], [\gamma, \alpha°]]$$

$$0.2-1.1 \equiv 1.3'-3.3'$$

$$[[\gamma, \alpha°], [\beta\alpha, id3]]$$

$$1.3'-3.3' \equiv 0.2-1.1$$

$$[[\beta\alpha, id3], [\gamma, \alpha°]]$$

$$0.3-1.1 \equiv 1.1'-3.1'$$

$$[[\gamma, \alpha°\beta°], [\beta\alpha, id1]]$$

$$1.1'-3.1' \equiv 0.3-1.1$$

$$[[\beta\alpha, id1], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.2'-3.1'$$

$$[[\gamma, \alpha°\beta°], [\beta\alpha, \alpha°]]$$

$$1.2'-3.1' \equiv 0.3-1.1$$

$$[[\beta\alpha, \alpha°], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.3'-3.1'$$

$$[[\gamma, \alpha°\beta°], [\beta\alpha, \alpha°\beta°]]$$

$$1.3'-3.1' \equiv 0.3-1.1$$

$$[[\beta\alpha, \alpha°\beta°], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.2'-3.2'$$

$$[[\gamma, \alpha°\beta°], [\beta\alpha, id2]]$$

$$1.2'-3.2' \equiv 0.3-1.1$$

$$[[\beta\alpha, id2], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.3'-3.2'$$

$$[[\gamma, \alpha°\beta°], [\beta\alpha, \beta°]]$$

$$1.3'-3.2' \equiv 0.3-1.1$$

$$[[\beta\alpha, \beta°], [\gamma, \alpha°\beta°]]$$

$$0.3-1.1 \equiv 1.3'-3.3'$$

$$[[\gamma, \alpha°\beta°], [\beta\alpha, id3]]$$

$$1.3'-3.3' \equiv 0.3-1.1$$

$$[[\beta\alpha, id3], [\gamma, \alpha°\beta°]]$$

$$0.2-1.2 \equiv 1.1'-3.1'$$

$$[[\gamma, id2], [\beta\alpha, id1]]$$

$$1.1'-3.1' \equiv 0.2-1.2$$

$$[[\beta\alpha, id1], [\gamma, id2]]$$

$$0.2-1.2 \equiv 1.2'-3.1'$$

$$[[\gamma, id2], [\beta\alpha, \alpha°]]$$

$$1.2'-3.1' \equiv 0.2-1.2$$

$$[[\beta\alpha, \alpha°], [\gamma, id2]]$$

$$0.2-1.2 \equiv 1.3'-3.1'$$

$$[[\gamma, id2], [\beta\alpha, \alpha°\beta°]]$$

$$1.3'-3.1' \equiv 0.2-1.2$$

$$[[\beta\alpha, \alpha°\beta°], [\gamma, id2]]$$

$$0.2-1.2 \equiv 1.2'-3.2'$$

$$[[\gamma, id2], [\beta\alpha, id2]]$$

$$1.2'-3.2' \equiv 0.2-1.2$$

$$[[\beta\alpha, id2], [\gamma, id2]]$$

$$0.2-1.2 \equiv 1.3'-3.2'$$

$$[[\gamma, id2], [\beta\alpha, \beta°]]$$

$$1.3'-3.2' \equiv 0.2-1.2$$

$$[[\beta\alpha, \beta°], [\gamma, id2]]$$

$$0.2-1.2 \equiv 1.3'-3.3'$$

$$[[\gamma, id2], [\beta\alpha, id3]]$$

$$1.3'-3.3' \equiv 0.2-1.2$$

$$[[\beta\alpha, id3], [\gamma, id2]]$$

$$0.3-1.2 \equiv 1.1'-3.1'$$

$$[[\gamma, \beta°], [\beta\alpha, id1]]$$

$$1.1'-3.1' \equiv 0.3-1.2$$

$$[[\beta\alpha, id1], [\gamma, \beta°]]$$

$$0.3-1.2 \equiv 1.2'-3.1'$$

$$[[\gamma, \beta°], [\beta\alpha, \alpha°]]$$

$$1.2'-3.1' \equiv 0.3-1.2$$

$$[[\beta\alpha, \alpha°], [\gamma, \beta°]]$$

$$0.3-1.2 \equiv 1.3'-3.1'$$

$$[[\gamma, \beta°], [\beta\alpha, \alpha°\beta°]]$$

$$1.3'-3.1' \equiv 0.3-1.2$$

$$[[\beta\alpha, \alpha°\beta°], [\gamma, \beta°]]$$

$0.3-1.2 \equiv 1.2'-3.2'$	$[[\gamma, \beta^\circ], [\beta\alpha, \text{id2}]]$	$1.2'-3.2' \equiv 0.3-1.2$	$[[\beta\alpha, \text{id2}], [\gamma, \beta^\circ]]$
$0.3-1.2 \equiv 1.3'-3.2'$	$[[\gamma, \beta^\circ], [\beta\alpha, \beta^\circ]]$	$1.3'-3.2' \equiv 0.3-1.2$	$[[\beta\alpha, \beta^\circ], [\gamma, \beta^\circ]]$
$0.3-1.2 \equiv 1.3'-3.3'$	$[[\gamma, \beta^\circ], [\beta\alpha, \text{id3}]]$	$1.3'-3.3' \equiv 0.3-1.2$	$[[\beta\alpha, \text{id3}], [\gamma, \beta^\circ]]$
$0.3-1.3 \equiv 1.1'-3.1'$	$[[\gamma, \text{id3}], [\beta\alpha, \text{id1}]]$	$1.1'-3.1' \equiv 0.3-1.3$	$[[\beta\alpha, \text{id1}], [\gamma, \text{id3}]]$
$0.3-1.3 \equiv 1.2'-3.1'$	$[[\gamma, \text{id3}], [\beta\alpha, \alpha^\circ]]$	$1.2'-3.1' \equiv 0.3-1.3$	$[[\beta\alpha, \alpha^\circ], [\gamma, \text{id3}]]$
$0.3-1.3 \equiv 1.3'-3.1'$	$[[\gamma, \text{id3}], [\beta\alpha, \alpha^\circ\beta^\circ]]$	$1.3'-3.1' \equiv 0.3-1.3$	$[[\beta\alpha, \alpha^\circ\beta^\circ], [\gamma, \text{id3}]]$
$0.3-1.3 \equiv 1.2'-3.2'$	$[[\gamma, \text{id3}], [\beta\alpha, \text{id2}]]$	$1.2'-3.2' \equiv 0.3-1.3$	$[[\beta\alpha, \text{id2}], [\gamma, \text{id3}]]$
$0.3-1.3 \equiv 1.3'-3.2'$	$[[\gamma, \text{id3}], [\beta\alpha, \beta^\circ]]$	$1.3'-3.2' \equiv 0.3-1.3$	$[[\beta\alpha, \beta^\circ], [\gamma, \text{id3}]]$
$0.3-1.3 \equiv 1.3'-3.3'$	$[[\gamma, \text{id3}], [\beta\alpha, \text{id3}]]$	$1.3'-3.3' \equiv 0.3-1.3$	$[[\beta v, \text{id3}], [\gamma, \text{id3}]]$

Now, we show the semiotic connections of the pre-semiotic sign relations:

<b>M/O ≡ M'/O'</b>		<b>M'/O' ≡ M/O</b>
1.1-2.1 ≡ 1.1'-2.1'	[[α, id1], [α, id1]]	1.1'-2.1' ≡ 1.1-2.1
1.1-2.1 ≡ 1.2'-2.1'	[[α, id1], [α, α°]]	1.2'-2.1' ≡ 1.1-2.1
1.1-2.1 ≡ 1.3'-2.1'	[[α, id1], [α, α°β°]]	1.3'-2.1' ≡ 1.1-2.1
1.1-2.1 ≡ 1.2'-2.2'	[[α, id1], [α, id2]]	1.2'-2.2' ≡ 1.1-2.1
1.1-2.1 ≡ 1.3'-2.2'	[[α, id1], [α, β°]]	1.3'-2.2' ≡ 1.1-2.1
1.1-2.1 ≡ 1.3'-2.3'	[[α, id1], [α, id3]]	1.3'-2.3' ≡ 1.1-2.1
1.2-2.1 ≡ 1.1'-2.1'	[[α, α°], [α, id1]]	1.1'-2.1' ≡ 1.2-2.1
1.2-2.1 ≡ 1.2'-2.1'	[[α, α°], [α, α°]]	1.2'-2.1' ≡ 1.2-2.1
1.2-2.1 ≡ 1.3'-2.1'	[[α, α°], [α, α°β°]]	1.3'-2.1' ≡ 1.2-2.1
1.2-2.1 ≡ 1.2'-2.2'	[[α, α°], [α, id2]]	1.2'-2.2' ≡ 1.2-2.1
1.2-2.1 ≡ 1.3'-2.2'	[[α, α°], [α, β°]]	1.3'-2.2' ≡ 1.2-2.1
1.2-2.1 ≡ 1.3'-2.3'	[[α, α°], [α, id3]]	1.3'-2.3' ≡ 1.2-2.1
1.3-2.1 ≡ 1.1'-2.1'	[[α, α°β°], [α, id1]]	1.1'-2.1' ≡ 1.3-2.1
1.3-2.1 ≡ 1.2'-2.1'	[[α, α°β°], [α, α°]]	1.2'-2.1' ≡ 1.3-2.1
1.3-2.1 ≡ 1.3'-2.1'	[[α, α°β°], [α, α°β°]]	1.3'-2.1' ≡ 1.3-2.1
1.3-2.1 ≡ 1.2'-2.2'	[[α, α°β°], [α, id2]]	1.2'-2.2' ≡ 1.3-2.1
1.3-2.1 ≡ 1.3'-2.2'	[[α, α°β°], [α, β°]]	1.3'-2.2' ≡ 1.3-2.1
1.3-2.1 ≡ 1.3'-2.3'	[[α, α°β°], [α, id3]]	1.3'-2.3' ≡ 1.3-2.1
1.2-2.2 ≡ 1.1'-2.1'	[[α, id2], [α, id1]]	1.1'-2.1' ≡ 1.2-2.2
1.2-2.2 ≡ 1.2'-2.1'	[[α, id2], [α, α°]]	1.2'-2.1' ≡ 1.2-2.2
1.2-2.2 ≡ 1.3'-2.1'	[[α, id2], [α, α°β°]]	1.3'-2.1' ≡ 1.2-2.2
1.2-2.2 ≡ 1.2'-2.2'	[[α, id2], [α, id2]]	1.2'-2.2' ≡ 1.2-2.2
1.2-2.2 ≡ 1.3'-2.2'	[[α, id2], [α, β°]]	1.3'-2.2' ≡ 1.2-2.2
1.2-2.2 ≡ 1.3'-2.3'	[[α, id2], [α, id3]]	1.3'-2.3' ≡ 1.2-2.2
1.3-2.2 ≡ 1.1'-2.1'	[[α, β°], [α, id1]]	1.1'-2.1' ≡ 1.3-2.2
1.3-2.2 ≡ 1.2'-2.1'	[[α, β°], [α, α°]]	1.2'-2.1' ≡ 1.3-2.2
1.3-2.2 ≡ 1.3'-2.1'	[[α, β°], [α, α°β°]]	1.3'-2.1' ≡ 1.3-2.2
1.3-2.2 ≡ 1.2'-2.2'	[[α, β°], [α, id2]]	1.2'-2.2' ≡ 1.3-2.2
1.3-2.2 ≡ 1.3'-2.2'	[[α, β°], [α, β°]]	1.3'-2.2' ≡ 1.3-2.2



$1.3\text{-}2.3 \equiv 2.2'\text{-}3.2'$	$[[\alpha, \text{id3}], [\beta, \text{id2}]]$	$2.2'\text{-}3.2' \equiv 1.3\text{-}2.3$	$[[\beta, \text{id2}], [\alpha, \text{id3}]]$
$1.3\text{-}2.3 \equiv 2.3'\text{-}3.2'$	$[[\alpha, \text{id3}], [\beta, \beta^\circ]]$	$2.3'\text{-}3.2' \equiv 1.3\text{-}2.3$	$[[\beta, \beta^\circ], [\alpha, \text{id3}]]$
$1.3\text{-}2.3 \equiv 2.3'\text{-}3.3'$	$[[\alpha, \text{id3}], [\beta, \text{id3}]]$	$2.3'\text{-}3.3' \equiv 1.3\text{-}2.3$	$[[\beta, \text{id3}], [\alpha, \text{id3}]]$

<b>M/O ≡ M'/I'</b>		<b>M'/I' ≡ M/O</b>
1.1-2.1 ≡ 1.1'-3.1'	[[α, id1], [βα, id1]]	1.1'-3.1' ≡ 1.1-2.1
1.1-2.1 ≡ 1.2'-3.1'	[[α, id1], [βα, α°]]	1.2'-3.1' ≡ 1.1-2.1
1.1-2.1 ≡ 1.3'-3.1'	[[α, id1], [βα, α°β°]]	1.3'-3.1' ≡ 1.1-2.1
1.1-2.1 ≡ 1.2'-3.2'	[[α, id1], [βα, id2]]	1.2'-3.2' ≡ 1.1-2.1
1.1-2.1 ≡ 1.3'-3.2'	[[α, id1], [βα, β°]]	1.3'-3.2' ≡ 1.1-2.1
1.1-2.1 ≡ 1.3'-3.3'	[[α, id1], [βα, id3]]	1.3'-3.3' ≡ 1.1-2.1
1.2-2.1 ≡ 1.1'-3.1'	[[α, α°], [βα, id1]]	1.1'-3.1' ≡ 1.2-2.1
1.2-2.1 ≡ 1.2'-3.1'	[[α, α°], [βα, α°]]	1.2'-3.1' ≡ 1.2-2.1
1.2-2.1 ≡ 1.3'-3.1'	[[α, α°], [βα, α°β°]]	1.3'-3.1' ≡ 1.2-2.1
1.2-2.1 ≡ 1.2'-3.2'	[[α, α°], [βα, id2]]	1.2'-3.2' ≡ 1.2-2.1
1.2-2.1 ≡ 1.3'-3.2'	[[α, α°], [βα, β°]]	1.3'-3.2' ≡ 1.2-2.1
1.2-2.1 ≡ 1.3'-3.3'	[[α, α°], [βα, id3]]	1.3'-3.3' ≡ 1.2-2.1
1.3-2.1 ≡ 1.1'-3.1'	[[α, α°β°], [βα, id1]]	1.1'-3.1' ≡ 1.3-2.1
1.3-2.1 ≡ 1.2'-3.1'	[[α, α°β°], [βα, α°]]	1.2'-3.1' ≡ 1.3-2.1
1.3-2.1 ≡ 1.3'-3.1'	[[α, α°β°], [βα, α°β°]]	1.3'-3.1' ≡ 1.3-2.1
1.3-2.1 ≡ 1.2'-3.2'	[[α, α°β°], [βα, id2]]	1.2'-3.2' ≡ 1.3-2.1
1.3-2.1 ≡ 1.3'-3.2'	[[α, α°β°], [βα, β°]]	1.3'-3.2' ≡ 1.3-2.1
1.3-2.1 ≡ 1.3'-3.3'	[[α, α°β°], [βα, id3]]	1.3'-3.3' ≡ 1.3-2.1
1.2-2.2 ≡ 1.1'-3.1'	[[α, id2], [βα, id1]]	1.1'-3.1' ≡ 1.2-2.2
1.2-2.2 ≡ 1.2'-3.1'	[[α, id2], [βα, α°]]	1.2'-3.1' ≡ 1.2-2.2
1.2-2.2 ≡ 1.3'-3.1'	[[α, id2], [βα, α°β°]]	1.3'-3.1' ≡ 1.2-2.2
1.2-2.2 ≡ 1.2'-3.2'	[[α, id2], [βα, id2]]	1.2'-3.2' ≡ 1.2-2.2
1.2-2.2 ≡ 1.3'-3.2'	[[α, id2], [βα, β°]]	1.3'-3.2' ≡ 1.2-2.2
1.2-2.2 ≡ 1.3'-3.3'	[[α, id2], [βα, id3]]	1.3'-3.3' ≡ 1.2-2.2
1.3-2.2 ≡ 1.1'-3.1'	[[α, β°], [βα, id1]]	1.1'-3.1' ≡ 1.3-2.2
1.3-2.2 ≡ 1.2'-3.1'	[[α, β°], [βα, α°]]	1.2'-3.1' ≡ 1.3-2.2
1.3-2.2 ≡ 1.3'-3.1'	[[α, β°], [βα, α°β°]]	1.3'-3.1' ≡ 1.3-2.2
1.3-2.2 ≡ 1.2'-3.2'	[[α, β°], [βα, id2]]	1.2'-3.2' ≡ 1.3-2.2
1.3-2.2 ≡ 1.3'-3.2'	[[α, β°], [βα, β°]]	1.3'-3.2' ≡ 1.3-2.2
1.3-2.2 ≡ 1.3'-3.3'	[[α, β°], [βα, id3]]	1.3'-3.3' ≡ 1.3-2.2
1.3-2.3 ≡ 1.1'-3.1'	[[α, id3], [βα, id1]]	1.1'-3.1' ≡ 1.3-2.3
1.3-2.3 ≡ 1.2'-3.1'	[[α, id3], [βα, α°]]	1.2'-3.1' ≡ 1.3-2.3
1.3-2.3 ≡ 1.3'-3.1'	[[α, id3], [βα, α°β°]]	1.3'-3.1' ≡ 1.3-2.3
1.3-2.3 ≡ 1.2'-3.2'	[[α, id3], [βα, id2]]	1.2'-3.2' ≡ 1.3-2.3
1.3-2.3 ≡ 1.3'-3.2'	[[α, id3], [βα, β°]]	1.3'-3.2' ≡ 1.3-2.3
1.3-2.3 ≡ 1.3'-3.3'	[[α, id3], [βα, id3]]	1.3'-3.3' ≡ 1.3-2.3

$O/I \equiv O'/I'$	$O'/I' \equiv O/I$
$2.1\text{-}3.1 \equiv 2.1'\text{-}3.1'$	$[[\beta, \text{id}1], [\beta, \text{id}1]] \equiv 2.1\text{-}3.1$
$2.1\text{-}3.1 \equiv 2.2'\text{-}3.1'$	$[[\beta, \text{id}1], [\beta, \alpha^o]] \equiv 2.2\text{-}3.1' \equiv 2.1\text{-}3.1$
$2.1\text{-}3.1 \equiv 2.3'\text{-}3.1'$	$[[\beta, \text{id}1], [\beta, \alpha^o \beta^o]] \equiv 2.3\text{-}3.1' \equiv 2.1\text{-}3.1$

2.1-3.1 ≡ 2.2'-3.2'	[[β, id1], [β, id2]]	2.2'-3.2' ≡ 2.1-3.1	[[β, id2], [β, id1]]
2.1-3.1 ≡ 2.3'-3.2'	[[β, id1], [β, β°]]	2.3'-3.2' ≡ 2.1-3.1	[[β, β°], [β, id1]]
2.1-3.1 ≡ 2.3'-3.3'	[[β, id1], [β, id3]]	2.3'-3.3' ≡ 2.1-3.1	[[β, id3], [β, id1]]
2.2-3.1 ≡ 2.1'-3.1'	[[β, α°], [β, id1]]	2.1'-3.1' ≡ 2.2-3.1	[[β, id1], [β, α°]]
2.2-3.1 ≡ 2.2'-3.1'	[[β, α°], [β, α°]]	2.2'-3.1' ≡ 2.2-3.1	[[β, α°], [β, α°]]
2.2-3.1 ≡ 2.3'-3.1'	[[β, α°], [β, α°β°]]	2.3'-3.1' ≡ 2.2-3.1	[[β, α°β°], [β, α°]]
2.2-3.1 ≡ 2.2'-3.2'	[[β, α°], [β, id2]]	2.2'-3.2' ≡ 2.2-3.1	[[β, id2], [β, α°]]
2.2-3.1 ≡ 2.3'-3.2'	[[β, α°], [β, β°]]	2.3'-3.2' ≡ 2.2-3.1	[[β, β°], [β, α°]]
2.2-3.1 ≡ 2.3'-3.3'	[[β, α°], [β, id3]]	2.3'-3.3' ≡ 2.2-3.1	[[β, id3], [β, α°]]
2.3-3.1 ≡ 2.1'-3.1'	[[β, α°β°], [β, id1]]	2.1'-3.1' ≡ 2.3-3.1	[[β, id1], [β, α°β°]]
2.3-3.1 ≡ 2.2'-3.1'	[[β, α°β°], [β, α°]]	2.2'-3.1' ≡ 2.3-3.1	[[β, α°], [β, α°β°]]
2.3-3.1 ≡ 2.3'-3.1'	[[β, α°β°], [β, α°β°]]	2.3'-3.1' ≡ 2.3-3.1	[[β, α°β°], [β, α°β°]]
2.3-3.1 ≡ 2.2'-3.2'	[[β, α°β°], [β, id2]]	2.2'-3.2' ≡ 2.3-3.1	[[β, id2], [β, α°β°]]
2.3-3.1 ≡ 2.3'-3.2'	[[β, α°β°], [β, β°]]	2.3'-3.2' ≡ 2.3-3.1	[[β, β°], [β, α°β°]]
2.3-3.1 ≡ 2.3'-3.3'	[[β, α°β°], [β, id3]]	2.3'-3.3' ≡ 2.3-3.1	[[β, id3], [β, α°β°]]
2.2-3.2 ≡ 2.1'-3.1'	[[β, id2], [β, id1]]	2.1'-3.1' ≡ 2.2-3.2	[[β, id1], [β, id2]]
2.2-3.2 ≡ 2.2'-3.1'	[[β, id2], [β, α°]]	2.2'-3.1' ≡ 2.2-3.2	[[β, α°], [β, id2]]
2.2-3.2 ≡ 2.3'-3.1'	[[β, id2], [β, α°β°]]	2.3'-3.1' ≡ 2.2-3.2	[[β, α°β°], [β, id2]]
2.2-3.2 ≡ 2.2'-3.2'	[[β, id2], [β, id2]]	2.2'-3.2' ≡ 2.2-3.2	[[β, id2], [β, id2]]
2.2-3.2 ≡ 2.3'-3.2'	[[β, id2], [β, β°]]	2.3'-3.2' ≡ 2.2-3.2	[[β, β°], [β, id2]]
2.2-3.2 ≡ 2.3'-3.3'	[[β, id2], [β, id3]]	2.3'-3.3' ≡ 2.2-3.2	[[β, id3], [β, id2]]
2.3-3.2 ≡ 2.1'-3.1'	[[β, β°], [β, id1]]	2.1'-3.1' ≡ 2.3-3.2	[[β, id1], [β, β°]]
2.3-3.2 ≡ 2.2'-3.1'	[[β, β°], [β, α°]]	2.2'-3.1' ≡ 2.3-3.2	[[β, α°], [β, β°]]
2.3-3.2 ≡ 2.3'-3.1'	[[β, β°], [β, α°β°]]	2.3'-3.1' ≡ 2.3-3.2	[[β, α°β°], [β, β°]]
2.3-3.2 ≡ 2.2'-3.2'	[[β, β°], [β, id2]]	2.2'-3.2' ≡ 2.3-3.2	[[β, id2], [β, β°]]
2.3-3.2 ≡ 2.3'-3.2'	[[β, β°], [β, β°]]	2.3'-3.2' ≡ 2.3-3.2	[[β, β°], [β, β°]]
2.3-3.2 ≡ 2.3'-3.3'	[[β, β°], [β, id3]]	2.3'-3.3' ≡ 2.3-3.2	[[β, id3], [β, β°]]
2.3-3.3 ≡ 2.1'-3.1'	[[β, id3], [β, id1]]	2.1'-3.1' ≡ 2.3-3.3	[[β, id1], [β, id3]]
2.3-3.3 ≡ 2.2'-3.1'	[[β, id3], [β, α°]]	2.2'-3.1' ≡ 2.3-3.3	[[β, α°], [β, id3]]
2.3-3.3 ≡ 2.3'-3.1'	[[β, id3], [β, α°β°]]	2.3'-3.1' ≡ 2.3-3.3	[[β, α°β°], [β, id3]]
2.3-3.3 ≡ 2.2'-3.2'	[[β, id3], [β, id2]]	2.2'-3.2' ≡ 2.3-3.3	[[β, id2], [β, id3]]
2.3-3.3 ≡ 2.3'-3.2'	[[β, id3], [β, β°]]	2.3'-3.2' ≡ 2.3-3.3	[[β, β°], [β, id3]]
2.3-3.3 ≡ 2.3'-3.3'	[[β, id3], [β, id3]]	2.3'-3.3' ≡ 2.3-3.3	[[β, id3], [β, id3]]

### O/I ≡ M'/I'

2.1-3.1 ≡ 1.1'-3.1'	[[β, id1], [βα, id1]]
2.1-3.1 ≡ 1.2'-3.1'	[[β, id1], [βα, α°]]
2.1-3.1 ≡ 1.3'-3.1'	[[β, id1], [βα, α°β°]]
2.1-3.1 ≡ 1.2'-3.2'	[[β, id1], [βα, id2]]
2.1-3.1 ≡ 1.3'-3.2'	[[β, id1], [βα, β°]]
2.1-3.1 ≡ 1.3'-3.3'	[[β, id1], [βα, id3]]
2.2-3.1 ≡ 1.1'-3.1'	[[β, α°], [βα, id1]]

### M'/I' ≡ O/I

1.1'-3.1' ≡ 2.1-3.1	[[βα, id1], [β, id1]]
1.2'-3.1' ≡ 2.1-3.1	[[βα, α°], [β, id1]]
1.3'-3.1' ≡ 2.1-3.1	[[βα, α°β°], [β, id1]]
1.2'-3.2' ≡ 2.1-3.1	[[βα, id2], [β, id1]]
1.3'-3.2' ≡ 2.1-3.1	[[βα, β°], [β, id1]]
1.3'-3.3' ≡ 2.1-3.1	[[βα, id3], [β, id1]]
1.1'-3.1' ≡ 2.2-3.1	[[βα, id1], [β, α°]]

2.2-3.1 ≡ 1.2'-3.1'	[[β, α°], [βα, α°]]	1.2'-3.1' ≡ 2.2-3.1	[[βα, α°], [β, α°]]
2.2-3.1 ≡ 1.3'-3.1'	[[β, α°], [βα, α°β°]]	1.3'-3.1' ≡ 2.2-3.1	[[βα, α°β°], [β, α°]]
2.2-3.1 ≡ 1.2'-3.2'	[[β, α°], [βα, id2]]	1.2'-3.2' ≡ 2.2-3.1	[[βα, id2], [β, α°]]
2.2-3.1 ≡ 1.3'-3.2'	[[β, α°], [βα, β°]]	1.3'-3.2' ≡ 2.2-3.1	[[βα, β°], [β, α°]]
2.2-3.1 ≡ 1.3'-3.3'	[[β, α°], [βα, id3]]	1.3'-3.3' ≡ 2.2-3.1	[[βα, id3], [β, α°]]
2.3-3.1 ≡ 1.1'-3.1'	[[β, α°β°], [βα, id1]]	1.1'-3.1' ≡ 2.3-3.1	[[βα, id1], [β, α°β°]]
2.3-3.1 ≡ 1.2'-3.1'	[[β, α°β°], [βα, α°]]	1.2'-3.1' ≡ 2.3-3.1	[[βα, α°], [β, α°β°]]
2.3-3.1 ≡ 1.3'-3.1'	[[β, α°β°], [βα, α°β°]]	1.3'-3.1' ≡ 2.3-3.1	[[βα, α°β°], [β, α°β°]]
2.3-3.1 ≡ 1.2'-3.2'	[[β, α°β°], [βα, id2]]	1.2'-3.2' ≡ 2.3-3.1	[[βα, id2], [β, α°β°]]
2.3-3.1 ≡ 1.3'-3.2'	[[β, α°β°], [βα, β°]]	1.3'-3.2' ≡ 2.3-3.1	[[βα, β°], [β, α°β°]]
2.3-3.1 ≡ 1.3'-3.3'	[[β, α°β°], [βα, id3]]	1.3'-3.3' ≡ 2.3-3.1	[[βα, id3], [β, α°β°]]
2.2-3.2 ≡ 1.1'-3.1'	[[β, id2], [βα, id1]]	1.1'-3.1' ≡ 2.2-3.2	[[βα, id1], [β, id2]]
2.2-3.2 ≡ 1.2'-3.1'	[[β, id2], [βα, α°]]	1.2'-3.1' ≡ 2.2-3.2	[[βα, α°], [β, id2]]
2.2-3.2 ≡ 1.3'-3.1'	[[β, id2], [βα, α°β°]]	1.3'-3.1' ≡ 2.2-3.2	[[βα, α°β°], [β, id2]]
2.2-3.2 ≡ 1.2'-3.2'	[[β, id2], [βα, id2]]	1.2'-3.2' ≡ 2.2-3.2	[[βα, id2], [β, id2]]
2.2-3.2 ≡ 1.3'-3.2'	[[β, id2], [βα, β°]]	1.3'-3.2' ≡ 2.2-3.2	[[βα, β°], [β, id2]]
2.2-3.2 ≡ 1.3'-3.3'	[[β, id2], [βα, id3]]	1.3'-3.3' ≡ 2.2-3.2	[[βα, id3], [β, id2]]
2.3-3.2 ≡ 1.1'-3.1'	[[β, β°], [βα, id1]]	1.1'-3.1' ≡ 2.3-3.2	[[βα, id1], [β, β°]]
2.3-3.2 ≡ 1.2'-3.1'	[[β, β°], [βα, α°]]	1.2'-3.1' ≡ 2.3-3.2	[[βα, α°], [β, β°]]
2.3-3.2 ≡ 1.3'-3.1'	[[β, β°], [βα, α°β°]]	1.3'-3.1' ≡ 2.3-3.2	[[βα, α°β°], [β, β°]]
2.3-3.2 ≡ 1.2'-3.2'	[[β, β°], [βα, id2]]	1.2'-3.2' ≡ 2.3-3.2	[[βα, id2], [β, β°]]
2.3-3.2 ≡ 1.3'-3.2'	[[β, β°], [βα, β°]]	1.3'-3.2' ≡ 2.3-3.2	[[βα, β°], [β, β°]]
2.3-3.2 ≡ 1.3'-3.3'	[[β, β°], [βα, id3]]	1.3'-3.3' ≡ 2.3-3.2	[[βα, id3], [β, β°]]
2.3-3.3 ≡ 1.1'-3.1'	[[β, id3], [βα, id1]]	1.1'-3.1' ≡ 2.3-3.3	[[βα, id1], [β, id3]]
2.3-3.3 ≡ 1.2'-3.1'	[[β, id3], [βα, α°]]	1.2'-3.1' ≡ 2.3-3.3	[[βα, α°], [β, id3]]
2.3-3.3 ≡ 1.3'-3.1'	[[β, id3], [βα, α°β°]]	1.3'-3.1' ≡ 2.3-3.3	[[βα, α°β°], [β, id3]]
2.3-3.3 ≡ 1.2'-3.2'	[[β, id3], [βα, id2]]	1.2'-3.2' ≡ 2.3-3.3	[[βα, id2], [β, id3]]
2.3-3.3 ≡ 1.3'-3.2'	[[β, id3], [βα, β°]]	1.3'-3.2' ≡ 2.3-3.3	[[βα, β°], [β, id3]]
2.3-3.3 ≡ 1.3'-3.3'	[[β, id3], [βα, id3]]	1.3'-3.3' ≡ 2.3-3.3	[[βα, id3], [β, id3]]

$$M/I \equiv M'/O'$$

- 1.1-3.1 ≡ 1.1'-2.1'
- 1.1-3.1 ≡ 1.2'-2.1'
- 1.1-3.1 ≡ 1.3'-2.1'
- 1.1-3.1 ≡ 1.2'-2.2'
- 1.1-3.1 ≡ 1.3'-2.2'
- 1.1-3.1 ≡ 1.3'-2.3'
- 1.2-3.1 ≡ 1.1'-2.1'
- 1.2-3.1 ≡ 1.2'-2.1'
- 1.2-3.1 ≡ 1.3'-2.1'
- 1.2-3.1 ≡ 1.2'-2.2'
- 1.2-3.1 ≡ 1.3'-2.2'

$$M'/O' \equiv M/I$$

$[[\beta\alpha, \text{id}1], [\alpha, \text{id}1]]$	$1.1' - 2.1' \equiv 1.1 - 3.1$	$[[\alpha, \text{id}1], [\beta\alpha, \text{id}1]]$
$[[\beta\alpha, \text{id}1], [\alpha, \alpha^\circ]]$	$1.2' - 2.1' \equiv 1.1 - 3.1$	$[[\alpha, \alpha^\circ], [\beta\alpha, \text{id}1]]$
$[[\beta\alpha, \text{id}1], [\alpha, \alpha^\circ\beta^\circ]]$	$1.3' - 2.1' \equiv 1.1 - 3.1$	$[[\alpha, \alpha^\circ\beta^\circ], [\beta\alpha, \text{id}1]]$
$[[\beta\alpha, \text{id}1], [\alpha, \text{id}2]]$	$1.2' - 2.2' \equiv 1.1 - 3.1$	$[[\alpha, \text{id}2], [\beta\alpha, \text{id}1]]$
$[[\beta\alpha, \text{id}1], [\alpha, \beta^\circ]]$	$1.3' - 2.2' \equiv 1.1 - 3.1$	$[[\alpha, \beta^\circ], [\beta\alpha, \text{id}1]]$
$[[\beta\alpha, \text{id}1], [\alpha, \text{id}3]]$	$1.3' - 2.3' \equiv 1.1 - 3.1$	$[[\alpha, \text{id}3], [\beta\alpha, \text{id}1]]$
$[[\beta\alpha, \alpha^\circ], [\alpha, \text{id}1]]$	$1.1' - 2.1' \equiv 1.2 - 3.1$	$[[\alpha, \text{id}1], [\beta\alpha, \alpha^\circ]]$
$[[\beta\alpha, \alpha^\circ], [\alpha, \alpha^\circ]]$	$1.2' - 2.1' \equiv 1.2 - 3.1$	$[[\alpha, \alpha^\circ], [\beta\alpha, \alpha^\circ]]$
$[[\beta\alpha, \alpha^\circ], [\alpha, \alpha^\circ\beta^\circ]]$	$1.3' - 2.1' \equiv 1.2 - 3.1$	$[[\alpha, \alpha^\circ\beta^\circ], [\beta\alpha, \alpha^\circ]]$
$[[\beta\alpha, \alpha^\circ], [\alpha, \text{id}2]]$	$1.2' - 2.2' \equiv 1.2 - 3.1$	$[[\alpha, \text{id}2], [\beta\alpha, \alpha^\circ]]$
$[[\beta\alpha, \alpha^\circ], [\alpha, \beta^\circ]]$	$1.3' - 2.2' \equiv 1.2 - 3.1$	$[[\alpha, \beta^\circ], [\beta\alpha, \alpha^\circ]]$

1.2-3.1 ≡ 1.3'-2.3'	[[βα, α°], [α, id3]]	1.3'-2.3' ≡ 1.2-3.1	[[α, id3], [βα, α°]]
1.3-3.1 ≡ 1.1'-2.1'	[[βα, α°β°], [α, id1]]	1.1'-2.1' ≡ 1.3-3.1	[[α, id1], [βα, α°β°]]
1.3-3.1 ≡ 1.2'-2.1'	[[βα, α°β°], [α, α°]]	1.2'-2.1' ≡ 1.3-3.1	[[α, α°], [βα, α°β°]]
1.3-3.1 ≡ 1.3'-2.1'	[[βα, α°β°], [α, α°β°]]	1.3'-2.1' ≡ 1.3-3.1	[[α, α°β°], [βα, α°β°]]
1.3-3.1 ≡ 1.2'-2.2'	[[βα, α°β°], [α, id2]]	1.2'-2.2' ≡ 1.3-3.1	[[α, id2], [βα, α°β°]]
1.3-3.1 ≡ 1.3'-2.2'	[[βα, α°β°], [α, β°]]	1.3'-2.2' ≡ 1.3-3.1	[[α, β°], [βα, α°β°]]
1.3-3.1 ≡ 1.3'-2.3'	[[βα, α°β°], [α, id3]]	1.3'-2.3' ≡ 1.3-3.1	[[α, id3], [βα, α°β°]]
1.2-3.2 ≡ 1.1'-2.1'	[[βα, id2], [α, id1]]	1.1'-2.1' ≡ 1.2-3.2	[[α, id1], [βα, id2]]
1.2-3.2 ≡ 1.2'-2.1'	[[βα, id2], [α, α°]]	1.2'-2.1' ≡ 1.2-3.2	[[α, α°], [βα, id2]]
1.2-3.2 ≡ 1.3'-2.1'	[[βα, id2], [α, α°β°]]	1.3'-2.1' ≡ 1.2-3.2	[[α, α°β°], [βα, id2]]
1.2-3.2 ≡ 1.2'-2.2'	[[βα, id2], [α, id2]]	1.2'-2.2' ≡ 1.2-3.2	[[α, id2], [βα, id2]]
1.2-3.2 ≡ 1.3'-2.2'	[[βα, id2], [α, β°]]	1.3'-2.2' ≡ 1.2-3.2	[[α, β°], [βα, id2]]
1.2-3.2 ≡ 1.3'-2.3'	[[βα, id2], [α, id3]]	1.3'-2.3' ≡ 1.2-3.2	[[α, id3], [βα, id2]]
1.3-3.2 ≡ 1.1'-2.1'	[[βα, β°], [α, id1]]	1.1'-2.1' ≡ 1.3-3.2	[[α, id1], [βα, β°]]
1.3-3.2 ≡ 1.2'-2.1'	[[βα, β°], [α, α°]]	1.2'-2.1' ≡ 1.3-3.2	[[α, α°], [βα, β°]]
1.3-3.2 ≡ 1.3'-2.1'	[[βα, β°], [α, α°β°]]	1.3'-2.1' ≡ 1.3-3.2	[[α, α°β°], [βα, β°]]
1.3-3.2 ≡ 1.2'-2.2'	[[βα, β°], [α, id2]]	1.2'-2.2' ≡ 1.3-3.2	[[α, id2], [βα, β°]]
1.3-3.2 ≡ 1.3'-2.2'	[[βα, β°], [α, β°]]	1.3'-2.2' ≡ 1.3-3.2	[[α, β°], [βα, β°]]
1.3-3.2 ≡ 1.3'-2.3'	[[βα, β°], [α, id3]]	1.3'-2.3' ≡ 1.3-3.2	[[α, id3], [βα, β°]]
1.3-3.3 ≡ 1.1'-2.1'	[[βα, id3], [α, id1]]	1.1'-2.1' ≡ 1.3-3.3	[[α, id1], [βα, id3]]
1.3-3.3 ≡ 1.2'-2.1'	[[βα, id3], [α, α°]]	1.2'-2.1' ≡ 1.3-3.3	[[α, α°], [βα, id3]]
1.3-3.3 ≡ 1.3'-2.1'	[[βα, id3], [α, α°β°]]	1.3'-2.1' ≡ 1.3-3.3	[[α, α°β°], [βα, id3]]
1.3-3.3 ≡ 1.2'-2.2'	[[βα, id3], [α, id2]]	1.2'-2.2' ≡ 1.3-3.3	[[α, id2], [βα, id3]]
1.3-3.3 ≡ 1.3'-2.2'	[[βα, id3], [α, β°]]	1.3'-2.2' ≡ 1.3-3.3	[[α, β°], [βα, id3]]
1.3-3.3 ≡ 1.3'-2.3'	[[βα, id3], [α, id3]]	1.3'-2.3' ≡ 1.3-3.3	[[α, id3], [βα, id3]]

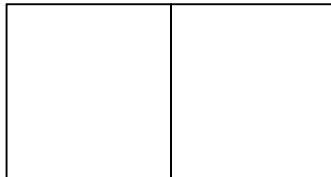
$M/I \equiv O'/I'$	$O'/I' \equiv M/I$
$1.1-3.1 \equiv 2.1'-3.1'$	$[[\beta\alpha, \text{id}1], [\beta, \text{id}1]] \quad 2.1'-3.1' \equiv 1.1-3.1$
$1.1-3.1 \equiv 2.2'-3.1'$	$[[\beta\alpha, \text{id}1], [\beta, \alpha^\circ]] \quad 2.2'-3.1' \equiv 1.1-3.1$
$1.1-3.1 \equiv 2.3'-3.1'$	$[[\beta\alpha, \text{id}1], [\beta, \alpha^\circ\beta^\circ]] \quad 2.3'-3.1' \equiv 1.1-3.1$
$1.1-3.1 \equiv 2.2'-3.2'$	$[[\beta\alpha, \text{id}1], [\beta, \text{id}2]] \quad 2.2'-3.2' \equiv 1.1-3.1$
$1.1-3.1 \equiv 2.3'-3.2'$	$[[\beta\alpha, \text{id}1], [\beta, \beta^\circ]] \quad 2.3'-3.2' \equiv 1.1-3.1$
$1.1-3.1 \equiv 2.3'-3.3'$	$[[\beta\alpha, \text{id}1], [\beta, \text{id}3]] \quad 2.3'-3.3' \equiv 1.1-3.1$
$1.2-3.1 \equiv 2.1'-3.1'$	$[[\beta\alpha, \alpha^\circ], [\beta, \text{id}1]] \quad 2.1'-3.1' \equiv 1.2-3.1$
$1.2-3.1 \equiv 2.2'-3.1'$	$[[\beta\alpha, \alpha^\circ], [\beta, \alpha^\circ]] \quad 2.2'-3.1' \equiv 1.2-3.1$
$1.2-3.1 \equiv 2.3'-3.1'$	$[[\beta\alpha, \alpha^\circ], [\beta, \alpha^\circ\beta^\circ]] \quad 2.3'-3.1' \equiv 1.2-3.1$
$1.2-3.1 \equiv 2.2'-3.2'$	$[[\beta\alpha, \alpha^\circ], [\beta, \text{id}2]] \quad 2.2'-3.2' \equiv 1.2-3.1$
$1.2-3.1 \equiv 2.3'-3.2'$	$[[\beta\alpha, \alpha^\circ], [\beta, \beta^\circ]] \quad 2.3'-3.2' \equiv 1.2-3.1$
$1.2-3.1 \equiv 2.3'-3.3'$	$[[\beta\alpha, \alpha^\circ], [\beta, \text{id}3]] \quad 2.3'-3.3' \equiv 1.2-3.1$
$1.3-3.1 \equiv 2.1'-3.1'$	$[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta, \text{id}1]] \quad 2.1'-3.1' \equiv 1.3-3.1$
$1.3-3.1 \equiv 2.2'-3.1'$	$[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta, \alpha^\circ]] \quad 2.2'-3.1' \equiv 1.3-3.1$
$1.3-3.1 \equiv 2.3'-3.1'$	$[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta, \alpha^\circ\beta^\circ]] \quad 2.3'-3.1' \equiv 1.3-3.1$



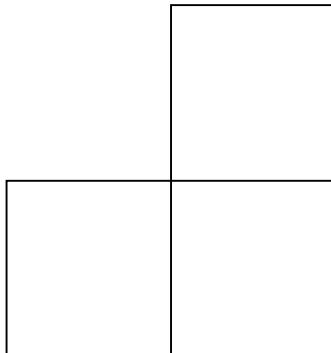
$1.2-3.2 \equiv 1.2'-3.1'$	$[[\beta\alpha, \text{id}2], [\beta\alpha, \alpha^\circ]]$	$1.2'-3.1' \equiv 1.2-3.2$	$[[\beta\alpha, \alpha^\circ], [\beta\alpha, \text{id}2]]$
$1.2-3.2 \equiv 1.3'-3.1'$	$[[\beta\alpha, \text{id}2], [\beta\alpha, \alpha^\circ\beta^\circ]]$	$1.3'-3.1' \equiv 1.2-3.2$	$[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta\alpha, \text{id}2]]$
$1.2-3.2 \equiv 1.2'-3.2'$	$[[\beta\alpha, \text{id}2], [\beta\alpha, \text{id}2]]$	$1.2'-3.2' \equiv 1.2-3.2$	$[[\beta\alpha, \text{id}2], [\beta\alpha, \text{id}2]]$
$1.2-3.2 \equiv 1.3'-3.2'$	$[[\beta\alpha, \text{id}2], [\beta\alpha, \beta^\circ]]$	$1.3'-3.2' \equiv 1.2-3.2$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \text{id}2]]$
$1.2-3.2 \equiv 1.3'-3.3'$	$[[\beta\alpha, \text{id}2], [\beta\alpha, \text{id}3]]$	$1.3'-3.3' \equiv 1.2-3.2$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \text{id}2]]$
$1.3-3.2 \equiv 1.1'-3.1'$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \text{id}1]]$	$1.1'-3.1' \equiv 1.3-3.2$	$[[\beta\alpha, \text{id}1], [\beta\alpha, \beta^\circ]]$
$1.3-3.2 \equiv 1.2'-3.1'$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \alpha^\circ]]$	$1.2'-3.1' \equiv 1.3-3.2$	$[[\beta\alpha, \alpha^\circ], [\beta\alpha, \beta^\circ]]$
$1.3-3.2 \equiv 1.3'-3.1'$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \alpha^\circ\beta^\circ]]$	$1.3'-3.1' \equiv 1.3-3.2$	$[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta\alpha, \beta^\circ]]$
$1.3-3.2 \equiv 1.2'-3.2'$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \text{id}2]]$	$1.2'-3.2' \equiv 1.3-3.2$	$[[\beta\alpha, \text{id}2], [\beta\alpha, \beta^\circ]]$
$1.3-3.2 \equiv 1.3'-3.2'$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \beta^\circ]]$	$1.3'-3.2' \equiv 1.3-3.2$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \beta^\circ]]$
$1.3-3.2 \equiv 1.3'-3.3'$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \text{id}3]]$	$1.3'-3.3' \equiv 1.3-3.2$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \beta^\circ]]$
$1.3-3.3 \equiv 1.1'-3.1'$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \text{id}1]]$	$1.1'-3.1' \equiv 1.3-3.3$	$[[\beta\alpha, \text{id}1], [\beta\alpha, \text{id}3]]$
$1.3-3.3 \equiv 1.2'-3.1'$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \alpha^\circ]]$	$1.2'-3.1' \equiv 1.3-3.3$	$[[\beta\alpha, \alpha^\circ], [\beta\alpha, \text{id}3]]$
$1.3-3.3 \equiv 1.3'-3.1'$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \alpha^\circ\beta^\circ]]$	$1.3'-3.1' \equiv 1.3-3.3$	$[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta\alpha, \text{id}3]]$
$1.3-3.3 \equiv 1.2'-3.2'$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \text{id}2]]$	$1.2'-3.2' \equiv 1.3-3.3$	$[[\beta\alpha, \text{id}2], [\beta\alpha, \text{id}3]]$
$1.3-3.3 \equiv 1.3'-3.2'$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \beta^\circ]]$	$1.3'-3.2' \equiv 1.3-3.3$	$[[\beta\alpha, \beta^\circ], [\beta\alpha, \text{id}3]]$
$1.3-3.3 \equiv 1.3'-3.3'$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \text{id}3]]$	$1.3'-3.3' \equiv 1.3-3.3$	$[[\beta\alpha, \text{id}3], [\beta\alpha, \text{id}3]]$

6. In order to conclude, we show here a few basic pre-semiotic sign-configurations, which are to be compared to the semiotic sign-configurations in Toth (2008b, pp. 62 ss.):

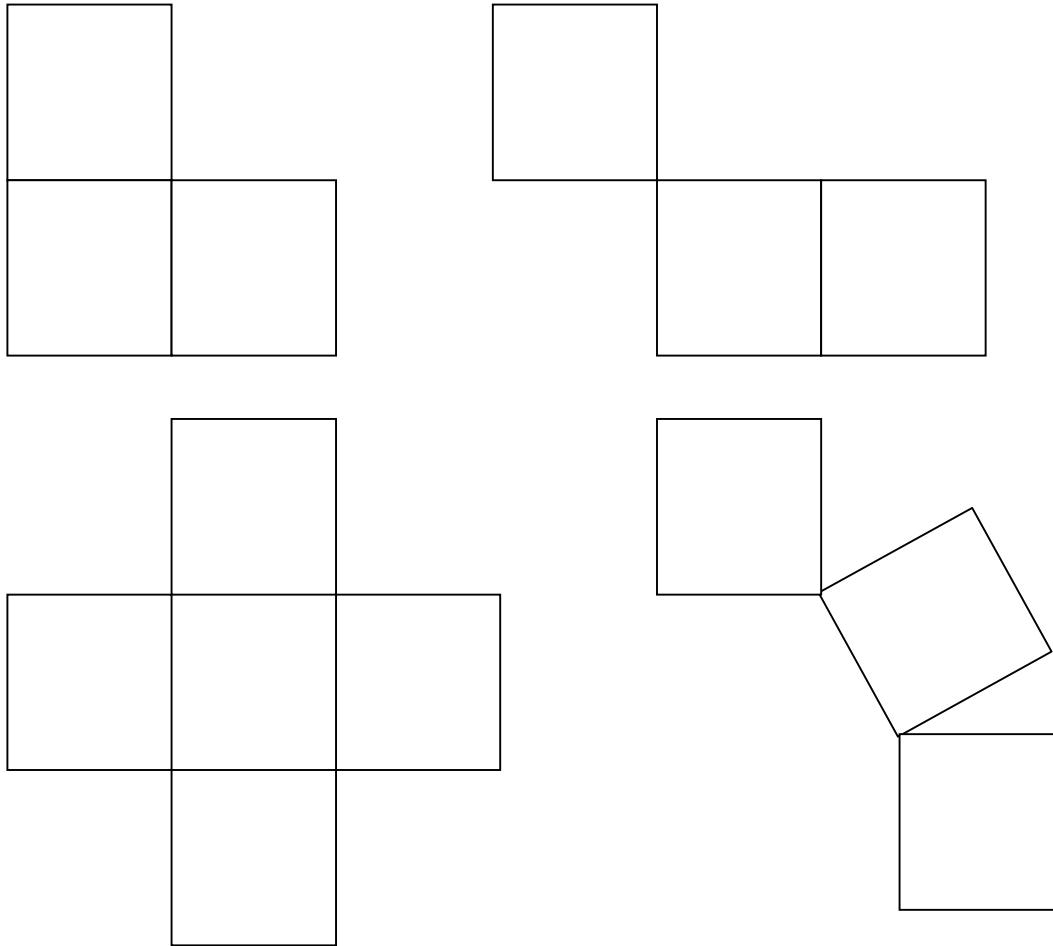
6.1. Type 1: Sign connections are pairs of dyadic sub-signs, i.e. the squares hang together by 2 vertices and 1 edge:



6.2. Type 2: Sign connections are single sub-signs, i.e. the squares hang together by 1 vertex and 0 edges:



6.3. Composite types: Sign connections are pairs of sub-signs as well as single sub-signs, i.e. the squares hang together by > 1 vertices and >3 edges. The configurations include both orthogonal and rotational connections (cf. Toth 2008f):



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