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## **Fundamentals for a general sign grammar of pre-semiotics**

1. The present study continues my book “Entwurf einer allgemeinen Zeichengrammatik/Outlines of a General Sign Grammar” (Toth 2008b), which is based on the previous works of Schnelle (1962), Bense (1975, pp. 78 ss.), and Stiebing (1978). As suggested in the title, at this place, we are interested in establishing a general framework of sign grammar for pre-semiotics, introduced in Toth (2008c, d, e) and other works. Especially, we shall focus on the interconnections between semiotic and ontological space (Bense 1975, p. 65) and its modeling in a semiotic-pre-semiotic sign grammar.

2. The pre-semiotic sign is a tetradic relation consisting of the four part-relations

$$(0), (0 \Rightarrow 1), ((0 \Rightarrow 1) \Rightarrow 2), (0 \Rightarrow 1 \Rightarrow 2 \Rightarrow 3)$$

i.e., it is a relation over a monadic, a dyadic, a triadic, and a tetradic relation; generally:

$$SR = (a, (a \Rightarrow b), ((a \Rightarrow b) \Rightarrow c), (a \Rightarrow b \Rightarrow c \Rightarrow d))$$

The possible sign values for a, b, and c, or 1, 2, and 3 are obtained by Cartesian multiplication of the four possible pre-semiotic prime-signs (0., 1., 2., 3.) in the rows and the three possible pre-semiotic prime-signs (.1, .2, .3) in the columns, as displayed in the pre-semiotic matrix:

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

In doing so, one gets the following sets of values for the four part-relations:

$$a = \{0.1, 0.2, 0.3\}$$

$$b = \{1.1, 1.2, 1.3\}$$

$$c = \{2.1, 2.2, 2.3\}$$

$$d = \{3.1, 3.2, 3.3\}$$

However, the pre-semiotic sign model as an extension of the Peircean sign model requires that a semiotic value be selected out of each of the four sets of values a, b, c, d and that the sign relation SR be ordered according to the following scheme of tetradicity:

SR = <3.w, 2.x, 1.y, 0.z> with w, x, y, z ∈ {1, 2, 3}

with special respect to the pre-semiotic inclusion order

$$w \leq x \leq y \leq z$$

By aid of these two constraints, the  $4^9 = 262144$  possible sign relations are reduced to the following 15 pre-semiotic sign classes:

1	(3.1 2.1 1.1 0.1)	9	(3.1 2.2 1.3 0.3)
2	(3.1 2.1 1.1 0.2)	10	(3.1 2.3 1.3 0.3)
3	(3.1 2.1 1.1 0.3)	11	(3.2 2.2 1.2 0.2)
4	(3.1 2.1 1.2 0.2)	12	(3.2 2.2 1.2 0.3)
5	(3.1 2.1 1.2 0.3)	13	(3.2 2.2 1.3 0.3)
6	(3.1 2.1 1.3 0.3)	14	(3.2 2.3 1.3 0.3)
7	(3.1 2.2 1.2 0.2)	15	(3.3 2.3 1.3 0.3)
8	(3.1 2.2 1.2 0.3)		

Thus, the abstract sign scheme underlying these 15 pre-semiotic sign classes can be noted as follows:

$$SR = (\square\square\square \square\square\square \square\square\square \square\square\square)$$

The four empty patterns of three variables are to be ordered according to decreasing indicated sign values (3.3, 3.2, 3.1; 2.3, 2.2, 2.1; 1.3, 1.2, 1.1; 0.3, 0.2, 0.1). Thus, according to the tetradicity principle, in the first 3-variables-pattern, a sign value from the set c = (3.1, 3.2, 3.3), in the second 3-variables-pattern, a sign value from the set b = (2.1, 2.2, 2.3), in the third 3-variables-pattern, a sign value from the set c = (1.1, 1.2, 1.3), and in the fourth 3-variables-pattern, a sign value from the set d = (0.1, 0.2, 0.3) has to be chosen. Note that the choice of the sign value from the set d depends on the choices for the sign values from the sets c, b, and a; the choice for c depends on b, and a, and the choice for b depends on a. In the abstract scheme, we therefore must and are allowed to assign four empty places by (■). In doing so, by aid of the sign scheme, the 15 pre-semiotic sign classes can be displayed as follows:

1	(3.1 2.1 1.1 0.1) = (□□■ □□■ □□■ □□■)
2	(3.1 2.1 1.1 0.2) = (□□■ □□■ □□■ □■□)
3	(3.1 2.1 1.1 0.3) = (□□■ □□■ □□■ ■□□)
4	(3.1 2.1 1.2 0.2) = (□□■ □□■ □■□ □■□)
5	(3.1 2.1 1.2 0.3) = (□□■ □□■ □■□ ■□□)
6	(3.1 2.1 1.3 0.3) = (□□■ □□■ ■□□ ■□□)
7	(3.1 2.2 1.2 0.2) = (□□■ □■□ □■□ □■□)
8	(3.1 2.2 1.2 0.3) = (□□■ □■□ □■□ ■□□)
9	(3.1 2.2 1.3 0.3) = (□□■ □■□ ■□□ ■□□)
10	(3.1 2.3 1.3 0.3) = (□□■ ■□□ ■□□ ■□□)
11	(3.2 2.2 1.2 0.2) = (□■□ □■□ □■□ □■□)
12	(3.2 2.2 1.2 0.3) = (□■□ □■□ □■□ ■□□)

- 13 (3.2 2.2 1.3 0.3) = (□■□ □■□ ■□□ ■□□)
- 14 (3.2 2.3 1.3 0.3) = (□■□ ■□□ ■□□ ■□□)
- 15 (3.3 2.3 1.3 0.3) = (■□□ ■□□ ■□□ ■□□)

In the following, we will use sign schemes – abstract one as well as assigned ones – in order to show how the semiotic operators work.

### 3.1. Bense (1971, S. 34) defined the following semiotic operators

o := (M ⇒ O) and  
 i := (O ⇒ I)

In addition to these two operators, a third one was introduced later: “A clear distinction between the designation function and the determination function, thus (M ⇒ O) and (O ⇒ I), allows to introduce the relation (I ⇒ M) as application function (a)” (Walther 1979, pp. 72s.):

a := (I ⇒ M)

In pre-semiotics, however, we have to introduce the following operator, which we will call “qualification”, and abbreviate it by m:

m: (Q ⇒ M)

Moreover, besides a := (I ⇒ M), there is a “contextualization function” c:

c := (I ⇒ Q).

Unlike the semiotic functions o, i, and a, the functions m and c are bridging functions between pre-signs and signs, or between semiotic and onotological spaces.

Besides these semiotic and pre-semiotic-semiotic operators, which are usually called “functions”, there are, according to Walther (1979, pp. 116 ss.) 9 more operators which apply both to semiotics and pre-semiotics.

### 3.2. Substitutor (/)

Example: (3.1 2.1 1.1 0.1) / (0.1/0.3) ≡  
 (□□■ □□■ □□■ □□■) / (0.1/0.3) = (□□■ □□■ □□■ ■□□)

### 3.3. Selector (>)

Example: (3.1 2.1 1.1 0.3), (1.1) > (1.2) ≡  
 (□□■ □□■ □□■ ■□□), (1.1) > (1.2) (□□■ □□■ □□□ ■□□)

Bense (1981, p. 108) still differentiated between separative ( $/$ ), abstractive ( $>$ ) and associative (X) selection. The first kind of selection is restricted to the medium relation, the second to the object relation, and the third to the interpretant relation of the triadic sign relation. In addition, we may introduce the “differentiating” selection operator, which works on the level of pre-semiotic quality. Note that all four operators apply only on trichotomies.

### 3.4. Coordinator ( $| \rightarrow$ )

Example: (2.1)  $| \rightarrow$  (1.1)

$$(\square\square\square\square\square\square\square\square\square\square), | \rightarrow (2.1, 1.1) = (\square\square\square\square\square\square\square\square\square\square)$$

Bense (1983, p. 57) further differentiates between founding ( $| \rightarrow$ ), reflexive ( $\leftrightarrow$ ), and analogue ( $> \rightarrow$ ) coordinator. In addition, we may introduce the “availability” coordinator, which works on the qualitative pre-semiotic level and coordinates between zeroness and firstness.

### 3.5. Creator (realizator) ( $>>$ )

Example: 3.1  
 $\wedge > 1.2$

$$0.2 \\ >> (0.2, 3.1) = (1.2) \equiv \\ >> ((\square\square\square\square\square\square\square\square\square\square), (\square\square\square\square\square\square\square\square\square\square)) = (\square\square\square\square\square\square\square\square\square\square),$$

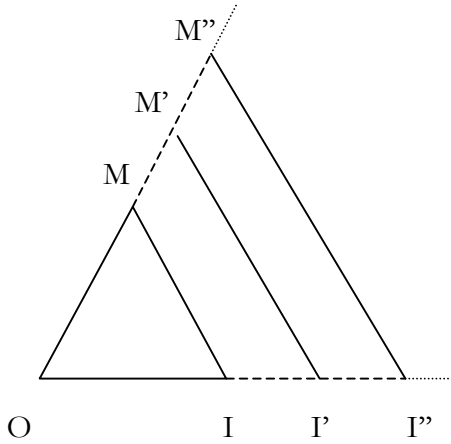
which means that an interpreting consciousness (3.1) selects from the available pre-semiotic qualities (1.2) in order to create or realize a medium (1.2).

### 3.6. Adjunctive ( $\cup$ )

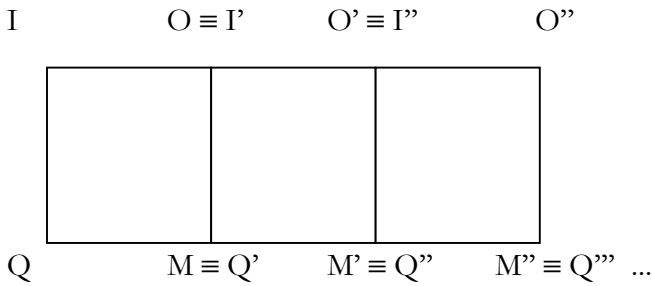
Example: (3.1 2.1 1.1 0.1)  $\cup$  (3.1 2.1 1.2 0.2)  $\cup$  ...  
 $(\square\square\square\square\square\square\square\square\square\square) \cup (\square\square\square\square\square\square\square\square\square\square) \cup \dots$

“Adjunction is a sign operation with serial, concatenating character” (Bense and Walther 1973, p. 11).

Display of an adjunction after Bense (1971, p. 53):



Using the tetradic-trichotomic pre-semiotic square sign model, we can display pre-semiotic adjunction as follows:

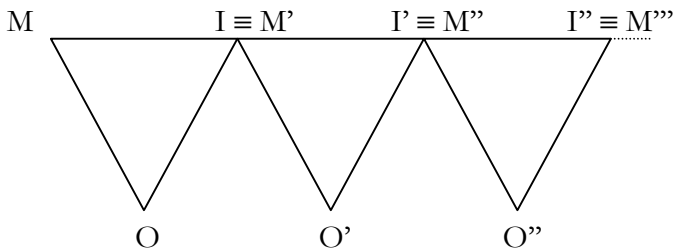


### 3.7. Superizator ( $\cap$ )

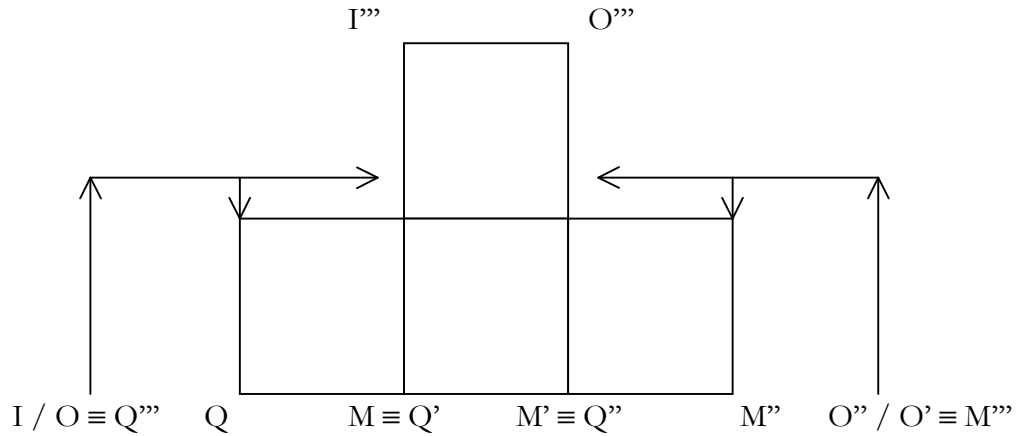
Example:  $(3.1\ 2.1\ 1.1\ 0.1) \cap (3.1\ 2.1\ 1.2\ 0.2) \cap \dots$   
 $(\square\square\square \square\square\square \square\square\square \square\square\square) \cap (\square\square\square \square\square\square \square\square\square \square\square\square) \cap \dots$

“Superization is a sign process in the sense of the comprising wholeness formation of a set of single signs to a gestalt, a structure, or a configuration” (Bense and Walther 1973, p. 106).

Display of a superization after Bense (1971, p. 54):







### 3.9 Thetic introduction ( $\vdash$ )

Note that only a sign relation with categorial number  $> 0$  can be thetically introduced (cf. Bense 1975, p. 65; Toth 2008c). Thus, only the triadic part-relation of the pre-semiotic sign relation  $PSR = (3.a\ 2.b\ 1.c\ 0.d)$  is thetically introduced.

Example:  $\vdash (2.1)$   
 $\vdash (2.1) (\square\square\square\square\square\square\square\square) = (\square\square\square\square\square\square\square\square)$

### 3.10 Autoreproductor ( $\lrcorner$ )

Example:  $(2.3) \lrcorner (2.3)$   
 $(2.3) \lrcorner (2.3) (\square\square\square\square\square\square\square\square) = (\square\square\square\square\square\square\square\square)$

Bense does not mention the dualizor, which Bense (1976, pp. 53 ss.) had introduced and which maps a sign class onto a reality thematic, amongst the semiotic operators.

### 3.11 Dualizor ( $\times$ )

Because of the asymmetry between tetrads and trichtomies in sign classes, and triads and tetratomies in reality thematics, we need a special new reality scheme in order to show a dualized sign class. The reason is that (1.0), (2.0), and (3.0) are not defined in sign classes, and that (0.1), (0.2), (0.3) are not defined in reality thematics, due to the non-quadratic matrix of  $SR_{4,3}$ . In order to construct a reality scheme, we proceed in the same way as we did for sign schemes, i.e. we order the variables for sub-signs in decreasing order.

Example:  $(3.1\ 2.1\ 1.1\ 0.1) \times (1.0\ 1.1\ 1.2\ 1.3)$   
 $(\square\square\square\square\square\square\square\square) \times (\square\square\square\square\square\square\square\square)$

- 1 (3.1 2.1 1.1 0.1) ≡ (□□■ □□■ □□■ □□■) × (□□□□ □□□□ ■■■■ □□□) ≡  
(1.0 1.1 1.2 1.3)
- 2 (3.1 2.1 1.1 0.2) ≡ (□□■ □□■ □□■ □□□) × (□□□□ □□□■ ■■■■ □□□) ≡  
(2.0 1.1 1.2 1.3)
- 3 (3.1 2.1 1.1 0.3) ≡ (□□■ □□■ □□■ ■□□) × (□□□■ □□□□ ■■■■ □□□) ≡  
(3.0 1.1 1.2 1.3)
- 4 (3.1 2.1 1.2 0.2) ≡ (□□■ □□■ □□□ □□□) × (□□□□ □□■ ■■■■ □□□) ≡  
(2.0 2.1 1.2 1.3)
- 5 (3.1 2.1 1.2 0.3) ≡ (□□■ □□■ □□□ ■□□) × (□□□■ □□□□ ■■■■ □□□) ≡  
(3.0 2.1 1.2 1.3)
- 6 (3.1 2.1 1.3 0.3) ≡ (□□■ □□■ ■□□ ■□□) × (□□■ ■□□□ ■■■■ □□□) ≡  
(3.0 3.1 1.2 1.3)
- 7 (3.1 2.2 1.2 0.2) ≡ (□□■ □□□ □□□ □□□) × (□□□□ □■ ■ ■□□□ □□□) ≡  
(2.0 2.1 2.2 1.3)
- 8 (3.1 2.2 1.2 0.3) ≡ (□□■ □□□ □□□ ■□□) × (□□□■ □■ ■ ■□□□ □□□) ≡  
(3.0 2.1 2.2 1.3)
- 9 (3.1 2.2 1.3 0.3) ≡ (□□■ □□□ ■□□ ■□□) × (□□■ ■□□□ ■■■■ □□□) ≡  
(3.0 3.1 2.2 1.3)
- 10 (3.1 2.3 1.3 0.3) ≡ (□□■ ■□□ ■□□ ■□□) × (□■ ■ ■□□□ ■■■■ □□□) ≡  
(3.0 3.1 3.2 1.3)
- 11 (3.2 2.2 1.2 0.2) ≡ (□■□ □■□ □■□ □■□) × (□□□□ ■■■■ □□□□ □□□) ≡  
(2.0 2.1 2.2 2.3)
- 12 (3.2 2.2 1.2 0.3) ≡ (□■□ □■□ □■□ ■□□) × (□□□■ ■■■■ □□□□ □□□) ≡  
(3.0 2.1 2.2 2.3)
- 13 (3.2 2.2 1.3 0.3) ≡ (□■□ □■□ ■□□ ■□□) × (□□■ ■■■■ □□□□ □□□) ≡  
(3.0 3.1 2.2 2.3)
- 14 (3.2 2.3 1.3 0.3) ≡ (□■□ ■□□ ■□□ ■□□) × (□■ ■ ■□□□ □□□□ □□□) ≡  
(3.0 3.1 3.2 2.3)
- 15 (3.3 2.3 1.3 0.3) ≡ (■□□ ■□□ ■□□ ■□□) × (■■■■ □□□□ □□□□ □□□) ≡  
(3.0 3.1 3.2 3.3)

### 3.12 Carry-on (Mitführung)

“Mitführung (carry-on) means that the ‘presentamen’ remains present gradually or partly in the ‘representamen’ (Bense 1979, p. 43). Thus, this operation of the pre-semiotic neverland between kenogrammatics and semiotics refers to the “thinning” of the world of objects on the one side and to the poly-affinity of sign classes and reality thematics on the other side (cf. Toth 2008a, pp. 166 ss.).



### 3.13. Additive Association

“Starting with the two configurations of the fundamental categorial three-digit order relations:

$$\begin{array}{ccc} 3. & 2. & 1. \\ .1 & .2 & .3 \end{array}$$

one gains by additive association the order of the sub-signs of the diagonal dual-invariant sign class-reality thematics (3.1 2.2 1.3)” (Bense 1981, p. 204). Displayed by aid of a structural sign scheme:

$$((\blacksquare \square \blacksquare \square), (\square \blacksquare \square \blacksquare)) = (\blacksquare \blacksquare \blacksquare \blacksquare) \approx (\square \square \square \square \blacksquare \square \square \square)$$

In the following, we introduce some more semiotic and pre-semiotic operators, which had been used for polycontextural semiotics (cf. Toth 2003, pp. 36 ss.):

### 3.14. Abolishment

Symbol:  $L_i$ : Abolishment of position  $i$   
 Example:  $L_1 (3.1 \ 2.2 \ 1.3 \ 0.3) = (\emptyset.1 \ 2.2 \ 1.3 \ 0.3)$   
 $L_1 (\blacksquare \blacksquare \blacksquare \blacksquare) = (\square \blacksquare \blacksquare \blacksquare)$

### 3.15. Assignment

Symbol:  $B_{ik}$ : Assignment of position  $i$  with value  $k$   
 Example:  $B_{22} (3.\emptyset \ 2.2 \ 1.3 \ 0.3) = (3.2 \ 2.2 \ 1.3 \ 0.3)$   
 $B_{22} (\blacksquare \square \blacksquare \blacksquare) = (\blacksquare \blacksquare \blacksquare \blacksquare)$

### 3.16. Nulling

Symbol:  $N_i$ : Nulling of position  $i$   
 Example:  $N_5 (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.2 \ \emptyset.3 \ 0.3)$   
 $N_5 (\blacksquare \blacksquare \blacksquare \blacksquare) = (\blacksquare \blacksquare \square \blacksquare)$

### 3.17. Maximization

Symbol:  $Max_i$ : Maximizing of position  $i$   
 Example:  $Max_4 (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.3 \ 1.3 \ 0.3)$   
 $Max_4 (\square \square \square \square) = (\square \square \square \blacksquare)$

### 3.18. Minimization

Symbol:  $Min_i$ : Minimizing of position  $i$   
 Example:  $Min_4 (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.1 \ 1.3 \ 0.3)$   
 $Min_4 (\square \square \square \square) = (\square \square \square \square)$

3.19. Assignment changing

Symbol:  $w_{ik}$ : Assignment changing  $w_i \rightarrow k$

Example:  $w_{22} (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.2 \ 2.2 \ 1.3 \ 0.3)$   
 $w_{22} (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square)$

3.20. Transposition

Symbol:  $T_{ik}$ : Transposition of  $w_i$  and  $w_k$

Example:  $w_{23} (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.2 \ 1.2 \ 1.3 \ 0.3)$   
 $w_{23} (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square)$

Permutation is an m-digit transposition:

Example:  $w_{312111} (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.1 \ 1.1 \ 0.3)$   
 $w_{312111} (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square)$

3.21. Reflexion

Symbol:  $R_{\square\square\square\dots}$ : Part-reflexion of all positions, marked by i

Example:  $R_{\square\square\square\dots} (3.1 \ 2.2 \ 1.3 \ 0.3) = *(3.1 \ 2.3 \ 1.2 \ 0.3)$  (irregular)  
 $R_{\square\square\square\dots} (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = *(\square\square\square \ \blacksquare\square\square \ \square\square\square \ \blacksquare\square\square)$

An m-digit reflexion  $R_m$  is a total reflexion:

Examples:  $R_6 (3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1 \ 2.2 \ 1.3 \ 03)$ ;  $R_6 (3.1 \ 2.1 \ 1.1 \ 0.3) = (1.1 \ 1.2 \ 1.3 \ 0.3)$ .  
 $R_6 (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square)$ ;  
 $R_6 (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) = (\square\square\square \ \square\square\square \ \blacksquare\square\square \ \blacksquare\square\square)$

Thus, the total reflector is identical with the dualizer introduced in 3.11. Hence, only the dual-identical triadic part-relation of the pre-semiotic sign class (3.1 2.2 1.3 0.3) is mapped onto itself by  $R_m$ .

Another form of reflexion, which we shall call mirroring, we get, if we do not start with the numerical form of the sign classes, but with their corresponding sign schemes. We shall mark the mirroring operator by “—”:

- 1  $(3.1 \ 2.1 \ 1.1 \ 0.1) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \text{ — } (\blacksquare\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \equiv$   
 $(3.3 \ 3.0 \ 2.1 \ 2.0 \ 1.2)$
- 2  $(3.1 \ 2.1 \ 1.1 \ 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \text{ — } (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \equiv$   
 $(3.2 \ 3.0 \ 2.1 \ 1.2)$
- 3  $(3.1 \ 2.1 \ 1.1 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) \text{ — } (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 3.0 \ 2.1 \ 1.2)$
- 4  $(3.1 \ 2.1 \ 1.2 \ 0.2) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) \text{ — } (\square\square\square \ \blacksquare\square\square \ \square\square\square \ \square\square\square) \equiv$   
 $(3.2 \ 2.3 \ 2.1 \ 1.2)$
- 5  $(3.1 \ 2.1 \ 1.2 \ 0.3) \equiv (\square\square\square \ \square\square\square \ \square\square\square \ \blacksquare\square\square) \text{ — } (\square\square\square \ \blacksquare\square\square \ \square\square\square \ \square\square\square) \equiv$   
 $(3.1 \ 2.3 \ 2.1 \ 1.2)$

- 6 (3.1 2.1 1.3 0.3)  $\equiv$  (□□■ □□■ ■□□ ■□□) — (□□■□ □■□□ □■□□ □□□)  $\equiv$   
(3.1 2.2 2.1 1.2)
- 7 (3.1 2.2 1.2 0.2)  $\equiv$  (□□■ □□□ □■□ □■□) — (□■□□ ■□□■ □■□□ □□□)  $\equiv$   
(3.2 2.3 2.0 1.2)
- 8 (3.1 2.2 1.2 0.3)  $\equiv$  (□□■ □□□ □■□ ■□□) — (□□■□ ■□□■ □■□□ □□□)  $\equiv$   
(3.1 2.3 2.0 1.2)
- 9 (3.1 2.2 1.3 0.3)  $\equiv$  (□□■ □□□ ■□□ ■□□) — (□□■□ □■□■ □■□□ □□□)  $\equiv$   
(3.1 2.2 2.0 1.2)
- 10 (3.1 2.3 1.3 0.3)  $\equiv$  (□□■ ■□□ ■□□ ■□□) — (□□■□ □■□□ ■■□□ □□□)  $\equiv$   
(3.1 2.2 1.3 1.2)
- 11 (3.2 2.2 1.2 0.2)  $\equiv$  (□■□ □■□ □■□ □■□) — (□■□□ ■□□■ □□■□ □□□)  $\equiv$   
(3.2 2.3 2.0 1.1)
- 12 (3.2 2.2 1.2 0.3)  $\equiv$  (□■□ □■□ □■□ ■□□) — (□□■□ ■□□■ □□■□ □□□)  $\equiv$   
(3.1 2.3 2.0 1.1)
- 13 (3.2 2.2 1.3 0.3)  $\equiv$  (□■□ □■□ ■□□ ■□□) — (□□■□ □■□■ □□■□ □□□)  $\equiv$   
(3.1 2.2 2.0 1.1)
- 14 (3.2 2.3 1.3 0.3)  $\equiv$  (□■□ ■□□ ■□□ ■□□) — (□□■□ □■□□ ■□□□ □□□)  $\equiv$   
(3.1 2.2 1.3 1.1)
- 15 (3.3 2.3 1.3 0.3)  $\equiv$  (■□□ ■□□ ■□□ ■□□) — (□□■□ □■□□ ■□□■ □□□)  $\equiv$   
(3.1 2.2 1.3 1.0)

Therefore, by mirror regular pre-semiotic sign classes, we get exclusively irregular ones, while mirroring regular semiotic classes leads to exclusively regular ones; cf. Toth 2008b, p. 18). Since in the latter system (SS10), mirroring operation is identical with symplerosis (cf. Toth 2007, p. 45), it follows, that in pre-semiotics, mirroring is not identical with any group theoretic binary operation.

### 3.22. Addition

Symbol: +

Example: (3.1 2.2 1.3 0.3) + (3.2 2.2 1.3 0.3) = (3.2 2.2 1.3 0.3)  
(□□■ □■□ ■□□ ■□□) + (□■□ □■□ ■□□ ■□□) = (□■□ □■□ ■□□ ■□□)

Thus, addition is identical with lattice-theoretic union (cf. Toth 2007, pp. 71 ss.).

### 3.23. Subtraction

Symbol: –

Example: (3.2 2.3 1.3 0.3) – (3.2 2.2 1.3 0.3) = (3.1 2.2 1.3 0.3)  
(□■□ ■□□ ■□□ ■□□) – (□■□ □■□ ■□□ ■□□) = (□□■ □■□ ■□□ ■□□)

Thus, subtraction is identical with lattice-theoretical intersection (cf. Toth 2007, pp. 71 ss.).

### 3.24. Splitting

Symbol:  $Z_{mi,j} = Z(\cap_i \cap_j)$ : Splitting in two part of lengths  $i$  and  $j$ ;  $i + j = m$

Example:  $Z_{2,4}(3.1 \ 2.2 \ 1.3 \ 0.3) = (3.1); (2.2 \ 1.3 \ 0.3)$

$Z_{2,4}(\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) = (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square); (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square)$

$Z_m$  is the splitting in merely single parts of length 1.

Example:  $Z_6(3.1 \ 2.2 \ 1.3 \ 0.3) = 3; 1; 2; 2; 1; 3; 0; 3$

$Z_6(\blacksquare\blacksquare\blacksquare \ \blacksquare\blacksquare\blacksquare \ \blacksquare\blacksquare\blacksquare) = (\blacksquare_3); (\blacksquare_1); (\blacksquare_2); (\blacksquare_2); (\blacksquare_1); (\blacksquare_3); (\blacksquare_0); (\blacksquare_3)$

Thus, total splitting is the operation which is the basis of the semiotic catastrophe, introduced by Arin (1981, pp. 328 ss.).

### 3.25. Normal-form Operator

By aid of normal-form operators ( $N_i$ ), irregular sign classes can be transformed into regular ones. Since a pre-semiotic sign class is regular, if  $(3.a \leq 2.b \leq 1.c \leq 0.d)$  where  $a, b, c \in \{1, 2, 3\}$  normal-form operators are mostly ambiguous.

Examples:  $N^*(3.2 \ 2.1 \ 1.3 \ 0.3) = (3.1 \ 2.1 \ 1.3 \ 0.3), (3.2 \ 2.2 \ 1.2 \ 0.3), (3.2 \ 2.2 \ 1.3 \ 0.3)$  or  $(3.2 \ 2.3 \ 1.3 \ 0.3)$ ;

but cf.  $N^*(3.3 \ 2.1 \ 1.1 \ 0.3) = N^*(3.3 \ 2.1 \ 1.2 \ 0.3) = \dots = N(3.2 \ 2.2 \ 1.3 \ 0.3) = \dots = N^*(3.3 \ 2.3 \ 1.2 \ 0.3) = (3.3 \ 2.3 \ 1.3 \ 0.3)$

$N^*(\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) = (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square), (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square), (\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square)$  or  $(\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square)$ ;

but cf.  $N^*(\blacksquare\square\square \ \square\square\square \ \square\square\square \ \square\square\square) = N^*(\blacksquare\square\square \ \square\square\square \ \square\square\square \ \square\square\square) = \dots = N(\square\square\square \ \square\square\square \ \square\square\square \ \square\square\square) = \dots = N^*(\blacksquare\square\square \ \square\square\square \ \square\square\square \ \square\square\square) = (\blacksquare\square\square \ \square\square\square \ \square\square\square \ \square\square\square)$

4. In this chapter, we want to have a look at the pre-semiotic and semiotic sign connections achieved by the operators introduced in chapter 3. First, we shall show the monadic pre-semiotic sign connections:

<b>Q ≡ Q'</b>		<b>Q' ≡ Q</b>	
0.1 ≡ 0.1'	⇔	[id0, id1]	0.1' ≡ 0.1 ⇔ [id0, id1]
0.2 ≡ 0.1'	⇔	[id0, α°]	0.1' ≡ 0.2 ⇔ [id0, α]
0.3 ≡ 0.1'	⇔	[id0, α°β°]	0.1' ≡ 0.3 ⇔ [id0, βα]
0.2 ≡ 0.2'	⇔	[id0, id2]	0.2' ≡ 0.2 ⇔ [id0, id2]
0.3 ≡ 0.2'	⇔	[id0, β°]	0.2' ≡ 0.3 ⇔ [id0, β]
0.3 ≡ 0.3'	⇔	[id0, id3]	0.3' ≡ 0.3 ⇔ [id0, id3]

<b>Q ≡ M'</b>		<b>M' ≡ Q</b>	
0.1 ≡ 1.1'	⇔	[γ, id1]	1.1' ≡ 0.1 ⇔ [γ°, id1]
0.2 ≡ 1.1'	⇔	[γ, α°]	1.1' ≡ 0.2 ⇔ [γ°, α]
0.3 ≡ 1.1'	⇔	[γ, α°β°]	1.1' ≡ 0.3 ⇔ [γ°, βα]
0.1 ≡ 1.2'	⇔	[γ, α]	1.2' ≡ 0.1 ⇔ [γ°, α°]

$0.2 \equiv 1.2'$	$\Leftrightarrow$	$[\gamma, \text{id}2]$	$1.2' \equiv 0.2$	$\Leftrightarrow$	$[\gamma^\circ, \text{id}2]$
$0.3 \equiv 1.2'$	$\Leftrightarrow$	$[\gamma, \beta^\circ]$	$1.2' \equiv 0.3$	$\Leftrightarrow$	$[\gamma^\circ, \beta]$
$0.1 \equiv 1.3'$	$\Leftrightarrow$	$[\gamma, \beta\alpha]$	$1.3' \equiv 0.1$	$\Leftrightarrow$	$[\gamma^\circ, \alpha^\circ\beta^\circ]$
$0.2 \equiv 1.3'$	$\Leftrightarrow$	$[\gamma, \beta]$	$1.3' \equiv 0.2$	$\Leftrightarrow$	$[\gamma^\circ, \beta^\circ]$
$0.3 \equiv 1.3'$	$\Leftrightarrow$	$[\gamma, \text{id}3]$	$1.3' \equiv 0.3$	$\Leftrightarrow$	$[\gamma^\circ, \text{id}3]$

**Q  $\equiv$  O'**

$0.1 \equiv 2.1'$	$\Leftrightarrow$	$[\delta, \text{id}1]$
$0.2 \equiv 2.1'$	$\Leftrightarrow$	$[\delta, \alpha^\circ]$
$0.3 \equiv 2.1'$	$\Leftrightarrow$	$[\delta, \alpha^\circ\beta^\circ]$
$0.1 \equiv 2.2'$	$\Leftrightarrow$	$[\delta, \alpha]$
$0.2 \equiv 2.2'$	$\Leftrightarrow$	$[\delta, \text{id}2]$
$0.3 \equiv 2.2'$	$\Leftrightarrow$	$[\delta, \beta^\circ]$
$0.1 \equiv 2.3'$	$\Leftrightarrow$	$[\delta, \beta\alpha]$
$0.2 \equiv 2.3'$	$\Leftrightarrow$	$[\delta, \beta]$
$0.3 \equiv 2.3'$	$\Leftrightarrow$	$[\delta, \text{id}3]$

**O'  $\equiv$  Q**

$2.1' \equiv 0.1$	$\Leftrightarrow$	$[\delta^\circ, \text{id}1]$
$2.1' \equiv 0.2$	$\Leftrightarrow$	$[\delta^\circ, \alpha]$
$2.1' \equiv 0.3$	$\Leftrightarrow$	$[\delta^\circ, \beta\alpha]$
$2.2' \equiv 0.1$	$\Leftrightarrow$	$[\delta^\circ, \alpha^\circ]$
$2.2' \equiv 0.2$	$\Leftrightarrow$	$[\delta^\circ, \text{id}2]$
$2.2' \equiv 0.3$	$\Leftrightarrow$	$[\delta^\circ, \beta]$
$2.3' \equiv 0.1$	$\Leftrightarrow$	$[\delta^\circ, \alpha^\circ\beta^\circ]$
$2.3' \equiv 0.2$	$\Leftrightarrow$	$[\delta^\circ, \beta^\circ]$
$2.3' \equiv 0.3$	$\Leftrightarrow$	$[\delta^\circ, \text{id}3]$

**Q  $\equiv$  P'**

$0.1 \equiv 3.1'$	$\Leftrightarrow$	$[\delta\gamma, \text{id}1]$
$0.2 \equiv 3.1'$	$\Leftrightarrow$	$[\delta\gamma, \alpha^\circ]$
$0.3 \equiv 3.1'$	$\Leftrightarrow$	$[\delta\gamma, \alpha^\circ\beta^\circ]$
$0.1 \equiv 3.2'$	$\Leftrightarrow$	$[\delta\gamma, \alpha]$
$0.2 \equiv 3.2'$	$\Leftrightarrow$	$[\delta\gamma, \text{id}2]$
$0.3 \equiv 3.2'$	$\Leftrightarrow$	$[\delta\gamma, \beta^\circ]$
$0.1 \equiv 3.3'$	$\Leftrightarrow$	$[\delta\gamma, \beta\alpha]$
$0.2 \equiv 3.3'$	$\Leftrightarrow$	$[\delta\gamma, \beta]$
$0.3 \equiv 3.3'$	$\Leftrightarrow$	$[\delta\gamma, \text{id}3]$

**P'  $\equiv$  Q**

$3.1' \equiv 0.1$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \text{id}1]$
$3.1' \equiv 0.2$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \alpha]$
$3.1' \equiv 0.3$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \beta\alpha]$
$3.2' \equiv 0.1$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \alpha^\circ]$
$3.2' \equiv 0.2$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \text{id}2]$
$3.2' \equiv 0.3$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \beta]$
$3.3' \equiv 0.1$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \alpha^\circ\beta^\circ]$
$3.3' \equiv 0.2$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \beta^\circ]$
$3.3' \equiv 0.3$	$\Leftrightarrow$	$[\gamma^\circ\delta^\circ, \text{id}3]$

**M  $\equiv$  M'**

$1.1 \equiv 1.1'$	$\Leftrightarrow$	$[\text{id}1, \text{id}1]$
$1.2 \equiv 1.1'$	$\Leftrightarrow$	$[\text{id}1, \alpha^\circ]$
$1.3 \equiv 1.1'$	$\Leftrightarrow$	$[\text{id}1, \alpha^\circ\beta^\circ]$
$1.2 \equiv 1.2'$	$\Leftrightarrow$	$[\text{id}1, \text{id}2]$
$1.3 \equiv 1.2'$	$\Leftrightarrow$	$[\text{id}1, \beta^\circ]$
$1.3 \equiv 1.3'$	$\Leftrightarrow$	$[\text{id}1, \text{id}3]$

**M'  $\equiv$  M**

$1.1' \equiv 1.1$	$\Leftrightarrow$	$[\text{id}1, \text{id}1]$
$1.1' \equiv 1.2$	$\Leftrightarrow$	$[\text{id}1, \alpha]$
$1.1' \equiv 1.3$	$\Leftrightarrow$	$[\text{id}1, \beta\alpha]$
$1.2' \equiv 1.2$	$\Leftrightarrow$	$[\text{id}1, \text{id}2]$
$1.2' \equiv 1.3$	$\Leftrightarrow$	$[\text{id}1, \beta]$
$1.3' \equiv 1.3$	$\Leftrightarrow$	$[\text{id}1, \text{id}3]$

**M  $\equiv$  O'**

$1.1 \equiv 2.1'$	$\Leftrightarrow$	$[\alpha, \text{id}1]$
$1.2 \equiv 2.1'$	$\Leftrightarrow$	$[\alpha, \alpha^\circ]$
$1.3 \equiv 2.1'$	$\Leftrightarrow$	$[\alpha, \alpha^\circ\beta^\circ]$
$1.1 \equiv 2.2'$	$\Leftrightarrow$	$[\alpha, \alpha]$
$1.2 \equiv 2.2'$	$\Leftrightarrow$	$[\alpha, \text{id}2]$
$1.3 \equiv 2.2'$	$\Leftrightarrow$	$[\alpha, \beta^\circ]$

**O'  $\equiv$  M**

$2.1' \equiv 1.1$	$\Leftrightarrow$	$[\alpha^\circ, \text{id}1]$
$2.1' \equiv 1.2$	$\Leftrightarrow$	$[\alpha^\circ, \alpha]$
$2.1' \equiv 1.3$	$\Leftrightarrow$	$[\alpha^\circ, \beta\alpha]$
$2.2' \equiv 1.1$	$\Leftrightarrow$	$[\alpha^\circ, \alpha^\circ]$
$2.2' \equiv 1.2$	$\Leftrightarrow$	$[\alpha^\circ, \text{id}2]$
$2.2' \equiv 1.3$	$\Leftrightarrow$	$[\alpha^\circ, \beta]$

$$\begin{array}{llll}
1.1 \equiv 2.3' & \Leftrightarrow & [\alpha, \beta\alpha] & 2.3' \equiv 1.1 & \Leftrightarrow & [\alpha^\circ, \alpha^\circ\beta^\circ] \\
1.2 \equiv 2.3' & \Leftrightarrow & [\alpha, \beta] & 2.3' \equiv 1.2 & \Leftrightarrow & [\alpha^\circ, \beta^\circ] \\
1.3 \equiv 2.3' & \Leftrightarrow & [\alpha, \text{id}3] & 2.3' \equiv 1.3 & \Leftrightarrow & [\alpha^\circ, \text{id}3]
\end{array}$$

**M ≡ P**

$$\begin{array}{ll}
1.1 \equiv 3.1' & \Leftrightarrow [\beta\alpha, \text{id}1] \\
1.2 \equiv 3.1' & \Leftrightarrow [\beta\alpha, \alpha^\circ] \\
1.3 \equiv 3.1' & \Leftrightarrow [\beta\alpha, \alpha^\circ\beta^\circ] \\
1.1 \equiv 3.2' & \Leftrightarrow [\beta\alpha, \alpha] \\
1.2 \equiv 3.2' & \Leftrightarrow [\beta\alpha, \text{id}2] \\
1.3 \equiv 3.2' & \Leftrightarrow [\beta\alpha, \beta^\circ] \\
1.1 \equiv 3.3' & \Leftrightarrow [\beta\alpha, \beta\alpha] \\
1.2 \equiv 3.3' & \Leftrightarrow [\beta\alpha, \beta] \\
1.3 \equiv 3.3' & \Leftrightarrow [\beta\alpha, \text{id}3]
\end{array}$$

**P ≡ M**

$$\begin{array}{ll}
3.1' \equiv 1.1 & \Leftrightarrow [\alpha^\circ\beta^\circ, \text{id}1] \\
3.1' \equiv 1.2 & \Leftrightarrow [\alpha^\circ\beta^\circ, \alpha] \\
3.1' \equiv 1.3 & \Leftrightarrow [\alpha^\circ\beta^\circ, \beta\alpha] \\
3.2' \equiv 1.1 & \Leftrightarrow [\alpha^\circ\beta^\circ, \alpha^\circ] \\
3.2' \equiv 1.2 & \Leftrightarrow [\alpha^\circ\beta^\circ, \text{id}2] \\
3.2' \equiv 1.3 & \Leftrightarrow [\alpha^\circ\beta^\circ, \beta] \\
3.3' \equiv 1.1 & \Leftrightarrow [\alpha^\circ\beta^\circ, \alpha^\circ\beta^\circ] \\
3.3' \equiv 1.2 & \Leftrightarrow [\alpha^\circ\beta^\circ, \beta^\circ] \\
3.3' \equiv 1.3 & \Leftrightarrow [\alpha^\circ\beta^\circ, \text{id}3]
\end{array}$$

**O ≡ M'**

$$\begin{array}{ll}
2.1 \equiv 1.1' & \Leftrightarrow [\alpha^\circ, \text{id}1] \\
2.2 \equiv 1.1' & \Leftrightarrow [\alpha^\circ, \alpha^\circ] \\
2.3 \equiv 1.1' & \Leftrightarrow [\alpha^\circ, \alpha^\circ\beta^\circ] \\
2.1 \equiv 1.2' & \Leftrightarrow [\alpha^\circ, \alpha] \\
2.2 \equiv 1.2' & \Leftrightarrow [\alpha^\circ, \text{id}2] \\
2.3 \equiv 1.2' & \Leftrightarrow [\alpha^\circ, \beta^\circ] \\
2.1 \equiv 1.3' & \Leftrightarrow [\alpha^\circ, \beta\alpha] \\
2.2 \equiv 1.3' & \Leftrightarrow [\alpha^\circ, \beta] \\
2.3 \equiv 1.3' & \Leftrightarrow [\alpha^\circ, \text{id}3]
\end{array}$$

**M' ≡ O**

$$\begin{array}{ll}
1.1' \equiv 2.1 & \Leftrightarrow [\alpha, \text{id}1] \\
1.1' \equiv 2.2 & \Leftrightarrow [\alpha, \alpha] \\
1.1' \equiv 2.3 & \Leftrightarrow [\alpha, \beta\alpha] \\
1.2' \equiv 2.1 & \Leftrightarrow [\alpha, \alpha^\circ] \\
1.2' \equiv 2.2 & \Leftrightarrow [\alpha, \text{id}2] \\
1.2' \equiv 2.3 & \Leftrightarrow [\alpha, \beta] \\
1.3' \equiv 2.1 & \Leftrightarrow [\alpha, \alpha^\circ\beta^\circ] \\
1.3' \equiv 2.2 & \Leftrightarrow [\alpha, \beta^\circ] \\
1.3' \equiv 2.3 & \Leftrightarrow [\alpha, \text{id}3]
\end{array}$$

**O ≡ O'**

$$\begin{array}{ll}
2.1 \equiv 2.1' & \Leftrightarrow [\text{id}2, \text{id}1] \\
2.2 \equiv 2.1' & \Leftrightarrow [\text{id}2, \alpha^\circ] \\
2.3 \equiv 2.1' & \Leftrightarrow [\text{id}2, \alpha^\circ\beta^\circ] \\
2.2 \equiv 2.2' & \Leftrightarrow [\text{id}2, \text{id}2] \\
2.3 \equiv 2.2' & \Leftrightarrow [\text{id}2, \beta^\circ] \\
2.3 \equiv 2.3' & \Leftrightarrow [\text{id}2, \text{id}3]
\end{array}$$

**O' ≡ O**

$$\begin{array}{ll}
2.1' \equiv 2.1 & \Leftrightarrow [\text{id}2, \text{id}1] \\
2.1' \equiv 2.2 & \Leftrightarrow [\text{id}2, \alpha] \\
2.1' \equiv 2.3 & \Leftrightarrow [\text{id}2, \beta\alpha] \\
2.2' \equiv 2.2 & \Leftrightarrow [\text{id}2, \text{id}2] \\
2.2' \equiv 2.3 & \Leftrightarrow [\text{id}2, \beta] \\
2.3' \equiv 2.3 & \Leftrightarrow [\text{id}2, \text{id}3]
\end{array}$$

**O ≡ P'**

$$\begin{array}{ll}
2.1 \equiv 3.1' & \Leftrightarrow [\beta, \text{id}1] \\
2.2 \equiv 3.1' & \Leftrightarrow [\beta, \alpha^\circ] \\
2.3 \equiv 3.1' & \Leftrightarrow [\beta, \alpha^\circ\beta^\circ] \\
2.1 \equiv 3.2' & \Leftrightarrow [\beta, \alpha] \\
2.2 \equiv 3.2' & \Leftrightarrow [\beta, \text{id}2] \\
2.3 \equiv 3.2' & \Leftrightarrow [\beta, \beta^\circ] \\
2.1 \equiv 3.3' & \Leftrightarrow [\beta, \beta\alpha] \\
2.2 \equiv 3.3' & \Leftrightarrow [\beta, \beta]
\end{array}$$

**P' ≡ O**

$$\begin{array}{ll}
3.1' \equiv 2.1 & \Leftrightarrow [\beta^\circ, \text{id}1] \\
3.1' \equiv 2.2 & \Leftrightarrow [\beta^\circ, \alpha] \\
3.1' \equiv 2.3 & \Leftrightarrow [\beta^\circ, \beta\alpha] \\
3.2' \equiv 2.1 & \Leftrightarrow [\beta^\circ, \alpha^\circ] \\
3.2' \equiv 2.2 & \Leftrightarrow [\beta^\circ, \text{id}2] \\
3.2' \equiv 2.3 & \Leftrightarrow [\beta^\circ, \beta] \\
3.3' \equiv 2.1 & \Leftrightarrow [\beta^\circ, \alpha^\circ\beta^\circ] \\
3.3' \equiv 2.2 & \Leftrightarrow [\beta^\circ, \beta^\circ]
\end{array}$$

$$2.3 \equiv 3.3' \Leftrightarrow [\beta, \text{id}3] \quad 3.3' \equiv 2.3 \Leftrightarrow [\beta^\circ, \text{id}3]$$

**I ≡ M'**

$$3.1 \equiv 1.1' \Leftrightarrow [\alpha^\circ\beta^\circ, \text{id}1]$$

$$3.2 \equiv 1.1' \Leftrightarrow [\alpha^\circ\beta^\circ, \alpha^\circ]$$

$$3.3 \equiv 1.1' \Leftrightarrow [\alpha^\circ\beta^\circ, \alpha^\circ\beta^\circ]$$

$$3.1 \equiv 1.2' \Leftrightarrow [\alpha^\circ\beta^\circ, \alpha]$$

$$3.2 \equiv 1.2' \Leftrightarrow [\alpha^\circ\beta^\circ, \text{id}2]$$

$$3.3 \equiv 1.2' \Leftrightarrow [\alpha^\circ\beta^\circ, \beta^\circ]$$

$$3.1 \equiv 1.3' \Leftrightarrow [\alpha^\circ\beta^\circ, \beta\alpha]$$

$$3.2 \equiv 1.3' \Leftrightarrow [\alpha^\circ\beta^\circ, \beta]$$

$$3.3 \equiv 1.3' \Leftrightarrow [\alpha^\circ\beta^\circ, \text{id}3]$$

**M' ≡ I**

$$1.1' \equiv 3.1 \Leftrightarrow [\beta\alpha, \text{id}1]$$

$$1.1' \equiv 3.2 \Leftrightarrow [\beta\alpha, \alpha]$$

$$1.1' \equiv 3.3 \Leftrightarrow [\beta\alpha, \beta\alpha]$$

$$1.2' \equiv 3.1 \Leftrightarrow [\beta\alpha, \alpha^\circ]$$

$$1.2' \equiv 3.2 \Leftrightarrow [\beta\alpha, \text{id}2]$$

$$1.2' \equiv 3.3 \Leftrightarrow [\beta\alpha, \beta]$$

$$1.3' \equiv 3.1 \Leftrightarrow [\beta\alpha, \alpha^\circ\beta^\circ]$$

$$1.3' \equiv 3.2 \Leftrightarrow [\beta\alpha, \beta^\circ]$$

$$1.3' \equiv 3.3 \Leftrightarrow [\beta\alpha, \text{id}3]$$

**I ≡ O'**

$$3.1 \equiv 2.1' \Leftrightarrow [\beta^\circ, \text{id}1]$$

$$3.2 \equiv 2.1' \Leftrightarrow [\beta^\circ, \alpha^\circ]$$

$$3.3 \equiv 2.1' \Leftrightarrow [\beta^\circ, \alpha^\circ\beta^\circ]$$

$$3.1 \equiv 2.2' \Leftrightarrow [\beta^\circ, \alpha]$$

$$3.2 \equiv 2.2' \Leftrightarrow [\beta^\circ, \text{id}2]$$

$$3.3 \equiv 2.2' \Leftrightarrow [\beta^\circ, \beta^\circ]$$

$$3.1 \equiv 2.3' \Leftrightarrow [\beta^\circ, \beta\alpha]$$

$$3.2 \equiv 2.3' \Leftrightarrow [\beta^\circ, \beta]$$

$$3.3 \equiv 2.3' \Leftrightarrow [\beta^\circ, \text{id}3]$$

**O' ≡ I**

$$2.1' \equiv 3.1 \Leftrightarrow [\beta, \text{id}1]$$

$$2.1' \equiv 3.2 \Leftrightarrow [\beta, \alpha]$$

$$2.1' \equiv 3.3 \Leftrightarrow [\beta, \beta\alpha]$$

$$2.2' \equiv 3.1 \Leftrightarrow [\beta, \alpha^\circ]$$

$$2.2' \equiv 3.2 \Leftrightarrow [\beta, \text{id}2]$$

$$2.2' \equiv 3.3 \Leftrightarrow [\beta, \beta]$$

$$2.3' \equiv 3.1 \Leftrightarrow [\beta, \alpha^\circ\beta^\circ]$$

$$2.3' \equiv 3.2 \Leftrightarrow [\beta, \beta^\circ]$$

$$2.3' \equiv 3.3 \Leftrightarrow [\beta, \text{id}3]$$

**I ≡ P**

$$3.1 \equiv 3.1' \Leftrightarrow [\text{id}3, \text{id}1]$$

$$3.2 \equiv 3.1' \Leftrightarrow [\text{id}3, \alpha^\circ]$$

$$3.3 \equiv 3.1' \Leftrightarrow [\text{id}3, \alpha^\circ\beta^\circ]$$

$$3.2 \equiv 3.2' \Leftrightarrow [\text{id}3, \text{id}2]$$

$$3.3 \equiv 3.2' \Leftrightarrow [\text{id}3, \beta^\circ]$$

$$3.3 \equiv 3.3' \Leftrightarrow [\text{id}3, \text{id}3]$$

**P ≡ I**

$$3.1' \equiv 3.1 \Leftrightarrow [\text{id}3, \text{id}1]$$

$$3.1' \equiv 3.2 \Leftrightarrow [\text{id}3, \alpha]$$

$$3.1' \equiv 3.3 \Leftrightarrow [\text{id}3, \beta\alpha]$$

$$3.2' \equiv 3.2 \Leftrightarrow [\text{id}3, \text{id}2]$$

$$3.2' \equiv 3.3 \Leftrightarrow [\text{id}3, \beta]$$

$$3.3' \equiv 3.3 \Leftrightarrow [\text{id}3, \text{id}3]$$

5. Second, we shall present the dyadic pre-semiotic sign connections. For the sake of clearness, we first deal with the pre-semiotic connections separately.

**Q/M ≡ Q'/M'**

$$0.1-1.1 \equiv 0.1'-1.1' \quad [[\gamma, \text{id}1], [\gamma, \text{id}1]]$$

$$0.1-1.1 \equiv 0.2'-1.1' \quad [[\gamma, \text{id}1], [\gamma, \alpha^\circ]]$$

$$0.1-1.1 \equiv 0.3'-1.1' \quad [[\gamma, \text{id}1], [\gamma, \alpha^\circ\beta^\circ]]$$

$$0.1-1.1 \equiv 0.2'-1.2' \quad [[\gamma, \text{id}1], [\gamma, \text{id}2]]$$

$$0.1-1.1 \equiv 0.3'-1.2' \quad [[\gamma, \text{id}1], [\gamma, \beta^\circ]]$$

$$0.1-1.1 \equiv 0.3'-1.3' \quad [[\gamma, \text{id}1], [\gamma, \text{id}3]]$$

$$0.2-1.1 \equiv 0.1'-1.1' \quad [[\gamma, \alpha^\circ], [\gamma, \text{id}1]]$$

**Q'/M' ≡ Q/M**

$$0.1'-1.1' \equiv 0.1-1.1 \quad [[\gamma, \text{id}1], [\gamma, \text{id}1]]$$

$$0.2'-1.1' \equiv 0.1-1.1 \quad [[\gamma, \alpha^\circ], [\gamma, \text{id}1]]$$

$$0.3'-1.1' \equiv 0.1-1.1 \quad [[\gamma, \alpha^\circ\beta^\circ], [\gamma, \text{id}1]]$$

$$0.2'-1.2' \equiv 0.1-1.1 \quad [[\gamma, \text{id}2], [\gamma, \text{id}1]]$$

$$0.3'-1.2' \equiv 0.1-1.1 \quad [[\gamma, \beta^\circ], [\gamma, \text{id}1]]$$

$$0.3'-1.3' \equiv 0.1-1.1 \quad [[\gamma, \text{id}3], [\gamma, \text{id}1]]$$

$$0.1'-1.1' \equiv 0.2-1.1 \quad [[\gamma, \text{id}1], [\gamma, \alpha^\circ]]$$









0.2-1.2 $\equiv$ 1.2'-2.1'	[[ $\gamma$ , id2], [ $\alpha$ , $\alpha^\circ$ ]]	1.2'-2.1' $\equiv$ 0.2-1.2	[[ $\alpha$ , $\alpha^\circ$ ], [ $\gamma$ , id2]]
0.2-1.2 $\equiv$ 1.3'-2.1'	[[ $\gamma$ , id2], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]	1.3'-2.1' $\equiv$ 0.2-1.2	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\gamma$ , id2]]
0.2-1.2 $\equiv$ 1.2'-2.2'	[[ $\gamma$ , id2], [ $\alpha$ , id2]]	1.2'-2.2' $\equiv$ 0.2-1.2	[[ $\alpha$ , id2], [ $\gamma$ , id2]]
0.2-1.2 $\equiv$ 1.3'-2.2'	[[ $\gamma$ , id2], [ $\alpha$ , $\beta^\circ$ ]]	1.3'-2.2' $\equiv$ 0.2-1.2	[[ $\alpha$ , $\beta^\circ$ ], [ $\gamma$ , id2]]
0.2-1.2 $\equiv$ 1.3'-2.3'	[[ $\gamma$ , id2], [ $\alpha$ , id3]]	1.3'-2.3' $\equiv$ 0.2-1.2	[[ $\alpha$ , id3], [ $\gamma$ , id2]]
0.3-1.2 $\equiv$ 1.1'-2.1'	[[ $\gamma$ , $\beta^\circ$ ], [ $\alpha$ , id1]]	1.1'-2.1' $\equiv$ 0.3-1.2	[[ $\alpha$ , id1], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.2 $\equiv$ 1.2'-2.1'	[[ $\gamma$ , $\beta^\circ$ ], [ $\alpha$ , $\alpha^\circ$ ]]	1.2'-2.1' $\equiv$ 0.3-1.2	[[ $\alpha$ , $\alpha^\circ$ ], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.2 $\equiv$ 1.3'-2.1'	[[ $\gamma$ , $\beta^\circ$ ], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]	1.3'-2.1' $\equiv$ 0.3-1.2	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.2 $\equiv$ 1.2'-2.2'	[[ $\gamma$ , $\beta^\circ$ ], [ $\alpha$ , id2]]	1.2'-2.2' $\equiv$ 0.3-1.2	[[ $\alpha$ , id2], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.2 $\equiv$ 1.3'-2.2'	[[ $\gamma$ , $\beta^\circ$ ], [ $\alpha$ , $\beta^\circ$ ]]	1.3'-2.2' $\equiv$ 0.3-1.2	[[ $\alpha$ , $\beta^\circ$ ], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.2 $\equiv$ 1.3'-2.3'	[[ $\gamma$ , $\beta^\circ$ ], [ $\alpha$ , id3]]	1.3'-2.3' $\equiv$ 0.3-1.2	[[ $\alpha$ , id3], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.3 $\equiv$ 1.1'-2.1'	[[ $\gamma$ , id3], [ $\alpha$ , id1]]	1.1'-2.1' $\equiv$ 0.3-1.3	[[ $\alpha$ , id1], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.2'-2.1'	[[ $\gamma$ , id3], [ $\alpha$ , $\alpha^\circ$ ]]	1.2'-2.1' $\equiv$ 0.3-1.3	[[ $\alpha$ , $\alpha^\circ$ ], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.3'-2.1'	[[ $\gamma$ , id3], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]	1.3'-2.1' $\equiv$ 0.3-1.3	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.2'-2.2'	[[ $\gamma$ , id3], [ $\alpha$ , id2]]	1.2'-2.2' $\equiv$ 0.3-1.3	[[ $\alpha$ , id2], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.3'-2.2'	[[ $\gamma$ , id3], [ $\alpha$ , $\beta^\circ$ ]]	1.3'-2.2' $\equiv$ 0.3-1.3	[[ $\alpha$ , $\beta^\circ$ ], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.3'-2.3'	[[ $\gamma$ , id3], [ $\alpha$ , id3]]	1.3'-2.3' $\equiv$ 0.3-1.3	[[ $\alpha$ , id3], [ $\gamma$ , id3]]

**Q/M  $\equiv$  O'/I'**

0.1-1.1 $\equiv$ 2.1'-3.1'	[[ $\gamma$ , id1], [ $\beta$ , id1]]
0.1-1.1 $\equiv$ 2.2'-3.1'	[[ $\gamma$ , id1], [ $\beta$ , $\alpha^\circ$ ]]
0.1-1.1 $\equiv$ 2.3'-3.1'	[[ $\gamma$ , id1], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
0.1-1.1 $\equiv$ 2.2'-3.2'	[[ $\gamma$ , id1], [ $\beta$ , id2]]
0.1-1.1 $\equiv$ 2.3'-3.2'	[[ $\gamma$ , id1], [ $\beta$ , $\beta^\circ$ ]]
0.1-1.1 $\equiv$ 2.3'-3.3'	[[ $\gamma$ , id1], [ $\beta$ , id3]]
0.2-1.1 $\equiv$ 2.1'-3.1'	[[ $\gamma$ , $\alpha^\circ$ ], [ $\beta$ , id1]]
0.2-1.1 $\equiv$ 2.2'-3.1'	[[ $\gamma$ , $\alpha^\circ$ ], [ $\beta$ , $\alpha^\circ$ ]]
0.2-1.1 $\equiv$ 2.3'-3.1'	[[ $\gamma$ , $\alpha^\circ$ ], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
0.2-1.1 $\equiv$ 2.2'-3.2'	[[ $\gamma$ , $\alpha^\circ$ ], [ $\beta$ , id2]]
0.2-1.1 $\equiv$ 2.3'-3.2'	[[ $\gamma$ , $\alpha^\circ$ ], [ $\beta$ , $\beta^\circ$ ]]
0.2-1.1 $\equiv$ 2.3'-3.3'	[[ $\gamma$ , $\alpha^\circ$ ], [ $\beta$ , id3]]
0.3-1.1 $\equiv$ 2.1'-3.1'	[[ $\gamma$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , id1]]
0.3-1.1 $\equiv$ 2.2'-3.1'	[[ $\gamma$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , $\alpha^\circ$ ]]
0.3-1.1 $\equiv$ 2.3'-3.1'	[[ $\gamma$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
0.3-1.1 $\equiv$ 2.2'-3.2'	[[ $\gamma$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , id2]]
0.3-1.1 $\equiv$ 2.3'-3.2'	[[ $\gamma$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , $\beta^\circ$ ]]
0.3-1.1 $\equiv$ 2.3'-3.3'	[[ $\gamma$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , id3]]
0.2-1.2 $\equiv$ 2.1'-3.1'	[[ $\gamma$ , id2], [ $\beta$ , id1]]
0.2-1.2 $\equiv$ 2.2'-3.1'	[[ $\gamma$ , id2], [ $\beta$ , $\alpha^\circ$ ]]
0.2-1.2 $\equiv$ 2.3'-3.1'	[[ $\gamma$ , id2], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
0.2-1.2 $\equiv$ 2.2'-3.2'	[[ $\gamma$ , id2], [ $\beta$ , id2]]
0.2-1.2 $\equiv$ 2.3'-3.2'	[[ $\gamma$ , id2], [ $\beta$ , $\beta^\circ$ ]]

**O'/I'  $\equiv$  Q/M**

2.1'-3.1' $\equiv$ 0.1-1.1	[[ $\beta$ , id1], [ $\gamma$ , id1]]
2.2'-3.1' $\equiv$ 0.1-1.1	[[ $\beta$ , $\alpha^\circ$ ], [ $\gamma$ , id1]]
2.3'-3.1' $\equiv$ 0.1-1.1	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\gamma$ , id1]]
2.2'-3.2' $\equiv$ 0.1-1.1	[[ $\beta$ , id2], [ $\gamma$ , id1]]
2.3'-3.2' $\equiv$ 0.1-1.1	[[ $\beta$ , $\beta^\circ$ ], [ $\gamma$ , id1]]
2.3'-3.3' $\equiv$ 0.1-1.1	[[ $\beta$ , id3], [ $\gamma$ , id1]]
2.1'-3.1' $\equiv$ 0.2-1.1	[[ $\beta$ , id1], [ $\gamma$ , $\alpha^\circ$ ]]
2.2'-3.1' $\equiv$ 0.2-1.1	[[ $\beta$ , $\alpha^\circ$ ], [ $\gamma$ , $\alpha^\circ$ ]]
2.3'-3.1' $\equiv$ 0.2-1.1	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\gamma$ , $\alpha^\circ$ ]]
2.2'-3.2' $\equiv$ 0.2-1.1	[[ $\beta$ , id2], [ $\gamma$ , $\alpha^\circ$ ]]
2.3'-3.2' $\equiv$ 0.2-1.1	[[ $\beta$ , $\beta^\circ$ ], [ $\gamma$ , $\alpha^\circ$ ]]
2.3'-3.3' $\equiv$ 0.2-1.1	[[ $\beta$ , id3], [ $\gamma$ , $\alpha^\circ$ ]]
2.1'-3.1' $\equiv$ 0.3-1.1	[[ $\beta$ , id1], [ $\gamma$ , $\alpha^\circ\beta^\circ$ ]]
2.2'-3.1' $\equiv$ 0.3-1.1	[[ $\beta$ , $\alpha^\circ$ ], [ $\gamma$ , $\alpha^\circ\beta^\circ$ ]]
2.3'-3.1' $\equiv$ 0.3-1.1	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\gamma$ , $\alpha^\circ\beta^\circ$ ]]
2.2'-3.2' $\equiv$ 0.3-1.1	[[ $\beta$ , id2], [ $\gamma$ , $\alpha^\circ\beta^\circ$ ]]
2.3'-3.2' $\equiv$ 0.3-1.1	[[ $\beta$ , $\beta^\circ$ ], [ $\gamma$ , $\alpha^\circ\beta^\circ$ ]]
2.3'-3.3' $\equiv$ 0.3-1.1	[[ $\beta$ , id3], [ $\gamma$ , $\alpha^\circ\beta^\circ$ ]]
2.1'-3.1' $\equiv$ 0.2-1.2	[[ $\beta$ , id1], [ $\gamma$ , id2]]
2.2'-3.1' $\equiv$ 0.2-1.2	[[ $\beta$ , $\alpha^\circ$ ], [ $\gamma$ , id2]]
2.3'-3.1' $\equiv$ 0.2-1.2	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\gamma$ , id2]]
2.2'-3.2' $\equiv$ 0.2-1.2	[[ $\beta$ , id2], [ $\gamma$ , id2]]
2.3'-3.2' $\equiv$ 0.2-1.2	[[ $\beta$ , $\beta^\circ$ ], [ $\gamma$ , id2]]



0.3-1.2 $\equiv$ 1.2'-3.2'	[[ $\gamma$ , $\beta^\circ$ ], [ $\beta\alpha$ , id2]]	1.2'-3.2' $\equiv$ 0.3-1.2	[[ $\beta\alpha$ , id2], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.2 $\equiv$ 1.3'-3.2'	[[ $\gamma$ , $\beta^\circ$ ], [ $\beta\alpha$ , $\beta^\circ$ ]]	1.3'-3.2' $\equiv$ 0.3-1.2	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.2 $\equiv$ 1.3'-3.3'	[[ $\gamma$ , $\beta^\circ$ ], [ $\beta\alpha$ , id3]]	1.3'-3.3' $\equiv$ 0.3-1.2	[[ $\beta\alpha$ , id3], [ $\gamma$ , $\beta^\circ$ ]]
0.3-1.3 $\equiv$ 1.1'-3.1'	[[ $\gamma$ , id3], [ $\beta\alpha$ , id1]]	1.1'-3.1' $\equiv$ 0.3-1.3	[[ $\beta\alpha$ , id1], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.2'-3.1'	[[ $\gamma$ , id3], [ $\beta\alpha$ , $\alpha^\circ$ ]]	1.2'-3.1' $\equiv$ 0.3-1.3	[[ $\beta\alpha$ , $\alpha^\circ$ ], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.3'-3.1'	[[ $\gamma$ , id3], [ $\beta\alpha$ , $\alpha^\circ\beta^\circ$ ]]	1.3'-3.1' $\equiv$ 0.3-1.3	[[ $\beta\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.2'-3.2'	[[ $\gamma$ , id3], [ $\beta\alpha$ , id2]]	1.2'-3.2' $\equiv$ 0.3-1.3	[[ $\beta\alpha$ , id2], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.3'-3.2'	[[ $\gamma$ , id3], [ $\beta\alpha$ , $\beta^\circ$ ]]	1.3'-3.2' $\equiv$ 0.3-1.3	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\gamma$ , id3]]
0.3-1.3 $\equiv$ 1.3'-3.3'	[[ $\gamma$ , id3], [ $\beta\alpha$ , id3]]	1.3'-3.3' $\equiv$ 0.3-1.3	[[ $\beta\alpha$ , id3], [ $\gamma$ , id3]]

Now, we show the semiotic connections of the pre-semiotic sign relations:

**M/O  $\equiv$  M'/O'**

1.1-2.1 $\equiv$ 1.1'-2.1'
1.1-2.1 $\equiv$ 1.2'-2.1'
1.1-2.1 $\equiv$ 1.3'-2.1'
1.1-2.1 $\equiv$ 1.2'-2.2'
1.1-2.1 $\equiv$ 1.3'-2.2'
1.1-2.1 $\equiv$ 1.3'-2.3'
1.2-2.1 $\equiv$ 1.1'-2.1'
1.2-2.1 $\equiv$ 1.2'-2.1'
1.2-2.1 $\equiv$ 1.3'-2.1'
1.2-2.1 $\equiv$ 1.2'-2.2'
1.2-2.1 $\equiv$ 1.3'-2.2'
1.2-2.1 $\equiv$ 1.3'-2.3'
1.3-2.1 $\equiv$ 1.1'-2.1'
1.3-2.1 $\equiv$ 1.2'-2.1'
1.3-2.1 $\equiv$ 1.3'-2.1'
1.3-2.1 $\equiv$ 1.2'-2.2'
1.3-2.1 $\equiv$ 1.3'-2.2'
1.3-2.1 $\equiv$ 1.3'-2.3'
1.2-2.2 $\equiv$ 1.1'-2.1'
1.2-2.2 $\equiv$ 1.2'-2.1'
1.2-2.2 $\equiv$ 1.3'-2.1'
1.2-2.2 $\equiv$ 1.2'-2.2'
1.2-2.2 $\equiv$ 1.3'-2.2'
1.2-2.2 $\equiv$ 1.3'-2.3'
1.3-2.2 $\equiv$ 1.1'-2.1'
1.3-2.2 $\equiv$ 1.2'-2.1'
1.3-2.2 $\equiv$ 1.3'-2.1'
1.3-2.2 $\equiv$ 1.2'-2.2'
1.3-2.2 $\equiv$ 1.3'-2.2'

**M'/O'  $\equiv$  M/O**

1.1'-2.1' $\equiv$ 1.1-2.1
1.2'-2.1' $\equiv$ 1.1-2.1
1.3'-2.1' $\equiv$ 1.1-2.1
1.2'-2.2' $\equiv$ 1.1-2.1
1.3'-2.2' $\equiv$ 1.1-2.1
1.3'-2.3' $\equiv$ 1.1-2.1
1.1'-2.1' $\equiv$ 1.2-2.1
1.2'-2.1' $\equiv$ 1.2-2.1
1.3'-2.1' $\equiv$ 1.2-2.1
1.2'-2.2' $\equiv$ 1.2-2.1
1.3'-2.2' $\equiv$ 1.2-2.1
1.3'-2.3' $\equiv$ 1.2-2.1
1.1'-2.1' $\equiv$ 1.3-2.1
1.2'-2.1' $\equiv$ 1.3-2.1
1.3'-2.1' $\equiv$ 1.3-2.1
1.2'-2.2' $\equiv$ 1.3-2.1
1.3'-2.2' $\equiv$ 1.3-2.1
1.3'-2.3' $\equiv$ 1.3-2.1
1.1'-2.1' $\equiv$ 1.2-2.2
1.2'-2.1' $\equiv$ 1.2-2.2
1.3'-2.1' $\equiv$ 1.2-2.2
1.2'-2.2' $\equiv$ 1.2-2.2
1.3'-2.2' $\equiv$ 1.2-2.2
1.3'-2.3' $\equiv$ 1.2-2.2
1.1'-2.1' $\equiv$ 1.3-2.2
1.2'-2.1' $\equiv$ 1.3-2.2
1.3'-2.1' $\equiv$ 1.3-2.2
1.2'-2.2' $\equiv$ 1.3-2.2
1.3'-2.2' $\equiv$ 1.3-2.2

1.3-2.2 $\equiv$ 1.3'-2.3'	[[ $\alpha$ , $\beta^\circ$ ], [ $\alpha$ , id3]]	1.3'-2.3' $\equiv$ 1.3-2.2	[[ $\alpha$ , id3], [ $\alpha$ , $\beta^\circ$ ]]
1.3-2.3 $\equiv$ 1.1'-2.1'	[[ $\alpha$ , id3], [ $\alpha$ , id1]]	1.1'-2.1' $\equiv$ 1.3-2.3	[[ $\alpha$ , id1], [ $\alpha$ , id3]]
1.3-2.3 $\equiv$ 1.2'-2.1'	[[ $\alpha$ , id3], [ $\alpha$ , $\alpha^\circ$ ]]	1.2'-2.1' $\equiv$ 1.3-2.3	[[ $\alpha$ , $\alpha^\circ$ ], [ $\alpha$ , id3]]
1.3-2.3 $\equiv$ 1.3'-2.1'	[[ $\alpha$ , id3], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]	1.3'-2.1' $\equiv$ 1.3-2.3	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\alpha$ , id3]]
1.3-2.3 $\equiv$ 1.2'-2.2'	[[ $\alpha$ , id3], [ $\alpha$ , id2]]	1.2'-2.2' $\equiv$ 1.3-2.3	[[ $\alpha$ , id2], [ $\alpha$ , id3]]
1.3-2.3 $\equiv$ 1.3'-2.2'	[[ $\alpha$ , id3], [ $\alpha$ , $\beta^\circ$ ]]	1.3'-2.2' $\equiv$ 1.3-2.3	[[ $\alpha$ , $\beta^\circ$ ], [ $\alpha$ , id3]]
1.3-2.3 $\equiv$ 1.3'-2.3'	[[ $\alpha$ , id3], [ $\alpha$ , id3]]	1.3'-2.3' $\equiv$ 1.3-2.3	[[ $\alpha$ , id3], [ $\alpha$ , id3]]

**M/O  $\equiv$  O'/I'**

1.1-2.1 $\equiv$ 2.1'-3.1'	[[ $\alpha$ , id1], [ $\beta$ , id1]]
1.1-2.1 $\equiv$ 2.2'-3.1'	[[ $\alpha$ , id1], [ $\beta$ , $\alpha^\circ$ ]]
1.1-2.1 $\equiv$ 2.3'-3.1'	[[ $\alpha$ , id1], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
1.1-2.1 $\equiv$ 2.2'-3.2'	[[ $\alpha$ , id1], [ $\beta$ , id2]]
1.1-2.1 $\equiv$ 2.3'-3.2'	[[ $\alpha$ , id1], [ $\beta$ , $\beta^\circ$ ]]
1.1-2.1 $\equiv$ 2.3'-3.3'	[[ $\alpha$ , id1], [ $\beta$ , id3]]
1.2-2.1 $\equiv$ 2.1'-3.1'	[[ $\alpha$ , $\alpha^\circ$ ], [ $\beta$ , id1]]
1.2-2.1 $\equiv$ 2.2'-3.1'	[[ $\alpha$ , $\alpha^\circ$ ], [ $\beta$ , $\alpha^\circ$ ]]
1.2-2.1 $\equiv$ 2.3'-3.1'	[[ $\alpha$ , $\alpha^\circ$ ], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
1.2-2.1 $\equiv$ 2.2'-3.2'	[[ $\alpha$ , $\alpha^\circ$ ], [ $\beta$ , id2]]
1.2-2.1 $\equiv$ 2.3'-3.2'	[[ $\alpha$ , $\alpha^\circ$ ], [ $\beta$ , $\beta^\circ$ ]]
1.2-2.1 $\equiv$ 2.3'-3.3'	[[ $\alpha$ , $\alpha^\circ$ ], [ $\beta$ , id3]]
1.3-2.1 $\equiv$ 2.1'-3.1'	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , id1]]
1.3-2.1 $\equiv$ 2.2'-3.1'	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , $\alpha^\circ$ ]]
1.3-2.1 $\equiv$ 2.3'-3.1'	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
1.3-2.1 $\equiv$ 2.2'-3.2'	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , id2]]
1.3-2.1 $\equiv$ 2.3'-3.2'	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , $\beta^\circ$ ]]
1.3-2.1 $\equiv$ 2.3'-3.3'	[[ $\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta$ , id3]]
1.2-2.2 $\equiv$ 2.1'-3.1'	[[ $\alpha$ , id2], [ $\beta$ , id1]]
1.2-2.2 $\equiv$ 2.2'-3.1'	[[ $\alpha$ , id2], [ $\beta$ , $\alpha^\circ$ ]]
1.2-2.2 $\equiv$ 2.3'-3.1'	[[ $\alpha$ , id2], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
1.2-2.2 $\equiv$ 2.2'-3.2'	[[ $\alpha$ , id2], [ $\beta$ , id2]]
1.2-2.2 $\equiv$ 2.3'-3.2'	[[ $\alpha$ , id2], [ $\beta$ , $\beta^\circ$ ]]
1.2-2.2 $\equiv$ 2.3'-3.3'	[[ $\alpha$ , id2], [ $\beta$ , id3]]
1.3-2.2 $\equiv$ 2.1'-3.1'	[[ $\alpha$ , $\beta^\circ$ ], [ $\beta$ , id1]]
1.3-2.2 $\equiv$ 2.2'-3.1'	[[ $\alpha$ , $\beta^\circ$ ], [ $\beta$ , $\alpha^\circ$ ]]
1.3-2.2 $\equiv$ 2.3'-3.1'	[[ $\alpha$ , $\beta^\circ$ ], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]
1.3-2.2 $\equiv$ 2.2'-3.2'	[[ $\alpha$ , $\beta^\circ$ ], [ $\beta$ , id2]]
1.3-2.2 $\equiv$ 2.3'-3.2'	[[ $\alpha$ , $\beta^\circ$ ], [ $\beta$ , $\beta^\circ$ ]]
1.3-2.2 $\equiv$ 2.3'-3.3'	[[ $\alpha$ , $\beta^\circ$ ], [ $\beta$ , id3]]
1.3-2.3 $\equiv$ 2.1'-3.1'	[[ $\alpha$ , id3], [ $\beta$ , id1]]
1.3-2.3 $\equiv$ 2.2'-3.1'	[[ $\alpha$ , id3], [ $\beta$ , $\alpha^\circ$ ]]
1.3-2.3 $\equiv$ 2.3'-3.1'	[[ $\alpha$ , id3], [ $\beta$ , $\alpha^\circ\beta^\circ$ ]]

**O'/I'  $\equiv$  M/O**

2.1'-3.1' $\equiv$ 1.1-2.1	[[ $\beta$ , id1], [ $\alpha$ , id1]]
2.2'-3.1' $\equiv$ 1.1-2.1	[[ $\beta$ , $\alpha^\circ$ ], [ $\alpha$ , id1]]
2.3'-3.1' $\equiv$ 1.1-2.1	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\alpha$ , id1]]
2.2'-3.2' $\equiv$ 1.1-2.1	[[ $\beta$ , id2], [ $\alpha$ , id1]]
2.3'-3.2' $\equiv$ 1.1-2.1	[[ $\beta$ , $\beta^\circ$ ], [ $\alpha$ , id1]]
2.3'-3.3' $\equiv$ 1.1-2.1	[[ $\beta$ , id3], [ $\alpha$ , id1]]
2.1'-3.1' $\equiv$ 1.2-2.1	[[ $\beta$ , id1], [ $\alpha$ , $\alpha^\circ$ ]]
2.2'-3.1' $\equiv$ 1.2-2.1	[[ $\beta$ , $\alpha^\circ$ ], [ $\alpha$ , $\alpha^\circ$ ]]
2.3'-3.1' $\equiv$ 1.2-2.1	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\alpha$ , $\alpha^\circ$ ]]
2.2'-3.2' $\equiv$ 1.2-2.1	[[ $\beta$ , id2], [ $\alpha$ , $\alpha^\circ$ ]]
2.3'-3.2' $\equiv$ 1.2-2.1	[[ $\beta$ , $\beta^\circ$ ], [ $\alpha$ , $\alpha^\circ$ ]]
2.3'-3.3' $\equiv$ 1.2-2.1	[[ $\beta$ , id3], [ $\alpha$ , $\alpha^\circ$ ]]
2.1'-3.1' $\equiv$ 1.3-2.1	[[ $\beta$ , id1], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]
2.2'-3.1' $\equiv$ 1.3-2.1	[[ $\beta$ , $\alpha^\circ$ ], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]
2.3'-3.1' $\equiv$ 1.3-2.1	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]
2.2'-3.2' $\equiv$ 1.3-2.1	[[ $\beta$ , id2], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]
2.3'-3.2' $\equiv$ 1.3-2.1	[[ $\beta$ , $\beta^\circ$ ], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]
2.3'-3.3' $\equiv$ 1.3-2.1	[[ $\beta$ , id3], [ $\alpha$ , $\alpha^\circ\beta^\circ$ ]]
2.1'-3.1' $\equiv$ 1.2-2.2	[[ $\beta$ , id1], [ $\alpha$ , id2]]
2.2'-3.1' $\equiv$ 1.2-2.2	[[ $\beta$ , $\alpha^\circ$ ], [ $\alpha$ , id2]]
2.3'-3.1' $\equiv$ 1.2-2.2	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\alpha$ , id2]]
2.2'-3.2' $\equiv$ 1.2-2.2	[[ $\beta$ , id2], [ $\alpha$ , id2]]
2.3'-3.2' $\equiv$ 1.2-2.2	[[ $\beta$ , $\beta^\circ$ ], [ $\alpha$ , id2]]
2.3'-3.3' $\equiv$ 1.2-2.2	[[ $\beta$ , id3], [ $\alpha$ , id2]]
2.1'-3.1' $\equiv$ 1.3-2.2	[[ $\beta$ , id1], [ $\alpha$ , $\beta^\circ$ ]]
2.2'-3.1' $\equiv$ 1.3-2.2	[[ $\beta$ , $\alpha^\circ$ ], [ $\alpha$ , $\beta^\circ$ ]]
2.3'-3.1' $\equiv$ 1.3-2.2	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\alpha$ , $\beta^\circ$ ]]
2.2'-3.2' $\equiv$ 1.3-2.2	[[ $\beta$ , id2], [ $\alpha$ , $\beta^\circ$ ]]
2.3'-3.2' $\equiv$ 1.3-2.2	[[ $\beta$ , $\beta^\circ$ ], [ $\alpha$ , $\beta^\circ$ ]]
2.3'-3.3' $\equiv$ 1.3-2.2	[[ $\beta$ , id3], [ $\alpha$ , $\beta^\circ$ ]]
2.1'-3.1' $\equiv$ 1.3-2.3	[[ $\beta$ , id1], [ $\alpha$ , id3]]
2.2'-3.1' $\equiv$ 1.3-2.3	[[ $\beta$ , $\alpha^\circ$ ], [ $\alpha$ , id3]]
2.3'-3.1' $\equiv$ 1.3-2.3	[[ $\beta$ , $\alpha^\circ\beta^\circ$ ], [ $\alpha$ , id3]]



**O/I ≡ M'/O'**

2.1-3.1 ≡ 1.1'-2.1'	[[β, id1], [α, id1]]
2.1-3.1 ≡ 1.2'-2.1'	[[β, id1], [α, α°]]
2.1-3.1 ≡ 1.3'-2.1'	[[β, id1], [α, α°β°]]
2.1-3.1 ≡ 1.2'-2.2'	[[β, id1], [α, id2]]
2.1-3.1 ≡ 1.3'-2.2'	[[β, id1], [α, β°]]
2.1-3.1 ≡ 1.3'-2.3'	[[β, id1], [α, id3]]
2.2-3.1 ≡ 1.1'-2.1'	[[β, α°], [α, id1]]
2.2-3.1 ≡ 1.2'-2.1'	[[β, α°], [α, α°]]
2.2-3.1 ≡ 1.3'-2.1'	[[β, α°], [α, α°β°]]
2.2-3.1 ≡ 1.2'-2.2'	[[β, α°], [α, id2]]
2.2-3.1 ≡ 1.3'-2.2'	[[β, α°], [α, β°]]
2.2-3.1 ≡ 1.3'-2.3'	[[β, α°], [α, id3]]
2.3-3.1 ≡ 1.1'-2.1'	[[β, α°β°], [α, id1]]
2.3-3.1 ≡ 1.2'-2.1'	[[β, α°β°], [α, α°]]
2.3-3.1 ≡ 1.3'-2.1'	[[β, α°β°], [α, α°β°]]
2.3-3.1 ≡ 1.2'-2.2'	[[β, α°β°], [α, id2]]
2.3-3.1 ≡ 1.3'-2.2'	[[β, α°β°], [α, β°]]
2.3-3.1 ≡ 1.3'-2.3'	[[β, α°β°], [α, id3]]
2.2-3.2 ≡ 1.1'-2.1'	[[β, id2], [α, id1]]
2.2-3.2 ≡ 1.2'-2.1'	[[β, id2], [α, α°]]
2.2-3.2 ≡ 1.3'-2.1'	[[β, id2], [α, α°β°]]
2.2-3.2 ≡ 1.2'-2.2'	[[β, id2], [α, id2]]
2.2-3.2 ≡ 1.3'-2.2'	[[β, id2], [α, β°]]
2.2-3.2 ≡ 1.3'-2.3'	[[β, id2], [α, id3]]
2.3-3.2 ≡ 1.1'-2.1'	[[β, β°], [α, id1]]
2.3-3.2 ≡ 1.2'-2.1'	[[β, β°], [α, α°]]
2.3-3.2 ≡ 1.3'-2.1'	[[β, β°], [α, α°β°]]
2.3-3.2 ≡ 1.2'-2.2'	[[β, β°], [α, id2]]
2.3-3.2 ≡ 1.3'-2.2'	[[β, β°], [α, β°]]
2.3-3.2 ≡ 1.3'-2.3'	[[β, β°], [α, id3]]
2.3-3.3 ≡ 1.1'-2.1'	[[β, id3], [α, id1]]
2.3-3.3 ≡ 1.2'-2.1'	[[β, id3], [α, α°]]
2.3-3.3 ≡ 1.3'-2.1'	[[β, id3], [α, α°β°]]
2.3-3.3 ≡ 1.2'-2.2'	[[β, id3], [α, id2]]
2.3-3.3 ≡ 1.3'-2.2'	[[β, id3], [α, β°]]
2.3-3.3 ≡ 1.3'-2.3'	[[β, id3], [α, id3]]

**O/I ≡ O'/I'**

2.1-3.1 ≡ 2.1'-3.1'	[[β, id1], [β, id1]]
2.1-3.1 ≡ 2.2'-3.1'	[[β, id1], [β, α°]]
2.1-3.1 ≡ 2.3'-3.1'	[[β, id1], [β, α°β°]]

**M'/O' ≡ O/I**

1.1'-2.1' ≡ 2.1-3.1	[[α, id1], [β, id1]]
1.2'-2.1' ≡ 2.1-3.1	[[α, α°], [β, id1]]
1.3'-2.1' ≡ 2.1-3.1	[[α, α°β°], [β, id1]]
1.2'-2.2' ≡ 2.1-3.1	[[α, id2], [β, id1]]
1.3'-2.2' ≡ 2.1-3.1	[[α, β°], [β, id1]]
1.3'-2.3' ≡ 2.1-3.1	[[α, id3], [β, id1]]
1.1'-2.1' ≡ 2.2-3.1	[[α, id1], [β, α°]]
1.2'-2.1' ≡ 2.2-3.1	[[α, α°], [β, α°]]
1.3'-2.1' ≡ 2.2-3.1	[[α, α°β°], [β, α°]]
1.2'-2.2' ≡ 2.2-3.1	[[α, id2], [β, α°]]
1.3'-2.2' ≡ 2.2-3.1	[[α, β°], [β, α°]]
1.3'-2.3' ≡ 2.2-3.1	[[α, id3], [β, α°]]
1.1'-2.1' ≡ 2.3-3.1	[[α, id1], [β, α°β°]]
1.2'-2.1' ≡ 2.3-3.1	[[α, α°], [β, α°β°]]
1.3'-2.1' ≡ 2.3-3.1	[[α, α°β°], [β, α°β°]]
1.2'-2.2' ≡ 2.3-3.1	[[α, id2], [β, α°β°]]
1.3'-2.2' ≡ 2.3-3.1	[[α, β°], [β, α°β°]]
1.3'-2.3' ≡ 2.3-3.1	[[α, id3], [β, α°β°]]
1.1'-2.1' ≡ 2.2-3.2	[[α, id1], [β, id2]]
1.2'-2.1' ≡ 2.2-3.2	[[α, α°], [β, id2]]
1.3'-2.1' ≡ 2.2-3.2	[[α, α°β°], [β, id2]]
1.2'-2.2' ≡ 2.2-3.2	[[α, id2], [β, id2]]
1.3'-2.2' ≡ 2.2-3.2	[[α, β°], [β, id2]]
1.3'-2.3' ≡ 2.2-3.2	[[α, id3], [β, id2]]
1.1'-2.1' ≡ 2.3-3.2	[[α, id1], [β, β°]]
1.2'-2.1' ≡ 2.3-3.2	[[α, α°], [β, β°]]
1.3'-2.1' ≡ 2.3-3.2	[[α, α°β°], [β, β°]]
1.2'-2.2' ≡ 2.3-3.2	[[α, id2], [β, β°]]
1.3'-2.2' ≡ 2.3-3.2	[[α, β°], [β, β°]]
1.3'-2.3' ≡ 2.3-3.2	[[α, id3], [β, β°]]
1.1'-2.1' ≡ 2.3-3.3	[[α, id1], [β, id3]]
1.2'-2.1' ≡ 2.3-3.3	[[α, α°], [β, id3]]
1.3'-2.1' ≡ 2.3-3.3	[[α, α°β°], [β, id3]]
1.2'-2.2' ≡ 2.3-3.3	[[α, id2], [β, id3]]
1.3'-2.2' ≡ 2.3-3.3	[[α, β°], [β, id3]]
1.3'-2.3' ≡ 2.3-3.3	[[α, id3], [β, id3]]

**O'/I' ≡ O/I**

2.1'-3.1' ≡ 2.1-3.1	[[β, id1], [β, id1]]
2.2'-3.1' ≡ 2.1-3.1	[[β, α°], [β, id1]]
2.3'-3.1' ≡ 2.1-3.1	[[β, α°β°], [β, id1]]







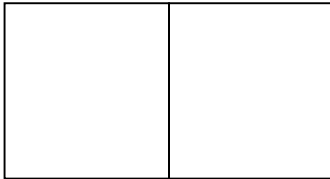




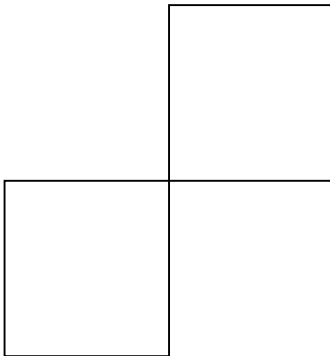
1.2-3.2 $\equiv$ 1.2'-3.1'	[[ $\beta\alpha$ , id2], [ $\beta\alpha$ , $\alpha^\circ$ ]]	1.2'-3.1' $\equiv$ 1.2-3.2	[[ $\beta\alpha$ , $\alpha^\circ$ ], [ $\beta\alpha$ , id2]]
1.2-3.2 $\equiv$ 1.3'-3.1'	[[ $\beta\alpha$ , id2], [ $\beta\alpha$ , $\alpha^\circ\beta^\circ$ ]]	1.3'-3.1' $\equiv$ 1.2-3.2	[[ $\beta\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta\alpha$ , id2]]
1.2-3.2 $\equiv$ 1.2'-3.2'	[[ $\beta\alpha$ , id2], [ $\beta\alpha$ , id2]]	1.2'-3.2' $\equiv$ 1.2-3.2	[[ $\beta\alpha$ , id2], [ $\beta\alpha$ , id2]]
1.2-3.2 $\equiv$ 1.3'-3.2'	[[ $\beta\alpha$ , id2], [ $\beta\alpha$ , $\beta^\circ$ ]]	1.3'-3.2' $\equiv$ 1.2-3.2	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , id2]]
1.2-3.2 $\equiv$ 1.3'-3.3'	[[ $\beta\alpha$ , id2], [ $\beta\alpha$ , id3]]	1.3'-3.3' $\equiv$ 1.2-3.2	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , id2]]
1.3-3.2 $\equiv$ 1.1'-3.1'	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , id1]]	1.1'-3.1' $\equiv$ 1.3-3.2	[[ $\beta\alpha$ , id1], [ $\beta\alpha$ , $\beta^\circ$ ]]
1.3-3.2 $\equiv$ 1.2'-3.1'	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , $\alpha^\circ$ ]]	1.2'-3.1' $\equiv$ 1.3-3.2	[[ $\beta\alpha$ , $\alpha^\circ$ ], [ $\beta\alpha$ , $\beta^\circ$ ]]
1.3-3.2 $\equiv$ 1.3'-3.1'	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , $\alpha^\circ\beta^\circ$ ]]	1.3'-3.1' $\equiv$ 1.3-3.2	[[ $\beta\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta\alpha$ , $\beta^\circ$ ]]
1.3-3.2 $\equiv$ 1.2'-3.2'	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , id2]]	1.2'-3.2' $\equiv$ 1.3-3.2	[[ $\beta\alpha$ , id2], [ $\beta\alpha$ , $\beta^\circ$ ]]
1.3-3.2 $\equiv$ 1.3'-3.2'	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , $\beta^\circ$ ]]	1.3'-3.2' $\equiv$ 1.3-3.2	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , $\beta^\circ$ ]]
1.3-3.2 $\equiv$ 1.3'-3.3'	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , id3]]	1.3'-3.3' $\equiv$ 1.3-3.2	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , $\beta^\circ$ ]]
1.3-3.3 $\equiv$ 1.1'-3.1'	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , id1]]	1.1'-3.1' $\equiv$ 1.3-3.3	[[ $\beta\alpha$ , id1], [ $\beta\alpha$ , id3]]
1.3-3.3 $\equiv$ 1.2'-3.1'	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , $\alpha^\circ$ ]]	1.2'-3.1' $\equiv$ 1.3-3.3	[[ $\beta\alpha$ , $\alpha^\circ$ ], [ $\beta\alpha$ , id3]]
1.3-3.3 $\equiv$ 1.3'-3.1'	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , $\alpha^\circ\beta^\circ$ ]]	1.3'-3.1' $\equiv$ 1.3-3.3	[[ $\beta\alpha$ , $\alpha^\circ\beta^\circ$ ], [ $\beta\alpha$ , id3]]
1.3-3.3 $\equiv$ 1.2'-3.2'	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , id2]]	1.2'-3.2' $\equiv$ 1.3-3.3	[[ $\beta\alpha$ , id2], [ $\beta\alpha$ , id3]]
1.3-3.3 $\equiv$ 1.3'-3.2'	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , $\beta^\circ$ ]]	1.3'-3.2' $\equiv$ 1.3-3.3	[[ $\beta\alpha$ , $\beta^\circ$ ], [ $\beta\alpha$ , id3]]
1.3-3.3 $\equiv$ 1.3'-3.3'	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , id3]]	1.3'-3.3' $\equiv$ 1.3-3.3	[[ $\beta\alpha$ , id3], [ $\beta\alpha$ , id3]]

6. In order to conclude, we show here a few basic pre-semiotic sign-configurations, which are to be compared to the semiotic sign-configurations in Toth (2008b, pp. 62 ss.):

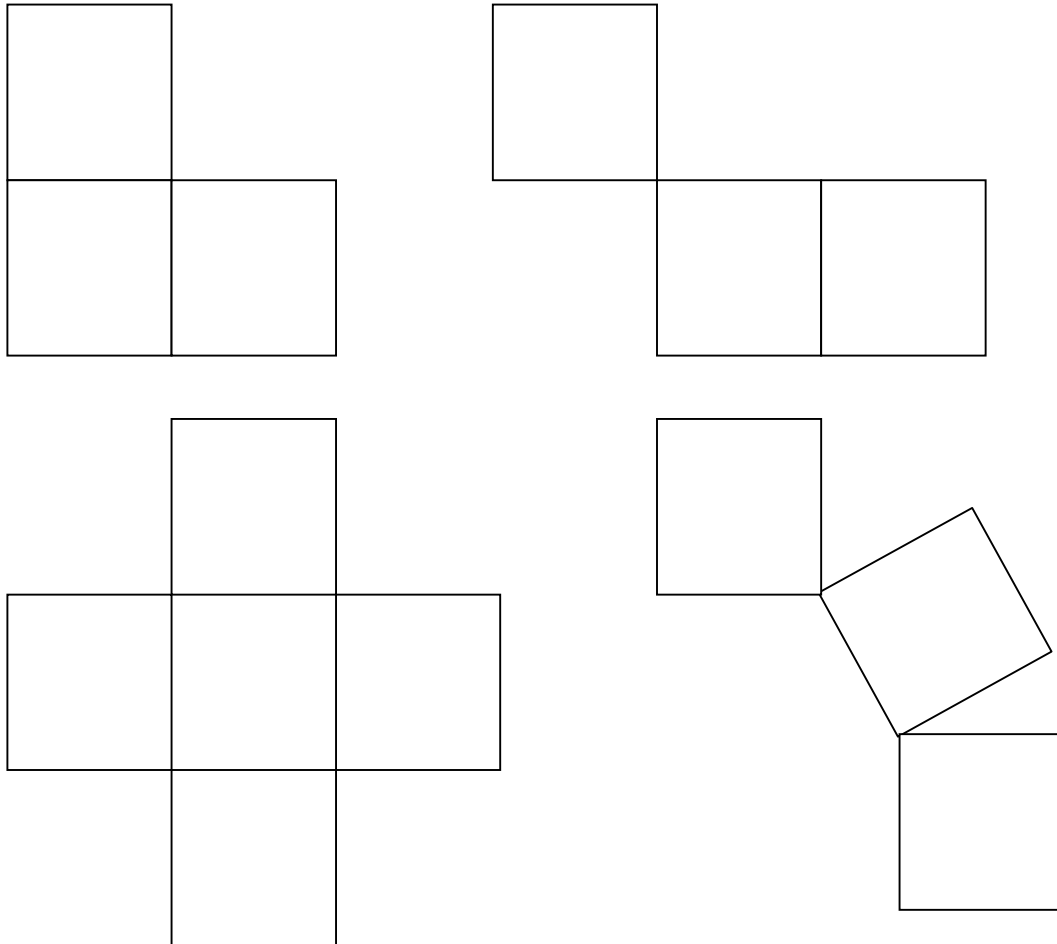
6.1. Type 1: Sign connections are pairs of dyadic sub-signs, i.e. the squares hang together by 2 vertices and 1 edge:



6.2. Type 2: Sign connections are single sub-signs, i.e. the squares hang together by 1 vertex and 0 edges:



6.3. Composite types: Sign connections are pairs of sub-signs as well as single sub-signs, i.e. the squares hang together by  $> 1$  vertices and  $>3$  edges. The configurations include both orthogonal and rotational connections (cf. Toth 2008f):



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