Prof. Dr. Alfred Toth

The category theoretic structure of pragmatic retrosemioses

1. Bense differentiated between virtual and effective signs: “We thus speak about a **virtual sign**, if only the formal triadic relation or the scheme of the abstract triadic relation is in focus, in opposition to the **effective sign** amongst which we shall understand the actual sign that is changing a situation in a space-time-place” (Bense 1975, p. 94). The transition from a virtual to an effective sign has to be recognized “as the **embedding** of the abstract triadic sign relation Z_v = R(M, O, I) in a concretely, both in space and in time fixed effective triadic relation Z_e = R(K, U, I_e) that results necessarily from the use and application situation as given by the environment of the sign” (1975, p. 94). In the effective sign relation, K stands for channel (Kanal), U for environment (Umgebung), and I_v for the external interpretant (interpreter). Moreover, “in the virtual sign Z_v, we differentiate between the relation of designation (M → O), the relation of denomination (O → I) and the relation of use (I → M). The relation of use, i.e. the retrograde relation of the sign-internal interpretant to the sign-internal medium (I_v → M) by a sign-external interpreter I_e, generates the effective sign relation from the virtual sign relation and thus the sign-external relation of the proper sign situation” (1975, p. 95). Thus, the relation of use (I_e → M) can be understood as pragmatic retrosemiosis (1975, p. 97).

2. In order to cope with the semiosic definition of the sign as consisting of the relation of designation (M → O), the relation of denomination (O → I) and the relation of use (I → M), it is not sufficient to ascribe semiotic morphisms strictly to the triadic sign values a, c and e of a general sign relation (a.b c.d e.f), but one has also to take into account the trichotomic sign values (b, d, f). Moreover, since the medium relation is represented in the object relation and both are represented in the interpretant relation of the sign (M. ((M → O) → I)) so that the monadic relation is contained in the dyadic and both are contained in the triadic relation, I had proposed to define semiotic category theoretic morphisms over the cross-relational pairs of [[a.c, b.d], [c.e, d.f], [a.e, b.f]] of the general sign relation (Toth 2008b, c, d, e). In the case of the semiosic definition of the sign consisting of the three semiotic relations, however, we have to start from an abstract sign relation (3.a 2.b 1.c) and ascribe semiotic morphisms to the following pairs of triadic-trichotomic sub-signs: [[3.2, a.b], [2.1, b.c], [1.c, c.a]. In doing so, the 10 sign classes can be written as follows:

1. (3.1 2.1 1.1) → ((1.1 2.1), (2.1 3.1), (3.1 1.1)) → [[α, id1], [β, id1], [α°β°, id1]]
2. (3.1 2.1 1.2) → ((1.2 2.1), (2.1 3.1), (3.1 1.2)) → [[α, α°], [β, id1], [α°β°, α]]
3. (3.1 2.1 1.3) → ((1.3 2.1), (2.1 3.1), (3.1 1.3)) → [[α, α°β°], [β, id1], [α°β°, βα]]
4. (3.1 2.2 1.2) → ((1.2 2.2), (2.2 3.1), (3.1 1.2)) → [[α, id2], [β, α°], [α°β°, α]]
5. (3.1 2.2 1.3) → ((1.3 2.2), (2.2 3.1), (3.1 1.3)) → [[α, β°], [β, α°], [α°β°, βα]]
6. (3.1 2.3 1.3) → ((1.3 2.3), (2.3 3.1), (3.1 1.3)) → [[α, id3], [β, α°β°], [α°β°, βα]]
7. (3.2 2.2 1.2) → ((1.2 2.2), (2.2 3.2), (3.2 1.2)) → [[α, id2], [β, id2], [α°β°, id2]]
8. (3.2 2.2 1.3) → ((1.3 2.2), (2.2 3.2), (3.2 1.3)) → [[α, β°], [β, α°], [α°β°, βα]]
9. (3.2 2.3 1.3) → (1.3 2.3), (2.3 3.2), (3.2 1.3)) → [α, id3], [β, β°], [α°β°, β]]

$$10.(3.3\ 2.3\ 1.3) \rightarrow \quad ((1.3\ 2.3), (2.3\ 3.3), (3.3\ 1.3)) \quad \rightarrow \quad [[\alpha, \mathrm{id}3], [\beta, \mathrm{id}3], \ [\alpha^\circ\beta^\circ, \mathrm{id}3]]$$

3. As one sees easily, the 10 sign classes can be grouped together according to their common relations of use:

$$1.\ (3.1\ 2.1\ 1.1) \rightarrow \quad ((1.1\ 2.1), (2.1\ 3.1), \boxed{(3.1\ 1.1}\ )) \rightarrow \quad [[\alpha, \mathrm{id}1], [\beta, \mathrm{id}1], \boxed{[\alpha^\circ\beta^\circ, \mathrm{id}1}\ ]]$$

$$2.\ (3.1\ 2.1\ 1.2) \rightarrow \quad ((1.2\ 2.1), (2.1\ 3.1), \boxed{(3.1\ 1.2}\ )) \rightarrow \quad [[\alpha, \alpha^\circ], [\beta, \mathrm{id}1], \ \boxed{[\alpha^\circ\beta^\circ, \alpha}]]$$
$$4.\ (3.1\ 2.2\ 1.2) \rightarrow \quad ((1.2\ 2.2), (2.2\ 3.1), \boxed{(3.1\ 1.2}\ )) \rightarrow \quad [[\alpha, \mathrm{id}2], [\beta, \alpha^\circ], \ \boxed{[\alpha^\circ\beta^\circ, \alpha}]]$$

$$3.\ (3.1\ 2.1\ 1.3) \rightarrow \quad ((1.3\ 2.1), (2.1\ 3.1), \boxed{(3.1\ 1.3}\ )) \rightarrow \quad [[\alpha, \alpha^\circ\beta^\circ], [\beta, \mathrm{id}1], \boxed{[\alpha^\circ\beta^\circ, \beta\alpha}\ ]]$$
$$5.\ (3.1\ 2.2\ 1.3) \rightarrow \quad ((1.3\ 2.2), (2.2\ 3.1), \boxed{(3.1\ 1.3}\ )) \rightarrow \quad [[\alpha, \beta^\circ], [\beta, \alpha^\circ], \quad \boxed{[\alpha^\circ\beta^\circ, \beta\alpha}\ ]]$$
$$6.\ (3.1\ 2.3\ 1.3) \rightarrow \quad ((1.3\ 2.3), (2.3\ 3.1), \boxed{(3.1\ 1.3}\ )) \rightarrow \quad [[\alpha, \mathrm{id}3], [\beta, \alpha^\circ\beta^\circ], \boxed{[\alpha^\circ\beta^\circ, \beta\alpha}\ ]]$$

$$7.\ (3.2\ 2.2\ 1.2) \rightarrow \quad ((1.2\ 2.2), (2.2\ 3.2), \boxed{(3.2\ 1.2}\ )) \rightarrow \quad [[\alpha, \mathrm{id}2], [\beta, \mathrm{id}2], \ \boxed{[\alpha^\circ\beta^\circ, \mathrm{id}2}]]$$

$$8.\ (3.2\ 2.2\ 1.3) \rightarrow \quad ((1.3\ 2.2), (2.2\ 3.2), \boxed{(3.2\ 1.3}\ )) \rightarrow \quad [[\alpha, \beta^\circ], [\beta, \alpha^\circ], \quad \boxed{[\alpha^\circ\beta^\circ, \beta]]}$$
$$9.\ (3.2\ 2.3\ 1.3) \rightarrow \quad (1.3\ 2.3), (2.3\ 3.2), \boxed{(3.2\ 1.3}\ )) \rightarrow \quad [\alpha, \mathrm{id}3], [\beta, \beta^\circ], \quad \boxed{[\alpha^\circ\beta^\circ, \beta]]}$$

$$10.(3.3\ 2.3\ 1.3) \rightarrow \quad ((1.3\ 2.3), (2.3\ 3.3), \boxed{(3.3\ 1.3}\ )) \rightarrow \quad [[\alpha, \mathrm{id}3], [\beta, \mathrm{id}3], \ \boxed{[\alpha^\circ\beta^\circ, \mathrm{id}3}\ ]]$$

We therefore find in the system of the 10 sign classes the following types of relations of use which determine the complete system of pragmatic retrosemioses:

$$(3.1 \begin{array}{l} \nearrow 1.1) \\ \rightarrow 1.2) \\ \searrow 1.3) \end{array} \qquad (3.2 \begin{array}{l} \nearrow 1.2) \\ \rightarrow 1.3) \end{array} \qquad (3.3 \ \rightarrow \ 1.3)$$

4. Bense further noticed, "that the retrosemioses, determined by the actual dyads, do not run inside of the trichotomies, like the Peircean 'replicas' do, but inside of the main triads" (1975, p. 115). Since we already had shown the category theoretic structure of replicas (cf. Toth 2008a, pp. 164 s.), we may here formalize exactly Bense's remark and complete the structure of pragmatic retrosemioses by the structure of replicas:

4.1. Replicas: trichotomic retrosemioses

4.1.1. $(3.1\ 2.1\ 1.2) \leftarrow (3.1\ 2.1\ 1.3) \equiv$
$[[\beta^\circ, \mathrm{id}1], [\alpha^\circ, \alpha]] \quad \leftarrow [[\beta^\circ, \mathrm{id}1], [\alpha^\circ, \beta\alpha]]$

4.1.2. $(3.1\ 2.2\ 1.2) \leftarrow (3.1\ 2.2\ 1.3) \quad \leftarrow (3.1\ 2.3\ 1.3) \equiv$
$[[\beta^\circ, \alpha], [\alpha^\circ, \mathrm{id}2]] \quad \leftarrow [[\beta^\circ, \alpha], [\alpha^\circ, \beta]] \quad \leftarrow [[\beta^\circ, \beta\alpha], [\alpha^\circ, \mathrm{id}3]]$

4.1.3. $(3.2\ 2.2\ 1.2) \leftarrow (3.2\ 2.2\ 1.3) \quad \leftarrow (3.2\ 2.3\ 1.3) \quad \leftarrow (3.3\ 2.3\ 1.3) \equiv$
$[[\beta^\circ, \mathrm{id}2], [\alpha^\circ, \mathrm{id}2]] \leftarrow [[\beta^\circ, \mathrm{id}2], [\alpha^\circ, \beta]] \leftarrow [[\beta^\circ, \beta], [\alpha^\circ, \mathrm{id}3]] \leftarrow [[\beta^\circ, \mathrm{id}3], [\alpha^\circ, \mathrm{id}3]]$

4.2. Functions of use: triadic retrosemioses

4.2.1. $(3.1 \rightarrow 1.1))$	$\equiv$	$[\alpha°\beta°, id1]]$	4.2.4. $(3.2 \rightarrow 1.2))$	$\equiv$	$[\alpha°\beta°, id2]]$
4.2.2 $(3.1 \rightarrow 1.2))$	$\equiv$	$[\alpha°\beta°, \alpha]]$	4.2.5. $(3.2 \rightarrow 1.3))$	$\equiv$	$[\alpha°\beta°, \beta]]$
4.2.3. $(3.1 \rightarrow 1.3))$	$\equiv$	$[\alpha°\beta°, \beta\alpha]]$	4.2.6. $(3.3 \rightarrow 1.3))$	$\equiv$	$[\alpha°\beta°, id3]]$

Thus, we have formalized the two main types of retrosemiosic relations of the complete triadic sign relation.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Toth, Alfred, Die kategorietheoretische und kategoriale Struktur der semiotischen Replizierung. In: Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008, pp. 164 s.

Toth, Alfred, Triples of morphisms for sign sets I. Ch. 2 (2008b)

Toth, Alfred, Triples of morphisms for sign sets II. Ch. 14 (2008c)

Toth, Alfred, Triples of morphisms for sign sets III. Ch. 16 (2008d)

Toth, Alfred, Fuzzy semiotic sets of second order (Triples of morphisms for sign sets IV). Ch. 30 (2008e)