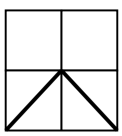
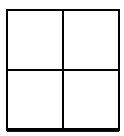
Prof. Dr. Alfred Toth

Semiotic Motzkin and Schröder paths

1. The Motzkin numbers describe the number of paths from the southwest corner of a grid to the southeast corner, using only steps northeast, east, and southeast. Motzkin numbers are also used in order to determine the number of different ways of drawing non-intersecting chords on a circle between n points (Motzkin 1948; Donaghey and Shapiro 1977).

2. In Toth (2008b), we have used grids to be mapped onto the respective semiotic matrices of $SR_{2,2}$, $SR_{3,3}$, $SR_{4,3}$, and $SR_{4,4}$, the intersections of the networks being associated with the sub-signs of the respective matrices.

In a 2 × 2 gird, there are 2 Motzkin paths:

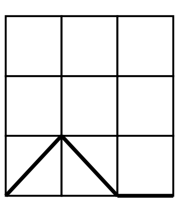
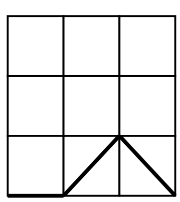
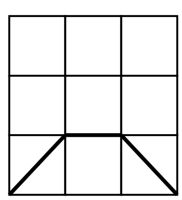
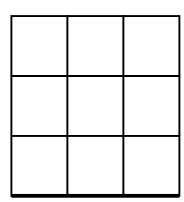
If this grid corresponds to the matrix of $SR_{3,3}$ or $SR_{4,3}$, we have:

1. ((3.1, 2.2), (2.2, 3.3)) ≡ [[β°, α], [β, β]]
2. ((3.1, 3.2), (3.2, 3.3)) ≡ [[id3, α], [id3, β]]

If this grid is a fragment of the matrix of $SR_{4,4}$, we get:

3. ((3.0, 2.1), (2.1, 3.2)) ≡ [[β°, γ], [β, α]]
4. ((3.0, 3.1), (3.1, 3.2)) ≡ [[id3, γ], [id3, α]]

In a 3 × 3 grid, there are 4 Motzkin paths:

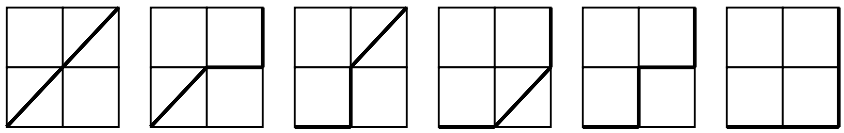
   

Since the 3 × 3 grid can only be a model of the matrix of $SR_{4,4}$, we have:

5. ((3.0, 2.1), (2.1, 3.2), (3.2, 3.3)) ≡ [[β°, γ], [β, α], [id3, β]]
6. ((3.0, 3.1), (3.1, 2.2), (2.2, 3.3)) ≡ [[id3, γ], [β°, α], [β, β]]
7. ((3.0, 2.1), (2.1, 2.2), (2.2, 3.3)) ≡ [[β°, γ], [id2, α], [β, β]]
8. ((3.0, 3.1), (3.1, 3.2), (3.2, 3.3)) ≡ [[id3, γ], [id3, α], [id3, β]]

3. The Schröder numbers describe the number of paths from the southwest corner of an $n \times n$ grid to the northeast corner using only single steps north, northeast, or east, that do not rise above the SW-NE diagonal (Weisstein 1999).

In a 2×2 grid, there are 6 Schröder paths:



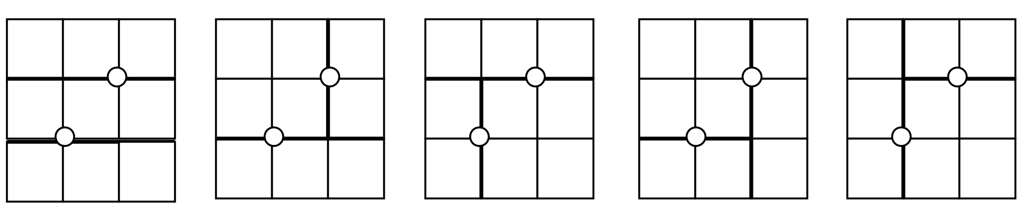
If this grid corresponds to the matrix of $SR_{3,3}$ or $SR_{4,4}$, we have:

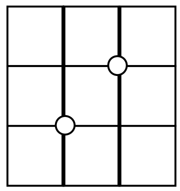
1. ((3.1, 2.2), (2.2, 1.3)) $\equiv [[\beta^\circ, \alpha], [\alpha^\circ, \beta]]$
2. ((3.1, 2.2), (2.2, 2.3), (2.3, 1.3)) $\equiv [[\beta^\circ, \alpha], [\mathrm{id}2, \beta]]$
3. ((3.1, 3.2), (3.2, 2.2), (2.2, 1.3)) $\equiv [[\mathrm{id}3, \alpha], [\beta^\circ, \mathrm{id}2], [\alpha^\circ, \beta]]$
4. ((3.1, 3.2), (3.2, 2.3), (2.3, 1.3)) $\equiv [[\mathrm{id}3, \alpha], [\beta^\circ, \beta], [\alpha^\circ, \mathrm{id}3]]$
5. ((3.1, 3.2), (3.2, 2.2), (2.2, 2.3), (2.3, 1.3)) $\equiv [[\mathrm{id}3, \alpha], [\beta^\circ, \mathrm{id}2], [\mathrm{id}2, \beta], [\alpha^\circ, \mathrm{id}3]]$
6. ((3.1, 3.2), (3.2, 3.3), (3.3, 2.3), (2.3, 1.3)) $\equiv [[\mathrm{id}3, \alpha], [\mathrm{id}3, \beta], [\beta^\circ, \mathrm{id}3], [\alpha^\circ, \mathrm{id}3]]$

If this grid is a fragment of the matrix of $SR_{4,3}$, we get:

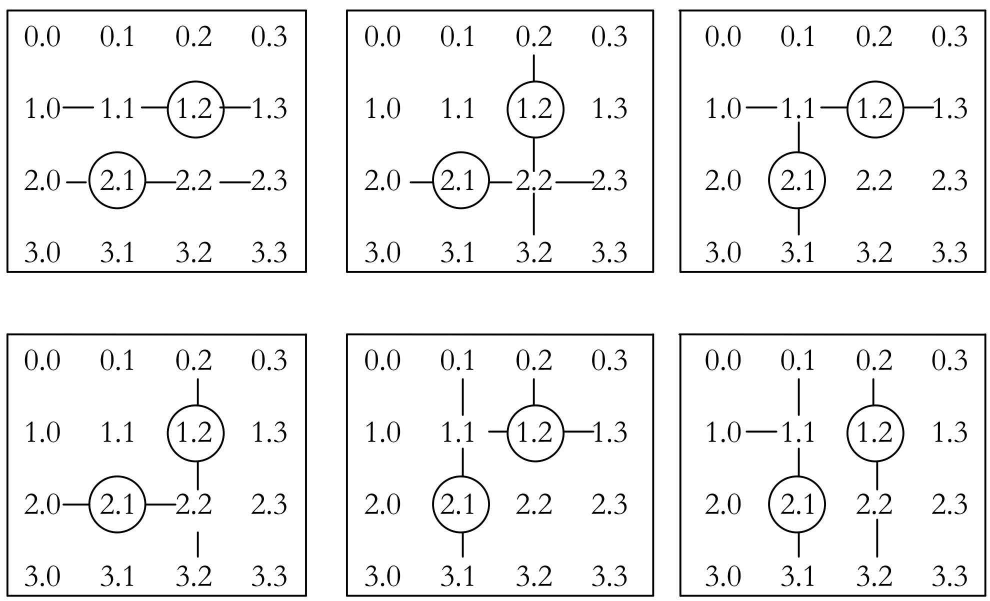
7. ((3.0, 2.1), (2.1, 1.2)) $\equiv [[\beta^\circ, \gamma], [\alpha^\circ, \alpha]]$
8. ((3.0, 2.1), (2.1, 2.2), (2.2, 1.2)) $\equiv [[\beta^\circ, \gamma], [\mathrm{id}2, \alpha], [\alpha^\circ, \mathrm{id}2]]$
9. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.2)) $\equiv [[\mathrm{id}3, \gamma], [\beta^\circ, \mathrm{id}1], [\alpha^\circ, \alpha]]$
10. ((3.0, 3.1), (3.1, 2.2), (2.2, 1.2)) $\equiv [[\mathrm{id}3, \gamma], [\beta^\circ, \alpha], [\alpha^\circ, \mathrm{id}2]]$
11. ((3.0, 3.1), (3.1, 2.1), (2.1, 2.2), (2.2, 1.2)) $\equiv [[\mathrm{id}3, \gamma], [\beta^\circ, \mathrm{id}1], [\mathrm{id}2, \alpha], [\alpha^\circ, \mathrm{id}2]]$
12. ((3.0, 3.1), (3.1, 3.2), (3.2, 2.2), (2.2, 1.2)) $\equiv [[\mathrm{id}3, \gamma], [\mathrm{id}3, \alpha], [\beta^\circ, \mathrm{id}2], [\alpha^\circ, \mathrm{id}2]]$

Schröder numbers can also be used in order to count the number to divide a rectangle into $n + 1$ smaller rectangles using n cuts. With the restriction that there are n points inside the rectangle, no two of these points falling on the same line parallel to either the x-axis or y-axis, and each cut intersects one of the points and divides only a single rectangle in two. In the following we show the 6 rectangulations of a 3×3 grid into 3 rectangles using two cuts (Weisstein 1999):





Thus, Schröder rectangulations can also be used to divide semiotic matrices into part-matrices containing only triadic, only trichotomic or mixed triadic-trichotomic sub-signs:



Therefore, in all 6 rectangulations, the sub-signs (1.2) and (2.1) mark the **semiotic border** of adjacent rectangles, a notion that we will further us in semiotic mereotopology (cf. Toth 2008a).

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