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Towards a semiotic mereology

1. Stanislaw Lesniewski developed "mereology" in 1927 to refer to a formal theory of part-whole relations as a concurrence to set theory, which he rejected. Thus, mereology seems to be an appropriate theory for semiotic relations, since the sign has been defined by Peirce and Bense as a triadic relation over a monadic, a dyadic and a triadic relation in which the triadic relation includes both the dyadic and the monadic, and the dyadic relation includes the monadic (Bense 1975, p. 82 ss.). In the present study, all I intend to establish are some basics for a semiotic mereology including some concepts for a semiotic mereotopology.

2.1. A mereological "system" is a first-order theory (with identity) whose universe of discourse consists of wholes and their respective parts, collectively called "objects". A mereological system requires at least one primitive binary relation, which is usually called "parthood" (or inclusion). We will write Pxy for "x is a part of y".

The sign class (3.1 2.1 1.3) consists of the monadic relation (1.3), which is part of the dyadic relation (2.1 1.3), of the dyadic relation (2.1 1.3), which is part of the triadic relation (3.1 2.2 1.3), and of the triadic relation (3.1 2.1 1.3) which is part of itself.

2.2. The following definitions follow immediately from the notion of Parthood (cf. Hovda 2006):

2.2.1. PPxy: "x is a proper part of y":

$$PPxy \leftrightarrow (Pxy \wedge \neg Pyx)$$

Only the triadic order (3.1 2.1 1.3) of the sign class (3.1 2.1 1.3) is a proper part of this sign class.

2.2.2. An object lacking proper parts is an atom. In semiotics, the atoms are therefore the sub-signs displayed in the semiotic matrix: (1.1), (1.2), (1.3), (2.1), (2.2), (2.3), (3.1), (3.2), (3.3). Since the mereological universe consists of all objects we wish to think about, it must contain all signs, too, for a sign is by definition nothing else than an object transformed by thetic introduction into a meta-object (Bense 1967, p. 9).

2.2.3. Oxy: "x and y overlap":

$$Oxy \leftrightarrow \exists z\ [Pzx \wedge Pzy]$$

E.g., the sign relations (3.1 2.1 1.3) overlaps both (3.1 2.1) and (2.1 1.3).

2.2.4. Uxy: “x and y underlap”:

$Uxy \leftrightarrow \exists z\ [Pxz \wedge Pyz]$

E.g., since (3.1 2.1) and (2.1 1.3) both are part of z = (3.1 2.1 1.3), they underlap it.

2.3. The axioms of parthood:

2.3.1. Reflexivity:

Pxx

Since each object can be introduced as a sign (2.2.2.), reflexivity of parthood means for semiotics, that each sign is part of itself, which is true, since the sign is self-containing pace the relation of thirdness.

2.3.2. Antisymmetry:

$(Pxy \wedge Pyx) \rightarrow x = y$

Antisymmetry guarantees that the sub-signs (1.2) ≠ (2.1), (1.3) ≠ (3.1), (2.3) ≠ (3.2) (cf. Toth 1996).

2.3.3. Transitivity:

$(Pxy \wedge Pyz) \rightarrow Pxz$

Transitivity guarantees that if there are two sub-signs (a.b) and (b.c), then also the sub-signs (a.c) are defined; e.g., if there are (1.2) and (2.3), then (1.3) is defined, too.

2.3.4. Weak Supplementation: If PPxy holds, there exists a z such that Pzy holds, but Ozx does not:

$PPxy \rightarrow \exists x\ [Pzy \wedge \neg Ozx]$

E.g., if ((a.b), (c.d)) is a dyadic relation, then there is an (a.b), so that ((e.f), (c.d)) holds, but there is no overlap ((e.f), (a.b)). F.ex., ((a.b), (c.d)) = ((2.1), (3.1)), and (e.f) = (3.3), then ((3.3), (3.1)) holds, but ((3.3), (2.1)) is no overlap.

2.3.5. Strong Supplementation: If Pyx holds, there exists a z such that Pzy holds but Ozx does not:

$\neg Pyx \rightarrow \exists z[Pzy \wedge \neg Ozx]$

E.g., x = (2.1), and y = (3.1), then there is a z, so that (z, (3.1)), but there is no overlap (z, (2.1)).

2.3.6. Atomistic Supplementation: If Pxy does not hold, then there exists an axiom z such that Pzx holds but Ozy does not:

$\neg Pxy \rightarrow \exists z[Pzx \wedge \neg Ozy \wedge \neg \exists v[PPvz]]$

Let be again x = (2.1), y = (3.1). So, if there is no dyadic relation ((2.1), (3.1)), then there is a z, so that (z, (2.1)), and there is neither an overlap (z, (3.1)) nor is there a v, so that v is a proper part of z.

2.3.7. Top: There exists a "universal object", designated W, such that PxW holds for any x:

$\exists W \forall x\ [PxW]$

The semiotic Top of the triadic-trichotomic sign relation $SR_{3,3}$ is the complete system of all 27 triadic sign classes, so that for PxW x can be a monadic, dyadic or triadic relation. One should be aware that the system of the 10 sign classes, which are constructed from the system of the 27 sign classes by restriction of the semiotic inclusion order (3.a 2.b 1.c), where $a \leq b \leq c$, is NOT W, since in this case, dyadic relation like (2.2 1.1), (3.2 3.1), or (1.2 3.3), and triadic relation like (3.3 2.2 1.1), (3.3 2.2 1.3), or (3.1 2.3 1.1) would not be part of W.

2.3.8. Bottom: There exists an atomic "null object", designated N, such that PNx holds for any s:

$\exists N \forall x\ [PNx]$

A semiotic "zero-sign" has been first introduced in Toth (2007, pp. 64 ss.) as an element of the power set of the relation of prime-signs S = {.1., .2., .3.}, $\underline{P}$ = {{1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}, ∅}.

2.3.9. Sum: If Uxy holds, there exists a z, called the "sum" or "fusion" of x and y, such that the parts of z are just those objects which are parts of either x or y:

$Uxy \rightarrow \exists z \forall v\ [Ovz \leftrightarrow (Ovx \vee Ovy)]$

E.g., if there is a semiotic underlap (3.1 2.1), then there is f. ex. an overlap (3.1 2.1 1.3), when either ((2.1), (1.3)) or ((3.1), (1.3)) holds.

2.3.10. Product: If Oxy holds, there exists a z, called the "product" of x and y, such that the parts of z are just those objects which are parts of both x and y:

$Oxy \rightarrow \exists z \forall v\ [Pvz \leftrightarrow (Pvx \wedge Pvy)]$

E.g., if there is a semiotic overlap ((3.1), (2.1)), then there is a z for all v, so that there is a dyadic relation (v, z), if there are both the relation (v, (3.1)) and (v, (2.1)).

2.3.11. Unrestricted Fusion: Let $\varphi(x)$ be a first-order formula in which x is a free variable. Then the fusion of all objects satisfying φ exists:

$$\exists x\ z\ [\varphi(x) \rightarrow \forall y\ [Oyz \leftrightarrow (\varphi(x) \wedge Oyx)]]$$

Unrestricted Fusion corresponds to the principle of unrestricted comprehension of naive set theory which gives rise to Russell' paradox. There is no mereological counterpart to this paradox simply because parthood, unlike set membership, is reflexive (cf. 2.3.1).

2.3.12. Unique Fusion: The fusions, whose existence 2.3.11. assert, are also unique.

2.3.13. Atomicity: All objects are either atoms or fusions of atoms:

$$\exists yz\ [Pyx \wedge \neg PPzy]$$

In semiotics, the atoms are the 9 monadic sub-signs, and the fusions are both the dyadic sign-relations and for the sign-relation $SR_{3,3}$ the triadic sign classes and reality thematics.

3.1. We will now further examine some issues of "classical" mereology by Hovda (2006). We introduce the sign $\leq$ for parthood.

Basically, a parthood structure is defined as a structure that satisfies the partial-ordering axioms, set-theoretic minimal upper bound (the existence of a join for each non-empty set), the axiom of weak supplementation, and the laws of distributivity, which governs the minimal upper bounds of two-element sets as binary joins. Where x + y denotes the z with minimal upper bound (z, {x, y}), the axiom is

$$x \leq y + z \rightarrow (x \leq y \vee x \leq z \vee \exists y' \leq y\ \exists z' \leq z\ (x = y' + z')).$$

For semiotics, let x = (1.3), y = (1.2), and z = (1.1), then we get:

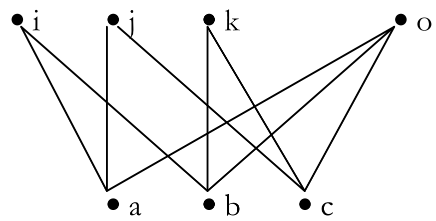
$$(1.3) \leq (1.2) + (1.1) \rightarrow ((1.3) \leq (1.2) \vee (1.3) \leq (1.1) \vee \exists y' \leq (1.2)\ \exists z' \leq (1.1)\ ((1.3) = y' + z'),$$

so, either y' = (1.1) or (1.2) and z' = (1.1), so that the final equation holds either as (1.3) = (1.1) + (1.3) or (1.3) = (1.2) + (1.3), because a triadic relation represents both a dyadic and a monadic, and a dyadic relation represents a monadic (cf. also Berger 1976).

The question naturally arises whether one can axiomatize classical mereology (according to Hovda 2006) in something like the manner of Tarski's compact axiomatization. Suppose we transitivity plus a universal closure for every instance of

$$\exists x\ \varphi_x \rightarrow \exists! z\ Fu_1\ (z, \varphi_x)$$

(with z not free in φ_x). This won't work, because there is a model of these axioms in which we have one element that is not parts of itself. Suppose then that we add reflexivity. Antisymmetry and weak supplementation may then be derived. Still, we get unwanted models. For example, consider (Hovda 2006, p. 20)



One can confirm that this is a model of the resulting system, as follows: Let D be the domain of M.6, and for each non-empty set $S \subseteq D$ write O(S) for the set $\{y \in D$: there is some $x \in S$ with $x \circ y\}$. Then we have

O({o}) = D

O({a}) = {a, i, j, o}	O({i}) = D\{c}
O({b}) = {b, i, k, o}	O({j}) = D\{b}
O({c}) = {c, j, k, o}	O({k}) = D\{a}

For a semiotic example, we may set a= (1.1), b = (1.2), c = (1.3). Then we get

O({o}) = {(1.1), (1.2), (1.3)}
O({a}) = {(1.1), (1.1 1.2), (1.1 1.3), (1.1 1.2 1.3)}
O({b}) = {(1.2), (1.1 1.2), (1.2 1.3), (1.1 1.2 1.3)}
O({c}) = {(1.3), (1.1 1.3), (1.2 1.3), (1.1 1.2 1.3)}
O({i}) = D\{(1.3)}
O({j}) = D\{(1.2)}
O({k}) = D\{(1.1)}

It is easy to check that M.6 is not a model of classical mereology.

3.2. We now show the close connection between classical mereology and Boolean algebra. "Basically, a complete Boolean algebra is a model of classical mereology with a single extra element called 0, an element that is a part of everything. In classical mereology, there is no 0, unless there is only one thing; one way to see this is that every object would then be a fusion of {0}; another is that weak supplementation fails, since 0 would be a proper part of everything else, but overlaps everything" (Hovda 2006, p. 21).

Nevertheless, we can define the existence of a neutral element by

$\forall x\ (0(x) \leftrightarrow \mathrm{Mub}(x, [y \mid y \neq y]))$

It is a consequence of this definition alone that

$\forall x \forall y\ (0(y) \rightarrow y \leq x)$

The definition of proper part, for which we will use the sign "$\ll$", remains the same. We use revised notions of overlap and disjointness as follows:

$s \bullet t := \exists x\ (\neg 0\ (x) \wedge x \leq s \wedge x \leq t)$

and

$s \wr t := \neg\ s \bullet t$ (Hovda 2006, p. 23)

Then we get the following neutral axioms, in which we use the abbreviation "Mub" for minimal upper bound:

Zero	$\forall x \forall y\ (0(x) \wedge 0(y) \rightarrow x = y)$
Transitivity	$\forall x \forall y \forall z\ (x \leq y \wedge y \leq z \rightarrow x \leq z)$
Weak Supplementation	$\forall x \forall y\ ((\neg 0(x) \wedge x \ll y \rightarrow \exists z\ (\neg 0(z) \wedge z \leq y \wedge x \wr z))$
Filtration	$\forall y \forall z\ ((\neg\ 0(y) \wedge y \leq z \wedge \mathrm{Mub}(z, \varphi_x)) \rightarrow \exists x(\varphi_x \wedge y \bullet x))$
Minimal Upper Bound:	$\exists x \varphi_x \rightarrow \exists z\ \mathrm{Mub}(z, \varphi_x)$ (Hovda 2006, p. 23)

For semiotics, let us assume that x = (1.1), y = (1.2) and z = (1.3). Then we have

ZeroU	$\forall(1.1)\forall(1.2)\ (0((1.1)) \wedge 0((1.2)y) \rightarrow (1.1) = (1.2)$. Thus, there is only one zero-sign $\varnothing$.
Transitivity	$\forall(1.1)\forall(1.2)\forall(1.3)\ ((1.1) \leq (1.2) \wedge (1.2) \leq (1.3) \rightarrow (1.1) \leq (1.3)$
$\mathrm{WeakSup}^N$	$\forall(1.1)\forall(1.2)\ ((\neg 0((1.1)) \wedge (1.1) \ll (1.2) \rightarrow \exists(1.3)\ (\neg 0((1.3)) \wedge (1.3) \leq (1.2) \wedge (1.1) \wr (1.3)))$
$\mathrm{Filtration}^N$	$\forall(1.2)\forall(1.3)\ ((\neg\ 0((1.2)) \wedge (1.2) \leq (1.3) \wedge \mathrm{Mub}((1.3), \varphi_x)) \rightarrow \exists(1.1)(\varphi_x \wedge (1.2) \bullet (1.1)))$
MubE	$\exists(1.1)\varphi_x \rightarrow \exists(1.3)\ \mathrm{Mub}((1.3), \varphi_x)$

In semiotics, the sub-signs are not to be considered variables, but expressions like $\forall(1.1)$ or $\exists(1.1)$ refer to the occurrence of the respective sub-signs in the sign classes or reality thematics of SS10 or SS27 (cf. Toth 2007, pp. 200 ss.)

3.3. Now, let us set up the axioms for complement and distributivity:

Complement	$\forall x\ \exists y\ (x + y = 1\ \wedge\ x \cdot y = 0)$
Distributivity	$\forall x \forall y \forall z\ (x + (y \cdot z) = (x + y) \cdot (x + z))$

In semiotics, the complement of the system of the 10 sign classes is the set of the "irregular" sign classes from the system of the 27 sign classes, i.e. all possible triadic trichotomic sign relation of the form (3.a 2.b 1.c) without inclusion restriction. Given x = (1.1), y = (1.2), z = (1.3), we have for semiotic distributivity: $((1.1) + ((1.2) \cdot (1.3)) = ((1.1) + (1.2)) \cdot ((1.1) + (1.3))) = ((1.2) \cdot (1.3)) = (1.2) = ((1.2) \cdot (1.3))$, which is correct.

One can then derive that the object asserted to exist by the axiom of complement is unique, using the following theorems to the left, for which we give semiotic examples to the right:

$x \cdot y = x \leftrightarrow x \leq y \leftrightarrow x + y = y$	$(1.1) \cdot (1.2) = (1.1) \leftrightarrow (1.1) \leq (1.2) \leftrightarrow (1.1) + (1.2) = (1.2)$
$(x \cdot y) + x = x$	$((1.1) \cdot (1.2)) + (1.1) = (1.1)$
$x \cdot (y + x) = x$	$(1.1) \cdot ((1.2) + (1.1)) = (1.1)$
$x = x + 0$	$(1.1) = (1.1) + 0$
$x = x \cdot 1$	$(1.1) = (1.1) \cdot 1$
$x \cdot y = y \cdot x$	$(1.1) \cdot (1.2) = (1.2) \cdot (1.1)$
$x + y = y + x$	$(1.1) + (1.2) = (1.2) + (1.1)$
$x + (y + z) = (x + y) + z$	$(1.1) + ((1.2) + (1.3)) = ((1.1) + (1.2)) + (1.3)$
$x \cdot (y \cdot z) = (x \cdot y) \cdot z$	$(1.1) \cdot ((1.2) \cdot (1.3)) = ((1.1) \cdot (1.2)) \cdot (1.3)$,

4. "In formal ontology, a branch of metaphysics, and in ontological computer science, mereotopology is a first-order theory, embodying mereological and topological concepts, of the relations among wholes, parts, parts of parts, and the boundaries between parts" (Cohn and Varzi 2003).

4.1. First, we have to point out a principle deficit of mathematical semiotics: It is impossible to define the notion of a semiotic closure in a non-trivial sense. In "pure" mathematics, a closure operator on a set A is a function c associating with each subset x of A a subset c(x) satisfying the following four constraints:

1. $\varnothing = c(\varnothing)$
2. $x \subseteq c(x)$
3. $c(c(x)) \subseteq c(x)$
4. $c(x) \cup c(y) = c(x \cup y)$

The empty sign $\varnothing$ is element of the power set $\underline{P}_S = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}, \varnothing\}$ of the sign-set $S = \{1, 2, 3\}$ (cf. Toth 2007, pp. 99 ss.).

However, in semiotics, empty signs are not thetically introduced and thus, strictly speaking (and paradoxically enough), not considered to be signs. Moreover, if we take, e.g., a trichotomic triad, i.e., any set of three sign classes, then how should we define the notions of interior, exterior, boundary, closure, open vs. closed semiotic set? We may well define them in many arbitrary ways, but no solution transcends triviality.

4.2. Therefore, it is impossible to establish a semiotic mereotopology by using the topology used in the standard versions of mereotopology (e.g., Varzi 1996, 1998; Cohn and Varzi 2003). However, since the classical mereological operations are compatible with the operations of Zermelo-Fraenkel set theory (ZFA), and since semiotics is compatible with ZFA, too, it follows that it is possible to establish a semiotic mereotopology strictly based on

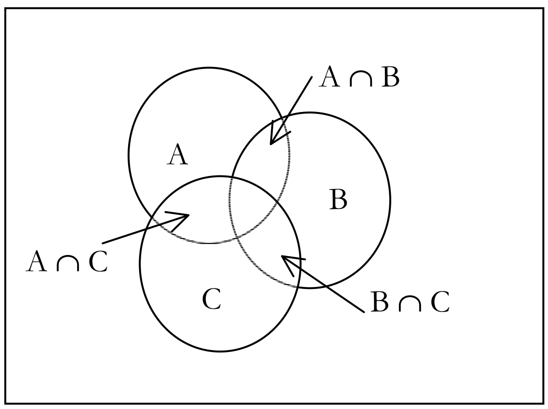
ZFA semiotics. Since the semiotic ZFA-laws are shown in Toth (2007, pp. 143 ss.), we restrict ourselves to just apply them to any arbitrary trichotomic triad (TrTr):

Let be

A = (3.1 2.1 1.1)
B = (3.1 2.2 1.3)
C = (3.3 2.3 1.3) (TrTr = {(3.1 2.1 1.1), (3.1 2.2 1.3), (3.3 2.3 1.3)})

Then,

$A \cap B = (3.1)$ $A \cup B = (3.1\ 2.2\ 2.1\ 1.3\ 1.1)$
$A \cap C = \varnothing$ $A \cup C = (3.3\ 3.1\ 2.3\ 2.1\ 1.3\ 1.1)$
$B \cap C = (1.3)$ $B \cup C = (3.3\ 3.1\ 2.3\ 2.2\ 1.3)$



Trichotomic Triad as Venn diagram

We define the notion of complement of a sign-set as we did in Toth (2007, pp. 14 ss.):

C(A) = (3.3 3.2 2.3 2.2 1.3 1.2)
C(B) = (3.3 3.2 2.3 2.1 1.2 1.1)
C(C) = (3.2 3.1 2.2 2.1 1.2 1.1)

Then we can establish all possible combinations of intersection (meet) and union (join) which correspond, as we have shown in part 1., to the mereological operations of overlap and fusion:

$C(A) \cap C(B) = (3.3\ 3.2\ 2.3\ 1.2)$ $C(A) \cup C(B) = (3.3\ 3.2\ 2.3\ 2.2\ 2.1\ 1.3\ 1.2\ 1.1)$
$C(A) \cap C(C) = (1.2)$ $C(A) \cup C(C) = (3.3\ 3.2\ 3.1\ 2.3\ 2.2\ 2.1\ 1.3\ 1.2\ 1.1)$
$C(B) \cap C(C) = (2.1\ 1.2\ 1.1)$ $C(B) \cup C(C) = (3.3\ 3.2\ 3.1\ 2.3\ 2.2\ 2.1\ 1.2\ 1.1)$

$A \cap C(B) = (2.1\ 1.1)$ $A \cup C(B) = (3.3\ 3.2\ 3.1\ 2.3\ 2.1\ 1.2\ 1.1)$
$A \cap C(C) = (3.2\ 2.1\ 1.1)\ (A \subset C(C))$ $A \cup C(C) = (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)$
$B \cap C(A) = (2.2\ 1.3)$ $B \cup C(A) = (3.3\ 3.2\ 3.1\ 2.3\ 2.2\ 1.3\ 1.2)$
$B \cap C(C) = (3.1\ 2.2)$ $B \cup C(C) = (3.2\ 3.1\ 2.2\ 2.1\ 1.3\ 1.2\ 1.1)$

$C \cap C(A) = (3.3\ 2.3\ 1.3)\ (C \subset C(A))$

$C \cap C(B) = (3.3\ 2.3)$

$C \cup C(A) = (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)$

$C \cup C(B) = (3.3\ 3.2\ 2.3\ 2.1\ 1.3\ 1.2\ 1.1)$

$(A \cap B) \cap C(A) = \varnothing$

$(A \cap B) \cap C(B) = \varnothing$

$(A \cap B) \cap C(C) = (3.1)$

$(A \cap C) \cap \text{everything} = \varnothing$

$(A \cap B) \cup C(A) = (3.3\ 3.2\ 3.1\ 2.3\ 2.2\ 1.3\ 1.2)$

$(A \cap B) \cup C(B) = (3.3\ 3.2\ 3.1\ 2.3\ 2.1\ 1.2\ 1.1)$

$(A \cap B) \cup C(C) = (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)$

$(A \cap C) \cup C(A) = (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)$

$(A \cap C) \cup C(B) = (3.3\ 3.2\ 2.3\ 2.1\ 1.2\ 1.1)$

$(A \cap C) \cup C(C) = (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)$

$(B \cap C) \cap C(A) = (1.3)$

$(B \cap C) \cap C(B) = \varnothing$

$(B \cap C) \cap C(C) = \varnothing$

$(B \cap C) \cup C(A) = (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)$

$(B \cap C) \cup C(B) = (3.3\ 3.2\ 2.3\ 2.1\ 1.3\ 1.2\ 1.1)$

$(B \cap C) \cup C(C) = (3.2\ 3.1\ 2.2\ 2.1\ 1.3\ 1.2\ 1.1)$

$(A \cap B) \cap (C(A) \cap C(B)) = \varnothing$

$(A \cap B) \cap (C(B) \cap C(C)) = \varnothing$

$(A \cap B) \cap (C(A) \cap C(C)) = \varnothing$

$(B \cap C) \cap (C(A) \cap C(B)) = \varnothing$

$(B \cap C) \cap (C(B) \cap C(C)) = \varnothing$

$(B \cap C) \cap (C(A) \cap C(C)) = \varnothing$

$(A \cap C) \cap (C(A) \cap C(B)) = \varnothing$

$(B \cap C) \cap (C(B) \cap C(C)) = \varnothing$

$(B \cap C) \cap (C(A) \cap C(C)) = \varnothing$

$(A \cap B) \cup (C(A) \cap C(B)) = (3.3\ 3.2\ 3.1\ 2.3\ 1.2)$

$(A \cap B) \cup (C(B) \cap C(C)) = (3.1\ 2.1\ 1.2\ 1.1)$

$(A \cap B) \cup (C(A) \cap C(C)) = (3.1\ 1.2)$

$(B \cap C) \cup (C(A) \cap C(B)) = (3.3\ 3.2\ 2.3\ 1.3\ 1.2)$

$(B \cap C) \cup (C(B) \cap C(C)) = (2.1\ 1.3\ 1.2\ 1.1)$

$(B \cap C) \cup (C(A) \cap C(C)) = (1.3\ 1.2)$

$(A \cap C) \cup (C(A) \cap C(B)) = (3.3\ 3.2\ 2.3\ 1.2)$

$(B \cap C) \cup (C(B) \cap C(C)) = (2.1\ 1.2\ 1.1)$

$(B \cap C) \cup (C(A) \cap C(C)) = (1.2)$

$A\ \Delta\ B = A \cup B - (A \cap B) = (2.2\ 2.1\ 1.3\ 1.1)$

$B\ \Delta\ C = B \cup C - (B \cap C) = (3.3\ 3.1\ 2.3\ 2.2)$

$A\ \Delta\ C = A \cup C - (A \cap C) = (3.3\ 3.1\ 2.3\ 2.1\ 1.3\ 1.1)$

$A \setminus (A \cap B) = (3.1\ 2.1\ 1.1) \setminus (3.1) = (2.1\ 1.1)$

$A \setminus (B \cap C) = (3.1\ 2.1\ 1.1) \setminus (1.3) = (3.1\ 2.1\ 1.1)$

$A \setminus (A \cap C) = (3.1\ 2.1\ 1.1) \setminus \varnothing = (3.1\ 2.1\ 1.1)$

$B \setminus (A \cap B) = (3.1\ 2.2\ 1.3) \setminus (3.1) = (2.2\ 1.3)$

$B \setminus (B \cap C) = (3.1\ 2.2\ 1.3) \setminus (1.3) = (3.1\ 2.2)$

$B \setminus (A \cap C) = (3.1\ 2.2\ 1.3) \setminus \varnothing = (3.1\ 2.2\ 1.3)$

$C \setminus (A \cap B) = (3.3\ 2.3\ 1.3) \setminus (3.1) = (3.3\ 2.3\ 1.3)$

$C \setminus (B \cap C) = (3.3\ 2.3\ 1.3) \setminus (1.3) = (3.3\ 2.3)$

$C \setminus (A \cap C) = (3.3\ 2.3\ 1.3) \setminus \varnothing = (3.3\ 2.3\ 1.3)$

$A \setminus (A \cap C(B)) = (3.1\ 2.1\ 1.1) \setminus ((3.1\ 2.1\ 1.1) \cap (3.3\ 3.2\ 2.3\ 2.1\ 1.2\ 1.1)) = (3.1)$

$A \setminus (A \cap C(C)) = (3.1\ 2.1\ 1.1) \setminus ((3.1\ 2.1\ 1.1) \cap (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)) = \varnothing$

$A \setminus (B \cap C(A)) = (3.1\ 2.1\ 1.1) \setminus ((3.1\ 2.2\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)) = (3.1\ 2.1\ 1.1)$

$A \setminus (B \cap C(C)) = (3.1\ 2.1\ 1.1) \setminus ((3.1\ 2.2\ 1.3) \cap (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)) = (2.1\ 1.1)$

$A \setminus (C \cap C(A)) = (3.1\ 2.1\ 1.1) \setminus ((3.3\ 2.3\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)) = (3.1\ 2.1\ 1.1)$

$A \setminus (C \cap C(B)) = (3.1\ 2.1\ 1.1) \setminus ((3.3\ 2.3\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.1\ 1.2\ 1.1)) = (3.1\ 2.1\ 1.1)$

$B \setminus (A \cap C(B)) = (3.1\ 2.2\ 1.3) \setminus ((3.1\ 2.1\ 1.1) \cap (3.3\ 3.2\ 2.3\ 2.1\ 1.2\ 1.1)) = (3.1\ 2.2\ 1.3)$
$B \setminus (A \cap C(C)) = (3.1\ 2.2\ 1.3) \setminus ((3.1\ 2.1\ 1.1) \cap (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)) = (2.2\ 1.3)$
$B \setminus (B \cap C(A)) = (3.1\ 2.2\ 1.3) \setminus ((3.1\ 2.2\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)) = (3.1)$
$B \setminus (B \cap C(C)) = (3.1\ 2.2\ 1.3) \setminus ((3.1\ 2.2\ 1.3) \cap (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)) = (1.3)$
$B \setminus (C \cap C(A)) = (3.1\ 2.2\ 1.3) \setminus ((3.3\ 2.3\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)) = (3.1\ 2.2)$
$B \setminus (C \cap C(B)) = (3.1\ 2.2\ 1.3) \setminus ((3.3\ 2.3\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.1\ 1.2\ 1.1)) = (3.1\ 2.2)$

$C \setminus (A \cap C(B)) = (3.3\ 2.3\ 1.3) \setminus ((3.1\ 2.1\ 1.1) \cap (3.3\ 3.2\ 2.3\ 2.1\ 1.2\ 1.1)) = (3.3\ 2.3\ 1.3)$
$C \setminus (A \cap C(C)) = (3.3\ 2.3\ 1.3) \setminus ((3.1\ 2.1\ 1.1) \cap (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)) = (3.3\ 2.3\ 1.3)$
$C \setminus (B \cap C(A)) = (3.3\ 2.3\ 1.3) \setminus ((3.1\ 2.2\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)) = (3.3\ 2.3)$
$C \setminus (B \cap C(C)) = (3.3\ 2.3\ 1.3) \setminus ((3.1\ 2.2\ 1.3) \cap (3.2\ 3.1\ 2.2\ 2.1\ 1.2\ 1.1)) = (3.3\ 2.3\ 1.3)$
$C \setminus (C \cap C(A)) = (3.3\ 2.3\ 1.3) \setminus ((3.3\ 2.3\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.2\ 1.3\ 1.2)) = \varnothing$
$C \setminus (C \cap C(B)) = (3.3\ 2.3\ 1.3) \setminus ((3.3\ 2.3\ 1.3) \cap (3.3\ 3.2\ 2.3\ 2.1\ 1.2\ 1.1)) = (1.3)$

Thus, by aid of semiotic overlap and semiotic fusion which are defined on semiotic set theory, we are able to construct a mereotopological semiotics, which is compatible with set theoretic semiotic topology constructed in Toth (2007, pp. 99 ss.).

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