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Mediation between morphisms and heteromorphisms in semiotic systems

1. In his new paper, Rudolf Kaehr (2009c) has defined the Diamond relation as follows:

Diamond relation DiamRel:

$R \in Cat, \; r \in Sat$

$$(R, r)^{(m)} \Longleftrightarrow Rel^{(m)} \,\|\, rel^{(m-1)}$$

Thus each relation R belongs, qua morphism, to a category, while each relation r belongs, qua heteromorphism, to a “saltatory”. Morphism and heteromorphism are not dual, but complementary, and so are category and saltatory.

2. However, in semiotics (cf. Kaehr 2009a, b), the unmediated 2-valued opposition between morphism and heteromorphism works only with sign classes that are constructed from 2-adic sub-signs of maximal contexture 3, e.g.:

$\times(2.1)_1 = (2.1)_1$	‖	$R(2.1)_1 = (1.2)_1$
$\times(2.2)_{1,2} = (2.2)_{1,2}$	‖	$R(2.2)_{1,2} = (2.2)_{2,1}$

“×” means here (monocontextural) dualization, “R” (polycontextural) reflection, thus dualization changes the order of the prime-signs constituting a sub-sign, while reflection also turns around the order of the contextures. So far, we have

Morphism: $(a.b)_i \rightarrow$ Heteromorphism: $(b.a)_i$
Morphism: $(a.b)_{i,j} \rightarrow$ Heteromorphisms: $(b.a)_{j,i}$

However, already here one possible mediation is lacking:

Morphism: $(a.b)_{i,j} \rightarrow$???: $(a.b)_{j,i}$

In other words: We need an operation "Y", which turns around only the contextures of a sub-sign, but not the sub-sign itself.

3. But now let us proceed to 4-contextural 3-adic semiotics. In order to make sure what we are speaking about, I present here again the 10 Peircean sign classes as 4-contextural sign classes:

$(3.1_{3,4}\ \ 2.1_{1,4}\ \ 1.1_{1,3,4})$
$(3.1_{3,4}\ \ 2.1_{1,4}\ \ 1.2_{1,4})$
$(3.1_{3,4}\ \ 2.1_{1,4}\ \ 1.3_{3,4})$
$(3.1_{3,4}\ \ 2.2_{1,2,4}\ \ 1.2_{1,4})$
$(3.1_{3,4}\ \ 2.2_{1,2,4}\ \ 1.3_{3,4})$
$(3.1_{3,4}\ \ 2.3_{2,4}\ \ 1.3_{3,4})$
$(3.2_{2,4}\ \ 2.2_{1,2,4}\ \ 1.2_{1,4})$
$(3.2_{2,4}\ \ 2.2_{1,2,4}\ \ 1.3_{3,4})$
$(3.2_{2,4}\ \ 2.3_{2,4}\ \ 1.3_{3,4})$
$(3.3_{2,3}\ \ 2.3_{2,4}\ \ 1.3_{3})$

As one sees, the genuine sub-signs (identitive morphisms) lie in 3 contextures, so that the maximal scheme for 4-contextural 3-adic sign classes is

$SCl(4,3) = (3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$

And here now the real problems with the semiotic adaptation of Diamond theory start:

1. A sub-sign like

$(a.b)_{i,j,k}$

as a morphism has no only its heteromorphisms

$(a.b)_{k,j,i}$,

but also 4 more "mediative" morphisms

$(a.b)_{i,k,j}$, $(a.b)_{k,j,i}$, $(a.b)_{j,i,k}$ and $(a.b)_{j,k,i}$.

2. By aid of our three operations above, we also get

$\times(a.b)_{i,j,k} = (b.a)_{i,j,k}$	$R(a.b)_{i,j,k} = (b.a)_{k,j,i}$	$Y(a.b)_{i,j,k} = (a.b)_{k,j,i}$
$\times(a.b)_{k,j,i} = (b.a)_{k,i,j}$	$R(a.b)_{k,j,i} = (b.a)_{i,j,k}$	$Y(a.b)_{k,j,i} = (a.b)_{i,j,k}$
$\times(a.b)_{i,k,j} = (b.a)_{i,k,j}$	$R(a.b)_{i,k,j} = (b.a)_{j,k,i}$	$Y(a.b)_{i,k,j} = (a.b)_{j,k,i}$
$\times(a.b)_{k,j,i} = (b.a)_{k,i,j}$	$R(a.b)_{k,j,i} = (b.a)_{i,j,k}$	$Y(a.b)_{k,j,i} = (a.b)_{i,j,k}$
$\times(a.b)_{j,i,k} = (b.a)_{j,i,k}$	$R(a.b)_{j,i,k} = (b.a)_{k,i,j}$	$Y(a.b)_{j,i,k} = (a.b)_{k,i,j}$
$\times(a.b)_{j,k,i} = (b.a)_{j,k,i}$	$R(a.b)_{j,k,i} = (b.a)_{i,k,j}$	$Y(a.b)_{j,k,i} = (a.b)_{i,k,j}$

3. Now, a 3-adic sign class consists per definitionem of three sub-signs:

$SCl(4,3) = (3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$

However, this means that we can permute first the sign class as such:

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$
$(3.a_{i,j,k}\ 1.c_{,j,k}\ 2.b_{,j,k})$
$(2.b_{,j,k}\ 3.a_{,j,k}\ 1.c_{i,j,k})$
$(2.b_{,j,k}\ 1.c_{,j,k}\ 3.a_{,j,k})$
$(1.c_{i,j,k}\ 3.a_{,j,k}\ 2.b_{,j,k})$
$(1.c_{,j,k}\ 2.b_{i,j,k}\ 3.a_{,j,k})$,

and second for also the contextures, and this for all three sub-signs. Therefore, we get exactly 126 permutations of (combinations of) sign classes and contextures per sign class (cf. Toth 2009). The combined permutations look for the first permutation, i.e. the sign class in "degenerative-retrosemiosic order":

$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{ijk})$		
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{ikj})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{ikj})$	
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{jik})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{jik})$	$(3.a_{ijk}\ 2.b_{jik}\ 1.c_{jik})$
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{jki})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{jki})$	$(3.a_{ijk}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{kij})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{kij})$	$(3.a_{ijk}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{kji})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{kji})$	$(3.a_{ijk}\ 2.b_{jik}\ 1.c_{kji})$

$(3.a_{ijk}\ 2.b_{jki}\ 1.c_{jki})$		
$(3.a_{ijk}\ 2.b_{jki}\ 1.c_{kij})$	$(3.a_{ijk}\ 2.b_{kij}\ 1.c_{kij})$	
$(3.a_{ijk}\ 2.b_{jki}\ 1.c_{kji})$	$(3.a_{ijk}\ 2.b_{kij}\ 1.c_{kji})$	$(3.a_{ijk}\ 2.b_{kji}\ 1.c_{kji})$
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{ijk})$		
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{ikj})$	$(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{ikj})$	
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{jik})$	$(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{jik})$	$(3.a_{ikj}\ 2.b_{jik}\ 1.c_{jik})$

$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{jki})$ $(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{jki})$ $(3.a_{ikj}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{kij})$ $(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{kij})$ $(3.a_{ikj}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{kji})$ $(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{kji})$ $(3.a_{ikj}\ 2.b_{jik}\ 1.c_{kji})$

$(3.a_{ikj}\ 2.b_{jki}\ 1.c_{jki})$
$(3.a_{ikj}\ 2.b_{jki}\ 1.c_{kij})$ $(3.a_{ikj}\ 2.b_{kij}\ 1.c_{kij})$
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$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{ijk})$
$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{ikj})$ $(3.a_{jik}\ 2.b_{ikj}\ 1.c_{ikj})$
$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{jik})$ $(3.a_{jik}\ 2.b_{ikj}\ 1.c_{jik})$ $(3.a_{jik}\ 2.b_{jik}\ 1.c_{jik})$
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$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{ijk})$
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$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{jik})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{jik})$ $(3.a_{kij}\ 2.b_{jik}\ 1.c_{jik})$
$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{jki})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{jki})$ $(3.a_{kij}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{kij})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{kij})$ $(3.a_{kij}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{kji})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{kji})$ $(3.a_{kij}\ 2.b_{jik}\ 1.c_{kji})$

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$(3.a_{kij}\ 2.b_{jki}\ 1.c_{kij})$ $(3.a_{kij}\ 2.b_{kij}\ 1.c_{kij})$
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$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{jik})$ $(3.a_{kji}\ 2.b_{ikj}\ 1.c_{jik})$ $(3.a_{kji}\ 2.b_{jik}\ 1.c_{jik})$
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$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{kij})$ $(3.a_{kji}\ 2.b_{ikj}\ 1.c_{kij})$ $(3.a_{kji}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{kji})$ $(3.a_{kji}\ 2.b_{ikj}\ 1.c_{kji})$ $(3.a_{kji}\ 2.b_{jik}\ 1.c_{kji})$

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Thus, we get for all 6 permutations 6 · 126 = 756 sign classes, and for all 10 sign classes therefore 7'560 sign classes. However, we must not forget the structural potential that lies in our three above operators, ×, R, and Y, so that at the end we have a semiotic system of no less **than 22'680 sign classes**.

4. But that is not all. In Toth (2008), based on Stiebing (1978), I had introduced 3-dimensional sign classes into semiotics. Monocontextural 3-dimensional sign classes have the following form

3-SCl = ((a.b.c) (d.e.f) (g.h.i)),

or, if we use Peirce's "normal form"

3-SCl = ((a.3.b) (c.2.d) (e.1.f)),

whereby one sees that (a, c, e) are the so-called "dimensional numbers". Because of the triadic form of each sub-sign, the geometrical model of 3-SCl is a cube, but we can still make it higher by adding more levels. Since, for the embedded

2-SCl = $((3.b_{i,j,k})\ (2.d_{i,j,k})\ (1.f_{i,j,k}))$,

we got 22'680 sign classes, and since there are n-levels, we do not only get

3! · 22'680 = 136'080, but

n! · 22'680 different sign classes (a, c, e ∈ {1, 2, 3, ..., n}

for sign classes constructed from 3-adic instead of 2-adic sub-signs.

5. However, the results obtained in this little contribution have enormous consequences for Diamond theory itself, because theoretically, we can surpass 3-adic sign classes and introduce 4-adic, 5-adic, 6-adic (, ..., -adic?) sign classes, the structural complexity of which grows astronomically because of the permutations, especially, if we also proceed to more than 4 contextures. Finally, we also can construct semiotic hypercubes and other nice high-dimensional polytopes that are not anymore based on cubic 3-sign classes, which are just made higher by adding more storeys, but by adding more dimensions. Since there are no formal restrictions concerning the order of dimensional numbers amongst themselves as well as in connection with prime-sign-numbers, already for a 4-dimensional (f.ex. tesseractic) sign model we have

4-SCl = ((a.b.3.c) (d.e.2.f) (g.h.1.i),
4-SCl = ((b.a.3.c) (e.d.2.f) (h.g.1.i)
(plus combinations)
4-SCl = ((3.a.b.c) (2.d.e.f) (1.g.h.i)
4-SCl = ((3.a.c.d) (2.d.f.e) (1.g.i.h)
(plus combinations)

If it is made clear, for which dimensions the variables of the dimensional numbers stand, we can also "scramble" them. Moreover, from the above constructions it results that a sign class can at the same time lie in more than 1 and maximally in 4 dimensions (if it is tesserarctic) as well as in several contextures (both qua sub-signs). That from here we have exciting connections to a quaternionic semiotics, I have already shown in a series of papers. Summa summarum, the incredibly huge amount of structural growth by introduction of contextures, permutations and dimensions in semiotics is hard to foresee.

However, to come back to Diamond theory, Kaehr has made clear that a diamond can not deal with more than a pair of morphism and hetero-morphism, the categorial/saltatorial equivalents for logical proposition and rejection. However, as we have shown already in chapter 1 of this paper, already in a 4-contextural semiotics, we have 4 mediative morphisms between morphisms and heteromorphisms. Therefore, the diamond model has to be substituted by another polygon. (And in high-dimensional semiotics even by a polytope?) Am I right that therefore, Leinster's n-category theory could give a model for a n-category/saltatory diamond theory (at least what concerns the semiotic dimensional numbers? And then: What about the set theoretic, arithmetic, logical and also topological consequences for the mediative morphisms of the original Diamond theory? However it will be, it seems to me that for once there may be an enormous input from semiotics for the future of "graphematics" (as Kaehr says), while up to now, semiotics has only profited from polycontexturals sciences, but never contributed to them.

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