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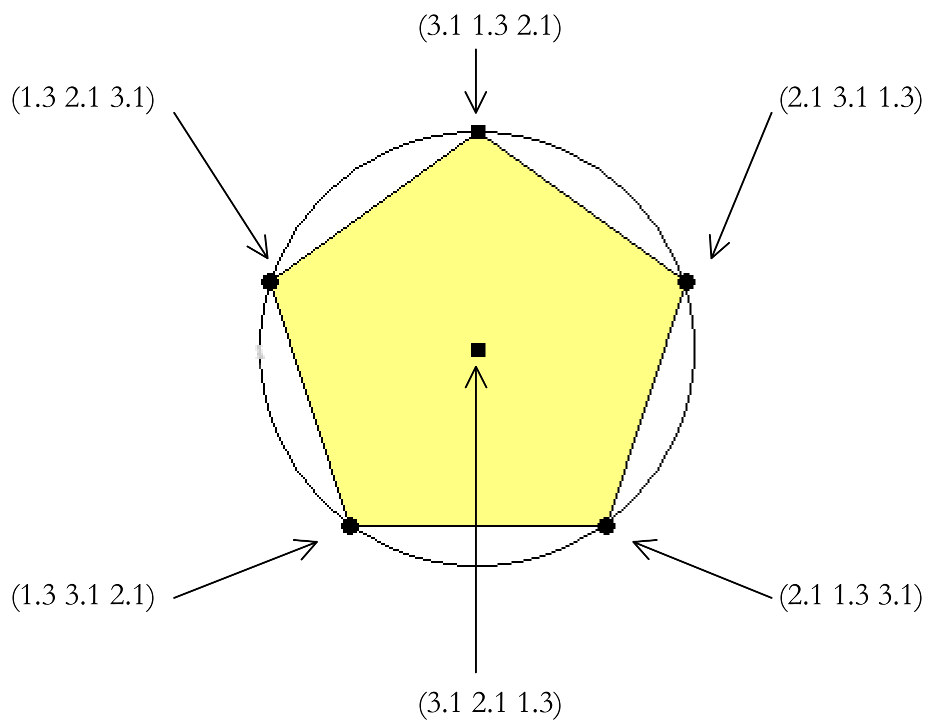
Convex and concave semiotic sets

1. In Euclidean space, an object is convex if for every pair of (arbitrarily chosen) points within the object, every point on the straight line-segment that joins them is also within the object. A set, which is not convex, is called non-convex. A set, whose complement is convex, is called concave.

2. We may now display the six transpositions of each sign class (and its dual reality thematic) in the shape of a pentagon, whereby the assignment of transpositions to the edges is arbitrary; all that is required is that the edges are pairwise different. One possibility is shown in the following example:



http://haftendorn.uni-lueneburg.de/mathe-lehramt.htm?show=http://haftendorn.uni-lueneburg.de/geo/grund/golden/fuenfeck.htm

While one transposition is in the center, the other five transpositions lie equidistant on a star-shaped figure whose connecting lines form the above regular pentagon. Therefore, the pentagon model of the semiotic transpositions forms a convex space and the lines connecting the outer five transpositions form a convex hull.

We may therefore introduce a convex semiotic hull operator CH, which generates the set of transpositions of a sign class (or more generally the set of all transpositions from one transposition). Since a sign relation can be introduced in its most abstract form as SR = (a.b c.d e.f), we get:

CH(a.b c.d e.f) = {(a.b c.d e.f), (a.b e.f c.d), (c.d a.b e.f), (c.d e.f a.b), (e.f a.b c.d), (e.f c.d a.b)}

Of course, we also have

CH(a.b c.d e.f) = {(a.b c.d e.f), ..., (e.f c.d a.b)}
CH(e.f c.d a.b) = {(e.f a.b c.d), ..., (a.b c.d e.f)}, etc.

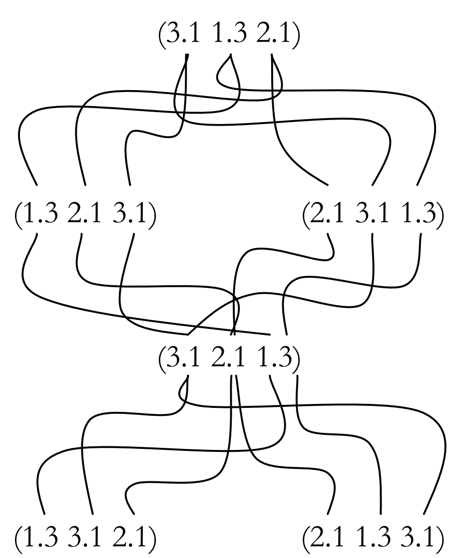
Now, let S be a sign class. Apparently, CH fulfills the requirements of a hull operator (Schwabhäuser 1971, p. 40):

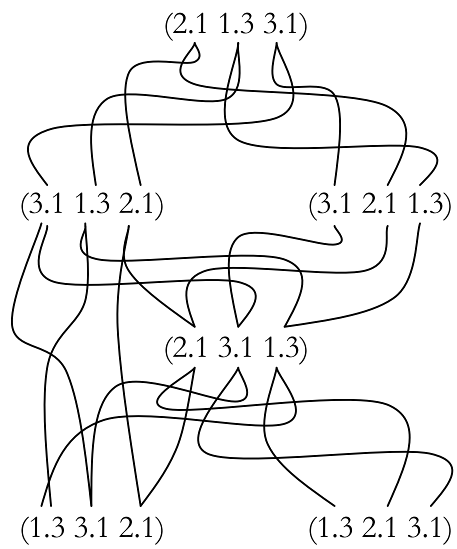
(1) $S \subset CH(S)$ (extensivity)
(2) If $S_1 \subset S_2$, then $CH(S_1) \subset CH(S_2)$ (monotony)
(3) $CH(CH(S)) \subset CH(S)$ (closure of CH),

since we have

(1) (3.1 2.1 1.3) ⊂ {(3.1 2.1 1.3), (3.1 1.3 2.1), (2.1 3.1 1.3), (2.1 1.3 3.1), (1.3 3.1 2.1), (1.3 2.1 3.1)}
(2) Let be S_1 = (2.1 3.1 1.3), S_2 = (3.1 2.1 1.3), then $S_1 \subset S_2$ and $CH(S_1) \subset CH(S_2)$
(3) Since CH(CH(S)) = CH(S), this requirement is fulfilled trivially.

3. However, semiotic sets are ordered sets (cf. Toth 2008). Therefore, from a pure semiotic point of view, (2.1 3.1 1.3) ⊂ (3.1 2.1 1.3) holds, but the different orders of the sub-signs in the transpositions has to be made clear. In order to visualize the semiotic relations inside of the convex set formed by the set of transpositions of a sign class, we may use the following type of diagram, giving only 2 possibilities out 6 + 5 + 4 + 3 + 2 + 1 = 21:





Thus, these diagrams show the complex inner structure of convex semiotic sets.

3. In 1992, Bogarin introduced the semiotic operation of symplerosis, "applied to any sign class, it creates one complementary to this sign class. As a result, the relation of complementarity between sign classes is also defined" (Bogarin 1992, p. 94). Thus, by aid of this operation (σ) that works on the group-theoretic exchanges of (.3.) → (.1.), (.1.) → (.3.), and (.2.) = (.2.) (cf. Toth 2007, pp. 37 ss.), we obtain for each sign class its complementary sign class (the two cases in bold shows the only sign classes that are identical with their complements):

σ(3.1 2.1 1.1) = (3.3 2.3 1.3)
σ(3.1 2.1 1.2) = (3.2 2.3 1.3)
σ(3.1 2.1 1.3) = (3.1 2.3 1.3)
σ(3.1 2.2 1.2) = (3.2 2.2 1.3)
σ(3.1 2.2 1.3) = (3.1 2.2 1.3)
σ(3.1 2.3 1.3) = (3.1 2.1 1.3)
σ(3.2 2.2 1.2) = (3.2 2.2 1.2)
σ(3.2 2.2 1.3) = (3.1 2.2 1.2)
σ(3.2 2.3 1.3) = (3.1 2.1 1.2)
σ(3.3 2.3 1.3) = (3.1 2.1 1.1)

Therefore, we can define a concave semiotic set as the set of transpositions of a complementary sign class. Further, we introduce a concave semiotic hull operator cH that produces the concave set to each sign class. If T stands for the semiotic operator that produces the transpositions of a sign class, we get

cH(a.b c.d e.f) = σT(a.b c.d e.f)

Therefore, the convex semiotic hull operator is nothing but

CH(a.b c.d e.f) = T(a.b c.d e.f)

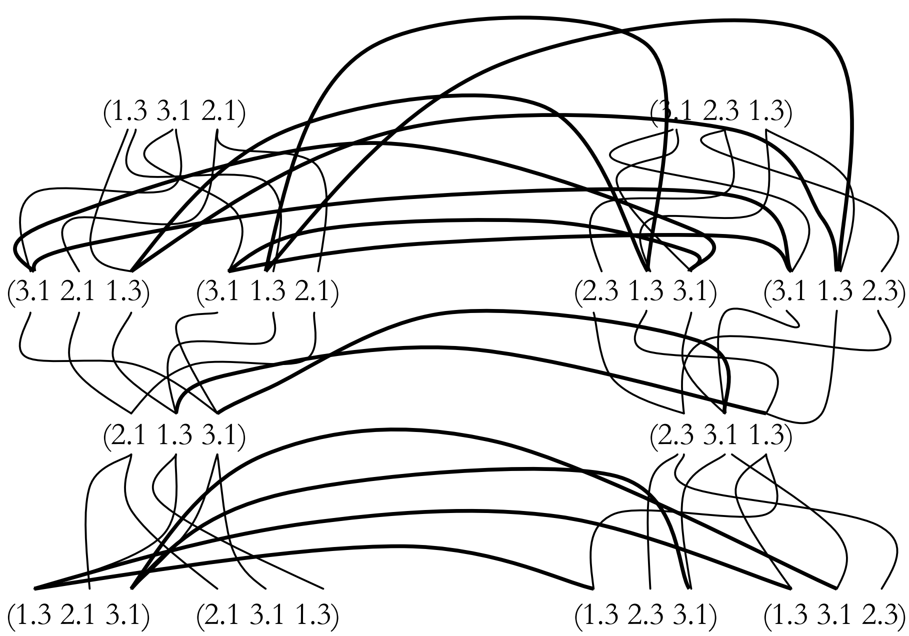
or because the underlying semiotic groups are cyclic

CH = σcH and
cH = σCH

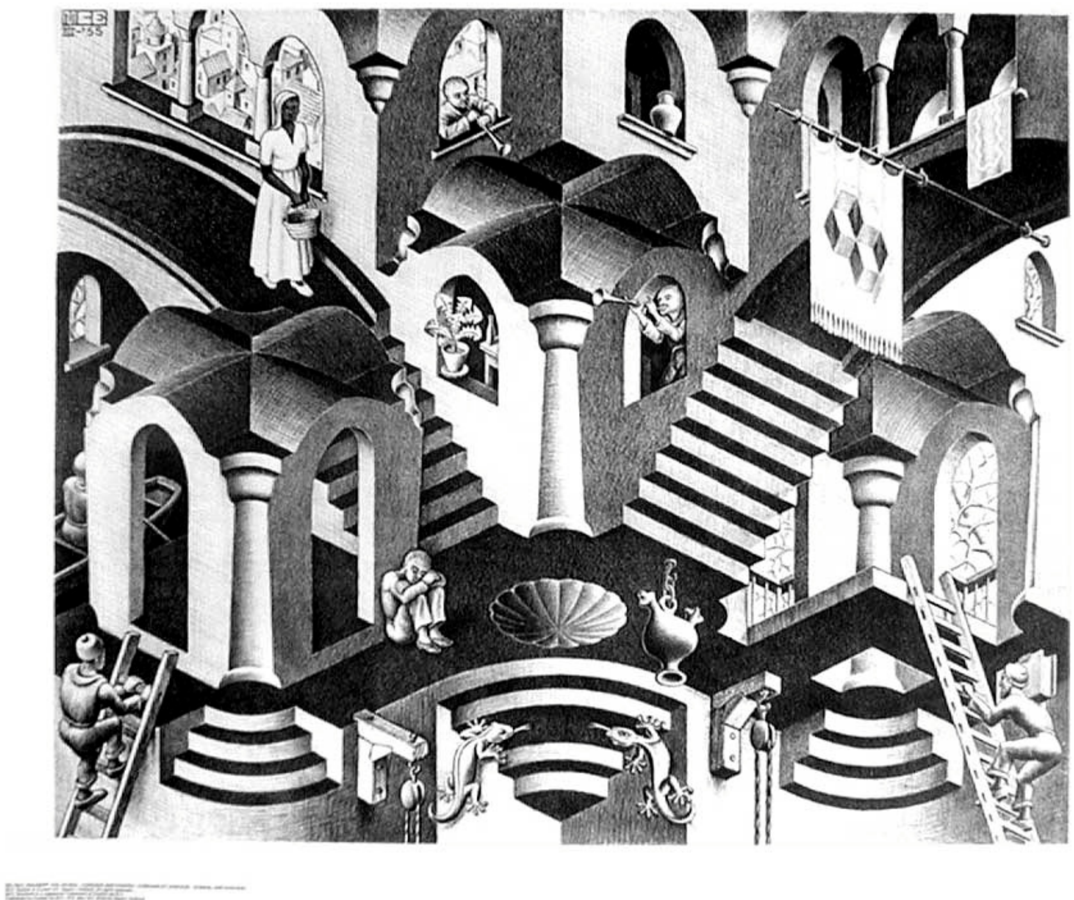
As examples, we show the first three concave semiotic sets of the 10 sign classes:

cH(3.1 2.1 1.1) = {(3.3 2.3 1.3), (3.3 1.3 2.3), (2.3 3.3 1.3), (2.3 1.3 3.3), (1.3 3.3 2.3), (1.3 2.3 3.3)}
cH(3.1 2.1 1.2) = {(3.2 2.3 1.3), (3.2 1.3 2.3), (2.3 3.2 1.3), (2.3 1.3 3.2), (1.3 3.2 2.3), (1.3 2.3 3.2)}
cH(3.1 2.1 1.3) = {(3.1 2.3 1.3), (3.1 1.3 2.3), (2.3 3.1 1.3), (2.3 1.3 3.1), (1.3 3.1 2.3), (1.3 2.3 3.1)}

4. On obvious reasons, the inner structure of a concave semiotic set is not principally different from the inner structure of a convex semiotic set. But since signs never appear alone, but in sign systems (Bense 1976, pp. 163 s.), we may now show the semiotic connections between a convex and a concave semiotic set (marked bold), giving again just 1 out of 21! combinations:



The above semiotic connections between convex and concave semiotic sets have found once more a genial anticipation in M.C. Escher's picture "Concave and Convex" from 1955. Escher himself described it as follows (translation by A.T.): "Side by side, there are three little houses with a cross-vault as roof. We see the left one from the outside, the right one from inside and the middle one arbitrarily from inside or outside. In this picture, several of such inversions are shown, one of which will be described here: two flute-playing boys. To the left, one of them looks through a window down to the roof of the middle house. If he climbs through his window, he can stand on the roof. If he then jumps on the front side down, he lands one floor lower on the dark floor in front of the house. However, the flute-player to the right sees the same cross-vault above his head as a roof, if he if he bends forward. If he wants to climb out of his window, he does not land on the soil, but falls in an infinitely deep abyss" (Escher 1989, p. 14):



M.C. Escher, "Concave and Convex", Lithograph (1955)
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