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Fuzzy semiotic sets

1. Fuzzy set theory has been introduced into theoretical semiotics by Nadin (1977, 1978, 1980, 1983), but never continued later. However, there are at least three good reasons to apply the concept of fuzzy sets to semiotics: 1. Fuzzy sets allow a semiotic analysis of reality thematics primarily independent of sign classes. 2. By aid of fuzzy sets, the continuous character of semiosis can be displayed much better than with ordinary sets. 3. Fuzzy set theory is compatible with category theory that had been introduced into theoretical semiotics already by Marty (1977) and Bense (1981, pp. 124 ss.).

2. In the following, we shall show that the system of structural realities as presented in the reality thematics of the sign classes can be understood as systems of fuzzy sets. The basic idea behind that is that the system of structural realities can be divided into two discrete groups:

- homogeneous reality thematics, f. ex.

$(3.1\ 2.1\ 1.1) \times (1.1\ \underline{1.2\ 1.3})$	M-them. M (complete M)
$(3.2\ 2.2\ 1.2) \times (2.1\ \underline{2.2\ 2.3})$	O-them. O (complete O)
$(3.3\ 2.3\ 1.3) \times (3.1\ \underline{3.2\ 3.3})$	I-them. I (complete I)

- heterogeneous reality thematics, f. ex.

$(3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3})$	M-them. O (2/3 M, 1/3 O)
$(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3})$	M-them. I (2/3 M, 1/3 I)
$(3.2\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 2.3)$	I-them. O (2/3 I, 1/3 O), etc.

The counting of the thematizing and thematized part-realities of the structural realities as thirds goes already back to Walther (1979, p. 108) and seems to be independent of the introduction of fuzzy sets into semiotics. According to Walther, the system of the 10 sign classes can thus be displayed as follows:

1. $(3.1\ 2.1\ 1.1) \times (1.1\ \underline{1.2\ 1.3})$	3/3 M	—	—
2. $(3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3})$	2/3 M	1/3 O	—
3. $(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3})$	2/3 M	—	1/3 I
4. $(3.1\ 2.2\ 1.2) \times (\underline{2.1\ 2.2}\ 1.3)$	—	2/3 O	1/3 M
5. $(3.1\ 2.2\ 1.3) \times (\underline{3.1}\ \underline{2.2}\ \underline{1.3})$	1/3 M	1/3 O	1/3 I
6. $(3.1\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 1.3)$	1/3 M	—	2/3 I
7. $(3.2\ 2.2\ 1.2) \times (2.1\ \underline{2.2\ 2.3})$	—	3/3 O	—
8. $(3.2\ 2.2\ 1.3) \times (3.1\ \underline{2.2\ 2.3})$	—	2/3 O	1/3 I
9. $(3.2\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 2.3)$	—	1/3 O	2/3 I
10. $(3.3\ 2.3\ 1.3) \times (3.1\ \underline{3.2\ 3.3})$	—	—	3/3 I

If we apply this classification to trichotomic triads, introduced by Walther (1981, 1982), we recognize that a certain number of reality thematics can be combined in such a way that their thematized realities present M, O, and I respectively and thus as a system a triadic sign relation, f. ex.

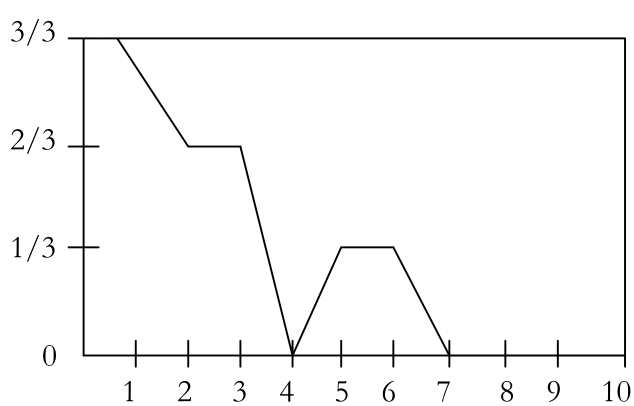
(3.1 2.1 1.1) × (1.1 <u>1.2 1.3</u>)	M-them. M	**3/3 M**	—	—
(3.1 2.1 1.2) × (2.1 <u>1.2 1.3</u>)	O-them. O	2/3 M	**1/3 O**	—
(3.1 2.1 1.3) × (3.1 <u>1.2 1.3</u>)	I-them. I	2/3 M	—	**1/3 I**

Therefore, the above trichotomic triad contains 7/3 M, 1/3 O and 1/3 I and thus 1/3 of each triad of a complete sign relation plus 6/3 of additional M.

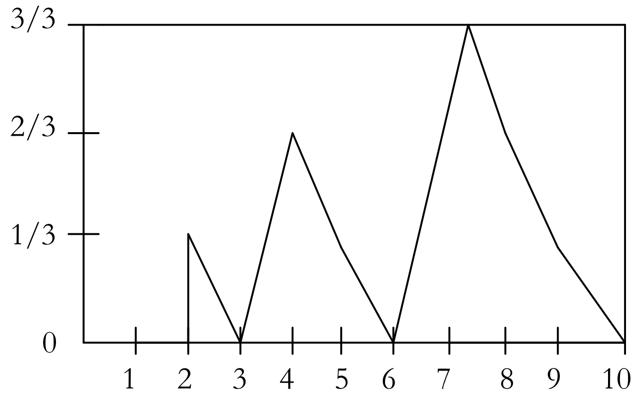
3. Already the above classification of reality thematics by thirds leads to the concept of a membership function for semiotic sets, but this has never been done up to now. Although the semiotic sets are in the above examples the reality thematics and thus the trichotomies of a sign relation, membership functions can of course be applied to the sign classes as the triads of sign relations as well. More precisely: A fuzzy set is a pair (A, μ) where A is a set and $\mu: A \rightarrow [0, 1]$. For each $x \in A$, $\mu(x)$ is the grade of membership of x. Thus, $x \in (A, \mu) \Leftrightarrow x \in A \wedge \mu(x) \neq 0$. An element mapping to the value 0 means that the member is not included in the fuzzy set, 1 describes a fully included member. Values strictly between 0 and 1 characterize the fuzzy members. Thus, if x = 1 or x = 0, the respective set is an ordinary set and μ its characteristic function (cf. Zadeh 1965, p. 339).

Let now A be the set of the reality thematics, $x_i \in A$ the single reality thematics according to the above table (i.e. i = 1 ... 10) and $\mu(x_i)$ their membership function according to the above classification of the structural realities by thirds. Then we can turn Walther's above diagram into the following three graphs in which the abscissa denotes the reality thematics and the ordinate $\mu(x_i)$:

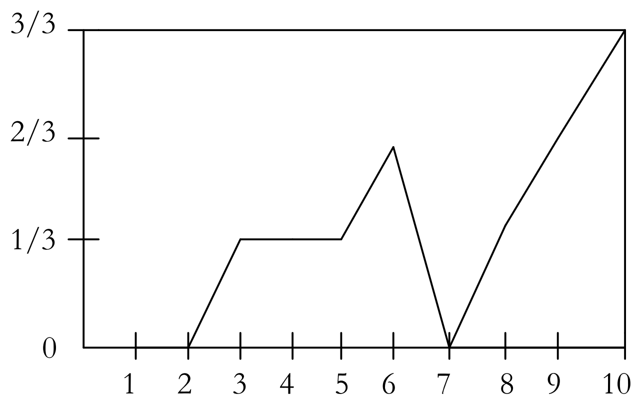
1. Semiotic fuzzy set for structural M ($M \subset A$)



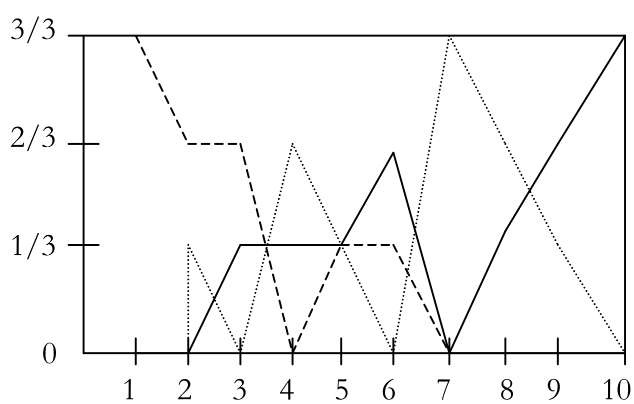
2. Semiotic fuzzy set for structural O (O ⊂ A)



3. Semiotic fuzzy set for structural I (I ⊂ A)



Thus, all three sub-sets M, O, I ⊂ A are non-convex. In the case of the union M ∪ O ∪ I = A, A is a non-convex set, too. In the following graph, the sub-set M is marked dashed, the sub-set O dotted, and the sub-set I straight:



4. However, in mathematical semiotics, two special problems arise. First, reality thematics and sign classes are ordered sets, and we therefore have to take account of that in introducing them as fuzzy sets. Second, each reality thematic and each sign class has 6 transpositions (cf. Toth 2008a, pp. 177 ss.). The second problem implies the first one insofar as transpositions differ from their "unmarked" sign classes and reality thematics only by the order of their constitutive sub-signs and thus by the order of their sub-sets. We may take account of both problems in drawing graphs whose abscissa represents the 6 transpositions of a sign class or reality thematic and whose ordinate represents the three (thematizing and thematized) part-realities of the structural realities. As example, we show the reality thematic $(3.1\ \underline{1.2\ 1.3})$ of the sign class (3.1 2.1 1.3) which presents the following transpositional realities:

$$(3.1\ 2.1\ 1.3) \times (3.1\ 1.2\ 1.3) \rightarrow \{(3.1\ \underline{1.2\ 1.3}), (3.1\ \underline{1.3\ 1.2}), (\underline{1.2}\ 3.1\ \underline{1.3}), (\underline{1.2\ 1.3}\ 3.1), (\underline{1.3}\ 3.1\ \underline{1.2}), (\underline{1.3\ 1.2}\ 3.1)\}$$

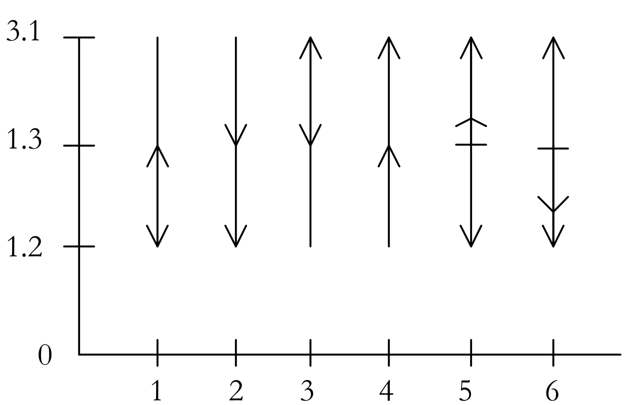
Generally, each semiotic dual system (a.b c.d e.f) × (f.e d.c b.a) (a, c, e ∈ {1., 2., 3.}, b, d, f ∈ {.1, .2, .3}) presents the following system of transpositional structural realities:

$$(a.b\ c.d\ e.f) \times (f.e\ d.c\ b.a) \rightarrow \{(f.e\ \underline{d.c\ b.a}), (f.e\ \underline{b.a\ d.c}), (\underline{d.c}\ f.e\ \underline{b.a}), (\underline{d.c\ b.a}\ f.e), (\underline{b.a}\ f.e\ \underline{d.c}), (\underline{b.a\ d.c}\ f.e)\}$$

Thus, the general structure of transpositional realities of all 10 reality thematics can be schematized as follows:

$Z \leftarrow (X, Y)$	$(X, Y) \rightarrow Z$	
$Z \leftarrow (Y, X)$	$Y \rightarrow Z \leftarrow X$	
$X \rightarrow Z \leftarrow Y$	$(Y, X) \rightarrow Z$	(X, Y, Z ∈ {(1.1), (1.2), (1.3), ..., (3.3)})

The graph for $\{(3.1\ \underline{1.2\ 1.3}), (3.1\ \underline{1.3\ 1.2}), (\underline{1.2}\ 3.1\ \underline{1.3}), (\underline{1.2\ 1.3}\ 3.1), (\underline{1.3}\ 3.1\ \underline{1.2}), (\underline{1.3\ 1.2}\ 3.1)\}$ is:



As one sees, this kind of graph leads to 6 sub-graphs that are nothing else than Hasse diagrams used for the visualization of posets (cf. Toth 2007a, pp. 84 ss.). Since the sign relations can be defined as posets (cf. Toth 1996; 2007a, pp. 85 ss.; 2008b), the above graphs have also the advantage that they are convertible into the language of category theory (cf.

Harris 1983). However, in order to take account for the above mentioned two semiotic problems, we will recur to the so-called “dynamic semiotic morphisms” introduced in Toth (2008a, pp. 159 ss.), because they fully consider the fact that a sign relation is a triadic relation over a dyadic and a monadic relation, too, and thus are capable of dealing with the intricate relational structures of sign classes and reality thematics.

For instance, in our sign class (3.1 2.1 1.3), we will not assign a semiotic “static” morphism to each sub-sign, because such morphisms would not take care of the general relational sign structure (3.a 2.b 1.c) with $a \leq b \leq c$. Therefore, we will assign dynamic morphisms to ((3.2), (1.1)) and ((2.1), (1.3)), which is legitimated by the fact that the triadic sign relation can be understood as concatenation of two dyads; in our example: (3.1 2.1 1.3) = (3.1 2.1) $\circ$ (2.1 1.3), cf. Walther (1979, p. 79). More generally, for the abstract sign relation ((a.b), ((c.d), (e.f)) we will assign two pairs of morphisms to ((a.c), (b.d)) and ((c.e), (d.f)).

We shall further agree that, in order to disambiguate cases like (1.1 1.2 1.3) and (1.1 1.3 1.2), which differ solely by the reverse order of the thematizing sub-signs, we will use the symbols “<” and “>” right above the first thematizing sub-sign in order to express that the sub-sign that carries the respective symbol is of lower or higher semiosic order than the following second thematizing sub-sign.

Moreover, we will use the notational system for reality thematics introduced in Toth (2007b, p. 177 ss.), in which the “basis” for a sub-sign gives its triadic value and the “exponent” its frequency; thus, “1^2” means the twice occurrence of a sub-sign of monadic value (1.), f. ex. (1.2 1.3). “$1^{2,<}$” thus means, f. e.x., (1.2 1.3), and “$1^{2,>}$” means, f. ex., (1.3 1.2). The arrows indicate the direction of thematization, whereby the general usual rule is that two sub-signs of the same triadic value thematize a sub-sign of the same (in homogeneous sign-class) or of different triadic value (in heterogeneous sign-classes).

It had been shown in Toth (2008a, pp. 272 ss.) that the system of the 10 sign classes and their dual reality thematics is only a fragment of the complete system of the 27 sign classes (and reality thematics). In the latter system, in the general sign structure (3.a 2.b 1.c), to a, b and c, all 9 sub-signs from the semiotic matrix can be assigned. Thus the semiotic order is total, namely partial like the order ($a \leq b \leq c$) of the system of the 10 sign classes, but in addition to that also total ($a \leq b$ or $b\leq a$). In other words, unlike the system of the 10 sign classes, the system of the 27 sign classes is solely restricted by the fact that in the abstract sign relation (a.b c.d e.f), a = 3., c = 2., and e = 1. Therefore, finally, we substitute the fragmentary system of the 10 sign classes by the complete system of the 27 sign classes in order to show all possible types of structural realities, amongst them structural realities which are “hidden” from the standpoint of the system of the 10 sign classes. We shall mark the 17 sign classes that do not belong to the “classical” system of 10 sign classes by an asterisk. In the following schemes, the first line shows the 6 transpositional realities of each reality thematic in the numerical version. The second line presents the structure of thematization of each transpositional reality. The third line displays all transpositional realities in the category theoretic version, using “dynamic” morphisms and thus disclosing their fuzziness. (The order of the transpositions is the same in all 3 lines.)

4.1. The structural realities of the sign class (3.1 2.1 1.1)

$1.1\ \underline{1.2\ 1.3}$	$\underline{1.2}\ 1.1\ \underline{1.3}$	$1.1\ \underline{1.3\ 1.2}$	$\underline{1.3}\ 1.1\ \underline{1.2}$	$\underline{1.2\ 1.3}\ 1.1$	$\underline{1.3\ 1.2}\ 1.1$
$1^1 \leftarrow 1^{2,<}$	$1^{1,<} \rightarrow 1^1 \leftarrow 1^1$	$1^1 \leftarrow 1^{2,>}$	$1^{1,>} \rightarrow 1^1 \leftarrow 1^1$	$1^{2,<} \rightarrow 1^1$	$1^{2,>} \rightarrow 1^1$

[[id1, α], [id1, β]]; [[id1, α°], [id1, βα]]; [[id1, βα], [id1, β°]]; [[id1, α°β°], [id1, α]]; [[id1, β], [id1, α°β°]]; [[id1, β°], [id1, α°]].

4.2. The structural realities of the sign class (3.1 2.1 1.2)

$2.1\ \underline{1.2\ 1.3}$	$\underline{1.2}\ 2.1\ \underline{1.3}$	$2.1\ \underline{1.3\ 1.2}$	$\underline{1.3}\ 2.1\ \underline{1.2}$	$\underline{1.2\ 1.3}\ 2.1$	$\underline{1.3\ 1.2}\ 2.1$
$2^1 \leftarrow 1^{2,<}$	$1^{1,<} \rightarrow 2^1 \leftarrow 1^1$	$2^1 \leftarrow 1^{2,>}$	$1^{1,>} \rightarrow 2^1 \leftarrow 1^1$	$1^{2,<} \rightarrow 2^1$	$1^{2,>} \rightarrow 2^1$

[[α°, α], [id1, β]]; [[α, α°], [α°, βα]]; [[α°, βα], [id1, β°]]; [[α, α°β°], [α°, α]]; [[id1, β], [α, α°β°]]; [[id1, β°], [α, α°]].

4.3. The structural realities of the sign class (3.1 2.1 1.3)

$3.1\ \underline{1.2\ 1.3}$	$\underline{1.2}\ 3.1\ \underline{1.3}$	$3.1\ \underline{1.3\ 1.2}$	$\underline{1.3}\ 3.1\ \underline{1.2}$	$\underline{1.2\ 1.3}\ 3.1$	$\underline{1.3\ 1.2}\ 3.1$
$3^1 \leftarrow 1^{2,<}$	$1^{1,<} \rightarrow 3^1 \leftarrow 1^1$	$3^1 \leftarrow 1^{2,>}$	$1^{1,>} \rightarrow 3^1 \leftarrow 1^1$	$1^{2,<} \rightarrow 3^1$	$1^{2,>} \rightarrow 3^1$

[[α°β°, α], [id1, β]]; [[βα, α°], [α°β°, βα]]; [[α°β°, βα], [id1, β°]]; [[βα, α°β°], [α°β°, α]]; [[id1, β], [βα, α°β°]]; [[id1, β°], [βα, α°]].

4.4. The structural realities of the sign class *(3.1 2.2 1.1)

$\underline{1.1}\ 2.2\ \underline{1.3}$	$2.2\ \underline{1.1\ 1.3}$	$\underline{1.1\ 1.3}\ 2.2$	$\underline{1.3\ 1.1}\ 2.1$	$2.2\ \underline{1.3\ 1.1}$	$\underline{1.3}\ 2.2\ \underline{1.1}$
$1^{1,<} \rightarrow 2^1 \leftarrow 1^1$	$1^1 \leftarrow 1^{2,<}$	$1^{2,<} \rightarrow 2^1$	$1^{2,>} \rightarrow 2^1$	$2^1 \leftarrow 1^{2,>}$	$1^{1,>} \rightarrow 2^1 \leftarrow 1^1$

[[α, α], [α°, β]]; [[α°, α°], [id1, βα]]; [[id1, βα], [α, β°]]; [[id1, α°β°], [α, id1]]; [[α°, β], [id1, α°β°]]; [[α, β°], [α°, α°]].

4.5. The structural realities of the sign class (3.1 2.2 1.2)

$\underline{2.1\ 2.2}\ 1.3$	$\underline{2.2\ 2.1}\ 1.3$	$\underline{2.1}\ 1.3\ \underline{2.2}$	$1.3\ \underline{2.1\ 2.2}$	$\underline{2.2}\ 1.3\ \underline{2.1}$	$1.3\ \underline{2.2\ 2.1}$
$2^{2,<} \rightarrow 1^1$	$2^{2,>} \rightarrow 1^1$	$2^{1,<} \rightarrow 1^1 \leftarrow 2^1$	$1^1 \leftarrow 2^{2,<}$	$2^{1,>} \rightarrow 1^1 \leftarrow 2^1$	$1^1 \leftarrow 2^{2,>}$

[[id2, α], [α°, β]]; [[id2, α°], [α°, βα]]; [[α°, βα], [α, β°]]; [[α, α°β°], [id2, α]]; [[α°, β], [α, α°β°]]; [[α, β°], [id2, α°]].

4.6. The structural realities of the sign class (3.1 2.2 1.3)

$\underline{3.1\ 2.2}\ 1.3$	$\underline{2.2\ 3.1}\ 1.3$	$\underline{3.1\ 1.3}\ 2.2$	$\underline{1.3\ 3.1}\ 2.2$	$\underline{2.2\ 1.3}\ 3.1$	$\underline{1.3\ 2.2}\ 3.1$
$3.1\ \underline{2.2\ 1.3}$	$2.2\ \underline{3.1\ 1.3}$	$3.1\ \underline{1.3\ 2.2}$	$1.3\ \underline{3.1\ 2.2}$	$2.2\ \underline{1.3\ 3.1}$	$1.3\ \underline{2.2\ 3.1}$
$\underline{3.1}\ 2.2\ \underline{1.3}$	$\underline{2.2}\ 3.1\ \underline{1.3}$	$\underline{3.1}\ 1.3\ \underline{2.2}$	$\underline{1.3}\ 3.1\ \underline{2.2}$	$\underline{2.2}\ 1.3\ \underline{3.1}$	$\underline{1.3}\ 2.2\ \underline{3.1}$

$3^1 \leftrightarrow 2^1 \rightarrow 1^1$	$2^1 \leftrightarrow 3^1 \rightarrow 1^1$	$3^1 \leftrightarrow 1^1 \rightarrow 2^1$	$1^1 \leftrightarrow 3^1 \rightarrow 2^1$	$2^1 \leftrightarrow 1^1 \rightarrow 3^1$	$1^1 \leftrightarrow 2^1 \rightarrow 3^1$
$3^1 \leftarrow 2^1 \leftrightarrow 1^1$	$2^1 \leftarrow 3^1 \leftrightarrow 1^1$	$3^1 \leftarrow 1^1 \leftrightarrow 2^1$	$1^1 \leftarrow 3^1 \leftrightarrow 2^1$	$2^1 \leftarrow 1^1 \leftrightarrow 3^1$	$1^1 \leftarrow 2^1 \leftrightarrow 3^1$
$3^1 \rightarrow 2^1 \leftarrow 1^1$	$2^1 \rightarrow 3^1 \leftarrow 1^1$	$3^1 \rightarrow 1^1 \leftarrow 2^1$	$1^1 \rightarrow 3^1 \leftarrow 2^1$	$2^1 \rightarrow 1^1 \leftarrow 3^1$	$1^1 \rightarrow 2^1 \leftarrow 3^1$

[[β°, α], [α°, β]]; [[β, α°], [α°β°, βα]]; [[α°β°, βα], [α, β°]]; [[βα, α°β°], [β°, α]; [[α°, β], [βα, α°β°]]; [[α, β°], [β, α°]].

Here, we have the first of several cases of triadic structural reality, whose condition is that all three trichotomic values of a sign class are different, i.e. (3.a 2.b 1.c) with a ≠ b ≠ c. In the system of the 10 sign classes, this happens only in the case of the dual-invariant sign class whose reality thematic is thus identical with the sign class itself.

4.7. The structural realities of the sign class *(3.1 2.3 1.1)

$\underline{1.1}$ 3.2 $\underline{1.3}$	3.2 $\underline{1.1\ 1.3}$	$\underline{1.1\ 1.3}$ 3.2	$\underline{1.3\ 1.1}$ 3.2	3.2 $\underline{1.3\ 1.1}$	$\underline{1.3}$ 3.2 $\underline{1.1}$
$1^{1,<} \rightarrow 3^1 \leftarrow 1^1$	$3^1 \leftarrow 1^{2,<}$	$1^{2,<} \rightarrow 3^1$	$1^{2,>} \rightarrow 3^1$	$3^1 \leftarrow 1^{2,>}$	$1^{1,>} \rightarrow 3^1 \leftarrow 1^1$

[[βα, α], [α°β°, β]]; [[α°β°, α°], [id1, βα]]; [[id1, βα], [βα, β°]]; [[id1, α°β°], [βα, α]; [[α°β°, β], [id1, α°β°]]; [[βα, β°], [α°β°, α°]].

4.8. The structural realities of the sign class *(3.1 2.3 1.2)

$\underline{2.1\ 3.2}$ 1.3	$\underline{3.2\ 2.1}$ 1.3	$\underline{2.1\ 1.3}$ 3.2	$\underline{1.3\ 2.1}$ 3.2	$\underline{3.2\ 1.3}$ 2.1	$\underline{1.3\ 3.2}$ 2.1
2.1 $\underline{3.2\ 1.3}$	3.2 $\underline{2.1\ 1.3}$	2.1 $\underline{1.3\ 3.2}$	1.3 $\underline{2.1\ 3.2}$	3.2 $\underline{1.3\ 2.1}$	1.3 $\underline{3.2\ 2.1}$
$\underline{2.1}$ 3.2 $\underline{1.3}$	$\underline{3.2}$ 2.1 $\underline{1.3}$	$\underline{2.1}$ 1.3 $\underline{3.2}$	$\underline{1.3}$ 2.1 $\underline{3.2}$	$\underline{3.2}$ 1.3 $\underline{2.1}$	$\underline{1.3}$ 3.2 $\underline{2.1}$

$2^1 \leftrightarrow 3^1 \rightarrow 1^1$	$3^1 \leftrightarrow 2^1 \rightarrow 1^1$	$2^1 \leftrightarrow 1^1 \rightarrow 3^1$	$1^1 \leftrightarrow 2^1 \rightarrow 3^1$	$3^1 \leftrightarrow 1^1 \rightarrow 2^1$	$1^1 \leftrightarrow 3^1 \rightarrow 2^1$
$2^1 \leftarrow 3^1 \leftrightarrow 1^1$	$3^1 \leftarrow 2^1 \leftrightarrow 1^1$	$2^1 \leftarrow 1^1 \leftrightarrow 3^1$	$1^1 \leftarrow 2^1 \leftrightarrow 3^1$	$3^1 \leftarrow 1^1 \leftrightarrow 2^1$	$1^1 \leftarrow 3^1 \leftrightarrow 2^1$
$2^1 \rightarrow 3^1 \leftarrow 1^1$	$3^1 \rightarrow 2^1 \leftarrow 1^1$	$2^1 \rightarrow 1^1 \leftarrow 3^1$	$1^1 \rightarrow 2^1 \leftarrow 3^1$	$3^1 \rightarrow 1^1 \leftarrow 2^1$	$1^1 \rightarrow 3^1 \leftarrow 2^1$

[[β, α], [α°β°, β]]; [[β°, α°], [α°, βα]]; [[α°, βα], [βα, β°]]; [[α, α°β°], [β, α]]; [[α°β°, β], [α, α°β°]]; [[βα, β°], [β°, α°]].

4.9. The structural realities of the sign class (3.1 2.3 1.3)

$\underline{3.1\ 3.2}$ 1.3	$\underline{3.2\ 3.1}$ 1.3	$\underline{3.1}$ 1.3 $\underline{3.2}$	1.3 $\underline{3.1\ 3.2}$	$\underline{3.2}$ 1.3 $\underline{3.1}$	1.3 $\underline{3.2\ 3.1}$
$3^{2,<} \rightarrow 1^1$	$3^{2,>} \rightarrow 1^1$	$3^{1,<} \rightarrow 1^1 \leftarrow 3^1$	$1^1 \leftarrow 3^{2,<}$	$3^{1,>} \rightarrow 1^1 \leftarrow 3^1$	$1^1 \leftarrow 3^{2,>}$

[[id3, α], [α°β°, β]]; [[id3, α°], [α°β°, βα]]; [[α°β°, βα], [βα, β°]]; [[βα, α°β°], [id3, α]]; [[α°β°, β], [βα, α°β°]]; [[βα, β°], [id3, α°]].

4.10. The structural realities of the sign class *(3.2 2.1 1.1)

$\underline{1.1\ 1.2}$ 2.3	$\underline{1.2\ 1.1}$ 2.3	$\underline{1.1}$ 2.3 $\underline{1.2}$	2.3 $\underline{1.1\ 1.2}$	$\underline{1.2}$ 2.3 $\underline{1.1}$	2.3 $\underline{1.2\ 1.1}$
$1^{2,<} \rightarrow 2^1$	$1^{2,>} \rightarrow 2^1$	$1^{1,<} \rightarrow 2^1 \leftarrow 1^1$	$2^1 \leftarrow 1^{2,<}$	$1^{1,>} \rightarrow 2^1 \leftarrow 1^1$	$2^1 \leftarrow 1^{2,>}$

[[id1, α], [α, β]]; [[id1, α°], [α, βα]]; [[α, βα], [α°, β°]]; [[α°, α°β°], [id1, α]]; [[α, β], [α°, α°β°]]; [[α°, β°], [id1, α°]].

4.11. The structural realities of the sign class *(3.2 2.1 1.2)

$\underline{2.1}$ 1.2 $\underline{2.3}$	1.2 $\underline{2.1\ 2.3}$	$\underline{2.1\ 2.3}$ 1.2	$\underline{2.3\ 2.1}$ 1.2	1.2 $\underline{2.3\ 2.1}$	$\underline{2.3}$ 1.2 $\underline{2.1}$
$2^{1,<} \rightarrow 1^1 \leftarrow 2^1$	$1^1 \leftarrow 2^{2,<}$	$2^{2,<} \rightarrow 1^1$	$2^{2,>} \rightarrow 1^1$	$1^1 \leftarrow 2^{2,>}$	$2^{1,>} \rightarrow 1^1 \leftarrow 2^1$

[[α°, α], [α, β]]; [[α, α°], [id2, βα]]; [[id2, βα], [α°, β°]]; [[id2, α°β°], [α°, α]]; [[α, β], [id2, α°β°]]; [[α°, β°], [α, α°]].

4.12. The structural realities of the sign class *(3.2 2.1 1.3)

$\underline{3.1\ 1.2}$ 2.3	$\underline{1.2\ 3.1}$ 2.3	$\underline{3.1\ 2.3}$ 1.2	$\underline{2.3\ 3.1}$ 1.2	$\underline{1.2\ 2.3}$ 3.1	$\underline{2.3\ 1.2}$ 3.1
3.1 $\underline{1.2\ 2.3}$	1.2 $\underline{3.1\ 2.3}$	3.1 $\underline{2.3\ 1.2}$	2.3 $\underline{3.1\ 1.2}$	1.2 $\underline{2.3\ 3.1}$	2.3 $\underline{1.2\ 3.1}$
$\underline{3.1}$ 1.2 $\underline{2.3}$	$\underline{1.2}$ 3.1 $\underline{2.3}$	$\underline{3.1}$ 2.3 $\underline{1.2}$	$\underline{2.3}$ 3.1 $\underline{1.2}$	$\underline{1.2}$ 2.3 $\underline{3.1}$	$\underline{2.3}$ 1.2 $\underline{3.1}$

$3^1 \leftrightarrow 1^1 \rightarrow 2^1$	$1^1 \leftrightarrow 3^1 \rightarrow 2^1$	$3^1 \leftrightarrow 2^1 \rightarrow 1^1$	$2^1 \leftrightarrow 3^1 \rightarrow 1^1$	$1^1 \leftrightarrow 2^1 \rightarrow 3^1$	$2^1 \leftrightarrow 1^1 \rightarrow 3^1$
$3^1 \leftarrow 1^1 \leftrightarrow 2^1$	$1^1 \leftarrow 3^1 \leftrightarrow 2^1$	$3^1 \leftarrow 2^1 \leftrightarrow 1^1$	$2^1 \leftarrow 3^1 \leftrightarrow 1^1$	$1^1 \leftarrow 2^1 \leftrightarrow 3^1$	$2^1 \leftarrow 1^1 \leftrightarrow 3^1$
$3^1 \rightarrow 1^1 \leftarrow 2^1$	$1^1 \rightarrow 3^1 \leftarrow 2^1$	$3^1 \rightarrow 2^1 \leftarrow 1^1$	$2^1 \rightarrow 3^1 \leftarrow 1^1$	$1^1 \rightarrow 2^1 \leftarrow 3^1$	$2^1 \rightarrow 1^1 \leftarrow 3^1$

[[α°β°, α], [α, β]]; [[βα, α°], [β°, βα]]; [[β°, βα], [α°, β°]]; [[β, α°β°], [α°β°, α]]; [[α, β], [β, α°β°]]; [[α°, β°], [βα, α°]].

4.13. The structural realities of the sign class *(3.2 2.2 1.1)

1.1 $\underline{2.2\ 2.3}$	$\underline{2.2}$ 1.1 $\underline{2.3}$	1.1 $\underline{2.3\ 2.2}$	$\underline{2.3}$ 1.1 $\underline{2.2}$	$\underline{2.2\ 2.3}$ 1.1	$\underline{2.3\ 2.2}$ 1.1
$1^1 \leftarrow 2^{2,<}$	$2^{1,<} \rightarrow 1^1 \leftarrow 2^1$	$1^1 \leftarrow 2^{2,>}$	$2^{1,>} \rightarrow 1^1 \leftarrow 2^1$	$2^{2,<} \rightarrow 1^1$	$2^{2,>} \rightarrow 1^1$

[[α, α], [id2, β]]; [[α°, α°], [α, βα]]; [[α, βα], [id2, β°]]; [[α°, α°β°], [α, α]]; [[id2, β], [α°, α°β°]]; [[id2, β°], [α°, α°]].

4.14. The structural realities of the sign class (3.2 2.2 1.2)

$\underline{2.1\ 2.2}$ 2.3	$\underline{2.2\ 2.1}$ 2.3	$\underline{2.1}$ 2.3 $\underline{2.2}$	2.3 $\underline{2.1\ 2.2}$	$\underline{2.2}$ 2.3 $\underline{2.1}$	2.3 $\underline{2.2\ 2.1}$
$2^{2,<} \rightarrow 2^1$	$2^{2,>} \rightarrow 2^1$	$2^{1,<} \rightarrow 2^1 \leftarrow 2^1$	$2^1 \leftarrow 2^{2,<}$	$2^{1,>} \rightarrow 2^1 \leftarrow 2^1$	$2^1 \leftarrow 2^{2,>}$

[[id2, α], [id2, β]]; [[id2, α°], [id2, βα]]; [[id2, βα], [id2, β°]]; [[id2, α°β°], [id2, α]]; [[id2, β], [id2, α°β°]]; [[id2, β°], [id2, α°]].

4.15. The structural realities of the sign class (3.2 2.2 1.3)

3.1 $\underline{2.2\ 2.3}$	$\underline{2.2}$ 3.1 $\underline{2.3}$	3.1 $\underline{2.3\ 2.2}$	$\underline{2.3}$ 3.1 $\underline{2.2}$	$\underline{2.2\ 2.3}$ 3.1	$\underline{2.3\ 2.2}$ 3.1
$3^1 \leftarrow 2^{2,<}$	$2^{1,<} \rightarrow 3^1 \leftarrow 2^1$	$3^1 \leftarrow 2^{2,>}$	$2^{1,>} \rightarrow 3^1 \leftarrow 2^1$	$2^{2,<} \rightarrow 3^1$	$2^{2,>} \rightarrow 3^1$

[[β°, α], [id2, β]]; [[β, α°], [β°, βα]]; [[β°, βα], [id2, β°]]; [[β, α°β°], [β°, α]]; [[id2, β], [β, α°β°]]; [[id2, β°], [β, α°]].

4.16. The structural realities of the sign class *(3.2 2.3 1.1)

$\underline{1.1\ 3.2}$ 2.3	$\underline{3.2\ 1.1}$ 2.3	$\underline{1.1\ 2.3}$ 3.2	$\underline{2.3\ 1.1}$ 3.2	$\underline{3.2\ 2.3}$ 1.1	$\underline{2.3\ 3.2}$ 1.1
1.1 $\underline{3.2\ 2.3}$	3.2 $\underline{1.1\ 2.3}$	1.1 $\underline{2.3\ 3.2}$	2.3 $\underline{1.1\ 3.2}$	3.2 $\underline{2.3\ 1.1}$	2.3 $\underline{3.2\ 1.1}$
$\underline{1.1}$ 3.2 $\underline{2.3}$	$\underline{3.2}$ 1.1 $\underline{2.3}$	$\underline{1.1}$ 2.3 $\underline{3.2}$	$\underline{2.3}$ 1.1 $\underline{3.2}$	$\underline{3.2}$ 2.3 $\underline{1.1}$	$\underline{2.3}$ 3.2 $\underline{1.1}$

$1^1 \leftrightarrow 3^1 \rightarrow 2^1$	$3^1 \leftrightarrow 1^1 \rightarrow 2^1$	$1^1 \leftrightarrow 2^1 \rightarrow 3^1$	$2^1 \leftrightarrow 1^1 \rightarrow 3^1$	$3^1 \leftrightarrow 2^1 \rightarrow 1^1$	$2^1 \leftrightarrow 3^1 \rightarrow 1^1$
$1^1 \leftarrow 3^1 \leftrightarrow 2^1$	$3^1 \leftarrow 1^1 \leftrightarrow 2^1$	$1^1 \leftarrow 2^1 \leftrightarrow 3^1$	$2^1 \leftarrow 1^1 \leftrightarrow 3^1$	$3^1 \leftarrow 2^1 \leftrightarrow 1^1$	$2^1 \leftarrow 3^1 \leftrightarrow 1^1$
$1^1 \rightarrow 3^1 \leftarrow 2^1$	$3^1 \rightarrow 1^1 \leftarrow 2^1$	$1^1 \rightarrow 2^1 \leftarrow 3^1$	$2^1 \rightarrow 1^1 \leftarrow 3^1$	$3^1 \rightarrow 2^1 \leftarrow 1^1$	$2^1 \rightarrow 3^1 \leftarrow 1^1$

[[βα, α], [β°, β]]; [[α°β°, α°], [α, βα]]; [[α, βα], [β, β°]]; [[α°, α°β°], [βα, α]]; [[β°, β], [α°, α°β°]]; [[β, β°], [α°β°, α°]].

4.17. The structural realities of the sign class *(3.2 2.3 1.2)

$\underline{2.1}$ 3.2 $\underline{2.3}$	3.2 $\underline{2.1\ 2.3}$	$\underline{2.1\ 2.3}$ 3.2	$\underline{2.3\ 2.1}$ 3.2	3.2 $\underline{2.3\ 2.1}$	$\underline{2.3}$ 3.2 $\underline{2.1}$
$2^{1,<} \rightarrow 3^1 \leftarrow 2^1$	$3^1 \leftarrow 2^{2,<}$	$2^{2,<} \rightarrow 3^1$	$2^{2,>} \rightarrow 3^1$	$3^1 \leftarrow 2^{2,>}$	$2^{1,>} \rightarrow 3^1 \leftarrow 2^1$

[[β, α], [β°, β]]; [[β°, α°], [id2, βα]]; [[id2, βα], [β, β°]]; [[id2, α°β°], [β, α]]; [[β°, β], [id2, α°β°]]; [[β, β°], [β°, α°]].

4.18. The structural realities of the sign class (3.2 2.3 1.3)

$\underline{3.1\ 3.2}$ 2.3	$\underline{3.2\ 3.1}$ 2.3	$\underline{3.1}$ 2.3 $\underline{3.2}$	2.3 $\underline{3.1\ 3.2}$	$\underline{3.2}$ 2.3 $\underline{3.1}$	2.3 $\underline{3.2\ 3.1}$
$3^{2,<} \rightarrow 2^1$	$3^{2,>} \rightarrow 2^1$	$3^{1,<} \rightarrow 2^1 \leftarrow 3^1$	$2^1 \leftarrow 3^{2,<}$	$3^{1,>} \rightarrow 2^1 \leftarrow 3^1$	$2^1 \leftarrow 3^{2,>}$

[[id3, α], [β°, β]]; [[id3, α°], [β°, βα]]; [[β°, βα], [β, β°]]; [[β, α°β°], [id3, α]]; [[β°, β], [β, α°β°]]; [[β, β°], [id3, α°]].

4.19. The structural realities of the sign class *(3.3 2.1 1.1)

$\underline{1.1\ 1.2}$ 3.3	$\underline{1.2\ 1.1}$ 3.3	$\underline{1.1}$ 3.3 $\underline{1.2}$	3.3 $\underline{1.1\ 1.2}$	$\underline{1.2}$ 3.3 $\underline{1.1}$	3.3 $\underline{1.2\ 1.1}$
$1^{2,<} \rightarrow 3^1$	$1^{2,>} \rightarrow 3^1$	$1^{1,<} \rightarrow 3^1 \leftarrow 1^1$	$3^1 \leftarrow 1^{2,<}$	$1^{1,>} \rightarrow 3^1 \leftarrow 1^1$	$3^1 \leftarrow 1^{2,>}$

[[id1, α], [βα, β]]; [[id1, α°], [βα, βα]]; [[βα, βα], [α°β°, β°]]; [[α°β°, α°β°], [id1, α]]; [[βα, β], [α°β°, α°β°]]; [[α°β°, β°], [id1, α°]].

4.20. The structural realities of the sign class *(3.3 2.1 1.2)

$\underline{2.1\ 1.2}\ 3.3$	$\underline{1.2\ 2.1}\ 3.3$	$\underline{2.1\ 3.3}\ 1.2$	$\underline{3.3\ 2.1}\ 1.2$	$\underline{1.2\ 3.3}\ 2.1$	$\underline{3.3\ 1.2}\ 2.1$
$2.1\ \underline{1.2\ 3.3}$	$1.2\ \underline{2.1\ 3.3}$	$2.1\ \underline{3.3\ 1.2}$	$3.3\ \underline{2.1\ 1.2}$	$1.2\ \underline{3.3\ 2.1}$	$3.3\ \underline{1.2\ 2.1}$
$\underline{2.1}\ 1.2\ \underline{3.3}$	$\underline{1.2}\ 2.1\ \underline{3.3}$	$\underline{2.1}\ 3.3\ \underline{1.2}$	$\underline{3.3}\ 2.1\ \underline{1.2}$	$\underline{1.2}\ 3.3\ \underline{2.1}$	$\underline{3.3}\ 1.2\ \underline{2.1}$

$2^1 \leftrightarrow 1^1 \rightarrow 3^1$	$1^1 \leftrightarrow 2^1 \rightarrow 3^1$	$2^1 \leftrightarrow 3^1 \rightarrow 1^1$	$3^1 \leftrightarrow 2^1 \rightarrow 1^1$	$1^1 \leftrightarrow 3^1 \rightarrow 2^1$	$3^1 \leftrightarrow 1^1 \rightarrow 2^1$
$2^1 \leftarrow 1^1 \leftrightarrow 3^1$	$1^1 \leftarrow 2^1 \leftrightarrow 3^1$	$2^1 \leftarrow 3^1 \leftrightarrow 1^1$	$3^1 \leftarrow 2^1 \leftrightarrow 1^1$	$1^1 \leftarrow 3^1 \leftrightarrow 2^1$	$3^1 \leftarrow 1^1 \leftrightarrow 2^1$
$2^1 \rightarrow 1^1 \leftarrow 3^1$	$1^1 \rightarrow 2^1 \leftarrow 3^1$	$2^1 \rightarrow 3^1 \leftarrow 1^1$	$3^1 \rightarrow 2^1 \leftarrow 1^1$	$1^1 \rightarrow 3^1 \leftarrow 2^1$	$3^1 \rightarrow 1^1 \leftarrow 2^1$

[[α°, α], [βα, β]]; [[α, α°], [β, βα]]; [[β, βα], [α°β°, β°]]; [[β°, α°β°], [α°, α]]; [[βα, β], [β°, α°β°]]; [[α°β°, β°], [α, α°]].

4.21. The structural realities of the sign class *(3.3 2.1 1.3)

$\underline{3.1}\ 1.2\ \underline{3.3}$	$1.2\ \underline{3.1\ 3.3}$	$\underline{3.1\ 3.3}\ 1.2$	$\underline{3.3\ 3.1}\ 1.2$	$1.2\ \underline{3.3\ 3.1}$	$\underline{3.3}\ 1.2\ \underline{3.1}$
$3^{1,<} \rightarrow 1^1 \leftarrow 3^1$	$1^1 \leftarrow 3^{2,<}$	$3^{2,<} \rightarrow 1^1$	$3^{2,>} \rightarrow 1^1$	$1^1 \leftarrow 3^{2,>}$	$3^{1,>} \rightarrow 1^1 \leftarrow 3^1$

[[α°β°, α], [βα, β]]; [[βα, α°], [id3, βα]]; [[id3, βα], [α°β°, β°]]; [[id3, α°β°], [α°β°, α]]; [[βα, β], [id3, α°β°]]; [[α°β°, β°], [βα, α°]].

4.22. The structural realities of the sign class *(3.3 2.2 1.1)

$\underline{1.1\ 2.2}\ 3.3$	$\underline{2.2\ 1.1}\ 3.3$	$\underline{1.1\ 3.3}\ 2.2$	$\underline{3.3\ 1.1}\ 2.2$	$\underline{2.2\ 3.3}\ 1.1$	$\underline{3.3\ 2.2}\ 1.1$
$1.1\ \underline{2.2\ 3.3}$	$2.2\ \underline{1.1\ 3.3}$	$1.1\ \underline{3.3\ 2.2}$	$3.3\ \underline{1.1\ 2.2}$	$2.2\ \underline{3.3\ 1.1}$	$3.3\ \underline{2.2\ 1.1}$
$\underline{1.1}\ 2.2\ \underline{3.3}$	$\underline{2.2}\ 1.1\ \underline{3.3}$	$\underline{1.1}\ 3.3\ \underline{2.2}$	$\underline{3.3}\ 1.1\ \underline{2.2}$	$\underline{2.2}\ 3.3\ \underline{1.1}$	$\underline{3.3}\ 2.2\ \underline{1.1}$

$1^1 \leftrightarrow 2^1 \rightarrow 3^1$	$2^1 \leftrightarrow 1^1 \rightarrow 3^1$	$1^1 \leftrightarrow 3^1 \rightarrow 2^1$	$3^1 \leftrightarrow 1^1 \rightarrow 2^1$	$2^1 \leftrightarrow 3^1 \rightarrow 1^1$	$3^1 \leftrightarrow 2^1 \rightarrow 1^1$
$1^1 \leftarrow 2^1 \leftrightarrow 3^1$	$2^1 \leftarrow 1^1 \leftrightarrow 3^1$	$1^1 \leftarrow 3^1 \leftrightarrow 2^1$	$3^1 \leftarrow 1^1 \leftrightarrow 2^1$	$2^1 \leftarrow 3^1 \leftrightarrow 1^1$	$3^1 \leftarrow 2^1 \leftrightarrow 1^1$
$1^1 \rightarrow 2^1 \leftarrow 3^1$	$2^1 \rightarrow 1^1 \leftarrow 3^1$	$1^1 \rightarrow 3^1 \leftarrow 2^1$	$3^1 \rightarrow 1^1 \leftarrow 2^1$	$2^1 \rightarrow 3^1 \leftarrow 1^1$	$3^1 \rightarrow 2^1 \leftarrow 1^1$

[[α, α], [β, β]]; [[α°, α°], [βα, βα]]; [[βα, βα], [β°, β°]]; [[α°β°, α°β°], [α, α]]; [[β, β], [α°β°, α°β°]]; [[β°, β°], [α°, α°]].

4.23. The structural realities of the sign class *(3.3 2.2 1.2)

$\underline{2.1\ 2.2}\ 3.3$	$\underline{2.2\ 2.1}\ 3.3$	$\underline{2.1}\ 3.3\ \underline{2.2}$	$3.3\ \underline{2.1\ 2.2}$	$\underline{2.2}\ 3.3\ \underline{2.1}$	$3.3\ \underline{2.2\ 2.1}$
$2^{2,<} \rightarrow 3^1$	$2^{2,>} \rightarrow 3^1$	$2^{1,<} \rightarrow 3^1 \leftarrow 2^1$	$3^1 \leftarrow 2^{2,<}$	$2^{1,>} \rightarrow 3^1 \leftarrow 2^1$	$3^1 \leftarrow 2^{2,>}$

[[id2, α], [β, β]]; [[id2, α°], [β, βα]]; [[β, βα], [β°, β°]]; [[β°, α°β°], [id2, α]]; [[β, β], [β°, α°β°]]; [[β°, β°], [id2, α°]].

4.24. The structural realities of the sign class *(3.3 2.2 1.3)

$\underline{3.1}$ 2.2 $\underline{3.3}$	2.2 $\underline{3.1\ 3.3}$	$\underline{3.1\ 3.3}$ 2.2	$\underline{3.3\ 3.1}$ 2.2	2.2 $\underline{3.3\ 3.1}$	$\underline{3.3}$ 2.2 $\underline{3.1}$
$3^{1,<} \rightarrow 2^1 \leftarrow 3^1$	$2^1 \leftarrow 3^{2,<}$	$3^{2,<} \rightarrow 2^1$	$3^{2,>} \rightarrow 2^1$	$2^1 \leftarrow 3^{2,>}$	$3^{1,>} \rightarrow 2^1 \leftarrow 3^1$

$[[\beta^\circ, \alpha], [\beta, \beta]]$; $[[\beta, \alpha^\circ], [id3, \beta\alpha]$; $[[id3, \beta\alpha], [\beta^\circ, \beta^\circ]]$; $[[id3, \alpha^\circ\beta^\circ], [\beta^\circ, \alpha]]$; $[[\beta, \beta], [id3, \alpha^\circ\beta^\circ]]$; $[[\beta^\circ, \beta^\circ], [\beta, \alpha^\circ]]$.

4.25. The structural realities of the sign class *(3.3 2.3 1.1)

1.1 $\underline{3.2\ 3.3}$	$\underline{3.2}$ 1.1 $\underline{3.3}$	1.1 $\underline{3.3\ 3.2}$	$\underline{3.3}$ 1.1 $\underline{3.2}$	$\underline{3.2\ 3.3}$ 1.1	$\underline{3.3\ 3.2}$ 1.1
$1^1 \leftarrow 3^{2,<}$	$3^{1,<} \rightarrow 1^1 \leftarrow 3^1$	$1^1 \leftarrow 3^{2,>}$	$3^{1,>} \rightarrow 1^1 \leftarrow 3^1$	$3^{2,<} \rightarrow 1^1$	$3^{2,>} \rightarrow 1^1$

$[[\beta\alpha, \alpha], [id3, \beta]]$; $[[\alpha^\circ\beta^\circ, \alpha^\circ], [\beta\alpha, \beta\alpha]]$; $[[\beta\alpha, \beta\alpha], [id3, \beta^\circ]]$; $[[\alpha^\circ\beta^\circ, \alpha^\circ\beta^\circ], [\beta\alpha, \alpha]]$; $[[id3, \beta], [\alpha^\circ\beta^\circ, \alpha^\circ\beta^\circ]]$; $[[id3, \beta^\circ], [\alpha^\circ\beta^\circ, \alpha^\circ]]$.

4.26. The structural realities of the sign class *(3.3 2.3 1.2)

2.1 $\underline{3.2\ 3.3}$	$\underline{3.2}$ 2.1 $\underline{3.3}$	2.1 $\underline{3.3\ 3.2}$	$\underline{3.3}$ 2.1 $\underline{3.2}$	$\underline{3.2\ 3.3}$ 2.1	$\underline{3.3\ 3.2}$ 2.1
$2^1 \leftarrow 3^{2,<}$	$3^{1,<} \rightarrow 2^1 \leftarrow 3^1$	$2^1 \leftarrow 3^{2,>}$	$3^{1,>} \rightarrow 2^1 \leftarrow 3^1$	$3^{2,<} \rightarrow 2^1$	$3^{2,>} \rightarrow 2^1$

$[[\beta, \alpha], [id3, \beta]]$; $[[\beta^\circ, \alpha^\circ], [\beta, \beta\alpha]]$; $[[\beta, \beta\alpha], [id3, \beta^\circ]]$; $[[\beta^\circ, \alpha^\circ\beta^\circ], [\beta, \alpha]]$; $[[id3, \beta], [\beta^\circ, \alpha^\circ\beta^\circ]]$; $[[id3, \beta^\circ], [\beta^\circ, \alpha^\circ]]$.

4.27. The structural realities of the sign class (3.3 2.3 1.3)

$\underline{3.1\ 3.2}$ 3.3	$\underline{3.2\ 3.1}$ 3.3	$\underline{3.1}$ 3.3 $\underline{3.2}$	3.3 $\underline{3.1\ 3.2}$	$\underline{3.2}$ 3.3 $\underline{3.1}$	3.3 $\underline{3.2\ 3.1}$
$3^2 \rightarrow 3^1$	$3^2 \rightarrow 3^1$	$3^1 \rightarrow 3^1 \leftarrow 3^1$	$3^1 \leftarrow 3^2$	$3^1 \rightarrow 3^1 \leftarrow 3^1$	$3^1 \leftarrow 3^2$
$3^{2,<} \rightarrow 3^1$	$3^{2,>} \rightarrow 3^1$	$3^{1,<} \rightarrow 3^1 \leftarrow 3^1$	$3^1 \leftarrow 3^{2,<}$	$3^{1,>} \rightarrow 3^1 \leftarrow 3^1$	$3^1 \leftarrow 3^{2,>}$

$[[id3, \alpha], [id3, \beta]]$; $[[id3, \alpha^\circ], [id3, \beta\alpha]]$; $[[id3, \beta\alpha], [id3, \beta^\circ]]$; $[[id3, \alpha^\circ\beta^\circ], [id3, \alpha]]$; $[[id3, \beta], [id3, \alpha^\circ\beta^\circ]]$; $[[id3, \beta^\circ], [id3, \alpha^\circ]]$.

5. Using dynamic category theoretic morphisms, also the transitions between the fuzzy semiotic sets can now easily be indicated; f. ex.

$[[id1, \alpha]$, ... $[id1, \beta]]$ _	$[[id1, \beta\alpha]$,	$[id1, \beta^\circ]]$	$[[id1, \beta]$,	$[id1, \alpha^\circ\beta^\circ]]$
$\downarrow$ID $\downarrow$D ... $\downarrow$ $\downarrow$A _	$\downarrow$ $\downarrow$D	$\downarrow$ID $\downarrow$DS	$\downarrow$ $\downarrow$D	$\downarrow$ID DA
$[[id1, \alpha^\circ]$,.... $[id1, \beta\alpha]]$	$[[id1, \alpha^\circ\beta^\circ]$,	$[id1, \alpha]]$	$[[id1, \beta^\circ]$,	$[id1, \alpha^\circ]]$

Thus, all we need to fully describe all possible transitions between fuzzy category theoretic semiotic sets, are the following four semiotic functors:

- ID: maps a morphism onto itself
- D: dualization; turns a category into its dual category, i.e. $X \rightarrow X^\circ$; $X^\circ \rightarrow X$

- A: adjunction: $X \rightarrow XY$, $Y \rightarrow YX$. For the difference between A and DA cf. $X = \alpha$, then $AX = \beta\alpha$, $DAX = \alpha°\beta°$; if $X = \beta$, then $AX = \beta\alpha$, $DAX = \alpha°\beta°$
- S: substitution: $X \rightarrow Y$; $Y \rightarrow X$, whereby $X, Y \in \{\alpha, \beta\}$

Bibliography

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Harris, Joe K., Invariants of Posets under F-Morphisms. PhD dissertation, University of Alabama, 1983

Marty, Robert, Catégories et foncteurs en sémiotique. In: Semiosis 6, 1977, pp. 5-15

Nadin, Mihai, Sign and fuzzy automata. In: Semiosis 1, 1977, pp. 19-26

Nadin, Mihai, Zeichen und Wert. In: Grundlagenstudien aus Kybernetik und Geisteswissenschaft 19/1, 1978, pp. 19-28

Nadin, Mihai, The logic of vagueness and the category of synechism. In: The Monist 63/3, 1980, pp. 351-363

Nadin, Mihai, Sign fuzzy processes and the value continuum. In: Borbé, Tasso (ed.), Semiotics Unfolding, vol. 1. Berlin/New York 1983, pp. 201-211

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007 (2007a)

Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2007 (2007b)

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)

Toth, Alfred, The transpositions of sign classes as partially ordered sets. Ch. 25 (2008b)

Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

Walther, Elisabeth, Vorläufige Bemerkungen zu Trichotomischen Triaden. In: Semiosis 21, 1981, pp. 29-39

Walther, Elisabeth, Nachtrag zu Trichotomischen Triaden. In: Semosis 27, 1982, pp. 15-20

Zadeh, Lotfi A., Fuzzy sets. In: Information and Control 8, 1965, pp. 338-353