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Fiberings of semiotic systems

1. If we map the triadic semiotic set

S = {.1., .2., .3.}

onto itself, we get from S × S = {.1., .2., .3.} × {.1., .2., .3.}

the following triadic-trichotomic semiotic matrix

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

for the usual triadic-trichotomic sign model $SR_{3,3}$, that is considered the basis of classical semiotics,

$SR_{3,3}$ = (3.a 2.b 1.c),

together with the following trichotomic inclusion order

a ≤ b ≤ c,

and we can construct on this basis the system SS10 of the 10 sign classes and their dual reality thematics

(3.1 2.1 1.1) × (1.1 1.2 1.3)
(3.1 2.1 1.2) × (2.1 1.2 1.3)
(3.1 2.1 1.3) × (3.1 1.2 1.3)
(3.1 2.2 1.2) × (2.1 2.2 1.3)
(3.1 2.2 1.3) × (3.1 2.2 1.3)
(3.1 2.3 1.3) × (3.1 3.2 1.3)
(3.2 2.2 1.2) × (2.1 2.2 2.3)
(3.2 2.2 1.3) × (3.1 2.2 2.3)
(3.2 2.3 1.3) × (3.1 3.2 2.3)
(3.3 2.3 1.3) × (3.1 3.2 3.3)

2. However, our recent results (Toth 2008a, pp. 166 ss.; 2008b) show, that this is not the real beginning of semiosis, since the sign connects as a function both the ontological and the epistemological space. "The introduction of a sign as a general scheme of invariance goes far beyond the semiotic basis theory. We start by recognizing that an object, which is introduced into a semiosis and which is designated or denominated, is not changed by such a presentational, representational and interpretative process, i.e., a sign holds back the invariances to which is refers" (Bense 1975, p. 40).

Therefore, we can characterize the trichotomic correlates of the medium M of a sign-relation by a determining invariant each:

$(O^0) \Rightarrow$ Qual: Invariance of the material **connection**;
$(O^0) \Rightarrow$ Sin: Invariance of the material **identification**;
$(O^0) \Rightarrow$ Leg: Invariance of the material **existence**

Consequently, also the other two semioses, which the sign, introduced as a medium, enters, can be traced back to the three trichotomically differentiable invariances of connection, identification and existence. Thus, semiotic is characterized through the three invariances of the medium (M), the designation function ($M \Rightarrow O$) and the denomination function ($O \Rightarrow I$). From that, it follows that the semiotic object (O) and the semiotic interpretant (I) are invariant, too. Medium, object, and interpretant relation show in their trichotomies **invariance of consistency** (firstness), **invariance of identification** (secondness), and **invariance of existence** (thirdness).

By aid of these schemes of semiotic invariance, presented objects are mapped onto "available" media. Bense (1975, pp. 45 s.) gives the following examples for this **first pre-semiotic transition**. The superposed "0" shows, that the respective objects and media have relational number 0, since, in this transition, they are not yet embedded into a triadic relation (Bense 1975, p. 65):

$O^0 \Rightarrow M^0$: three available media
$O^0 \Rightarrow M_1^0$: qualitative substrate: heat
$O^0 \Rightarrow M_2^0$: singular substrate: trail of smoke
$O^0 \Rightarrow M_3^0$: nominal substrate: name

On a **second level of pre-semiotic-semiotic transition**, the available media are mapped onto relational media, and the semiotic invariance scheme is "inherited":

$M^0 \Rightarrow M$: three relational media
$M_1^0 \Rightarrow (1.1)$: heat
$M_2^0 \Rightarrow (1.2)$: trail of smoke
$M_3^0 \Rightarrow (1.3)$: "fire"

Having the three trichotomic sub-signs of firstness (1.1, 1.2, 1.3), we have, of course, already entered the semiotic space, but how can the three available media M_i^0 be characterized themselves? Matthias Götz (1982, p. 28) suggested assuming a pre-semiotic level of "zeroness" and its sub-division in

0.1 Secancy
0.2 Semancy
0.3 Selectancy

Secancy is "a diaphragmatic condition, which first of all has to be designated as such in order to allow semiotic mediation, since undifferentiated items cannot be represented". Semancy is "the condition to allow form to be described as form", and "selectancy is the condition of a posterior application, if its is considered a selective procedure, or, more generally, the dealing with the object" (Götz 1982, p. 4).

3. Summing up the hitherto gained knowledge, we obtain the following scheme:

Presented objects → Available media → Relational media

World of objects → Pre-Semiotics → Semiotics

Ontological Space Semiotic Space

Scheme of invariance: → Inheritance of the scheme of invariance:

1. Consistency/Secancy → 0.1

2. Identification/Semancy → 0.2

3. Existence/Selectancy → 0.3

Through combination of the semiotic invariants of consistency, identification, and existence, or the pre-semiotic features secancy, semancy, and selectancy, respectively, we get the following pre-semiotic matrix:

	0.1	0.2	0.3
0.1	(0.1 0.1)	(0.1 0.2)	(0.1 0.3)
0.2	(0.2 0.1)	(0.2 0.2)	(0.2 0.3)
0.3	(0.3 0.1)	(0.3 0.2)	(0.3 0.3)

as a basis for the triadic-trichotomic semiotic matrix over $SR_{3,3}$, i.e. after the second transition is completed:

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3,

such that (0.1 0.1) → (1.1), (0.1 0.2) → (1.2), (0.1 0.3) → (1.3) via categorial reduction, and (0.2. 0.1) → (2.1), (0.2 0.2) → (2.2), (0.2 0.3) → (2.3); (0.3 0.1) → (3.1), (0.3 0.2) → (3.2), and (0.3 0.3) → (3.3) via categorial reduction and semiotic inheritance. In other words, the threeness or pre-semiotic triad of the scheme of invariance "Consistency-Identification-Existence" is iterated for each of the three invariances. At the same time, their features are inherited, such that from the three pre-semiotic triads, three pre-semiotic trichotomies are generated, whose categorial structure displays the same scheme of invariance:

Secancy-Consistency: $0.1 \rightarrow 1.1 \rightarrow 2.1 \rightarrow 3.1$
Semancy-Identification: $0.2 \rightarrow 1.2 \rightarrow 2.2 \rightarrow 3.2$
Selectancy-Existence: $0.3 \rightarrow 1.3 \rightarrow 2.3 \rightarrow 3.3$

4. As we have seen, the abyss between pre-semiotics and semiotic can be bridged, according to Bense, by two transitions, the pre-semiotic transition between objects and available media, and the pre-semiotic-semiotic transition between available and relation media. In order to establish pre-semiotic sign classes, Bense further introduced relational and categorial numbers, which together define the complete pre-semiotic sign-relation Z^{r}_{k} (Bense 1975, pp. 65 s.; Toth 2008c). Since the categorial number can only refer to relation sign-correlates, k is always > 0, while r is ≥ 0 (i.e., it can refer to available media as well). However, since r apparently refers to triadic pre-semiotic and semiotic values, we get the following tetradic-trichotomic sign relation

$SR_{4,3}$ = (0., .1., .2., .3.)

The dot only AFTER the zeroness indicates that 0 can only appear as triadic, but not as trichotomic value. Therefore, we get the following pre-semiotic matrix for $SR_{4,3}$:

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

with the following sign-relation in semiotic "normal form":

SR = (3.a 2.b 1.c 0.d)

and with the tetradic inclusion order

a ≥ b ≥ c ≥ d

and thus the following system SS15 over $SR_{4,3}$ of the 15 pre-semiotic sign classes and their dual reality thematics:

1 (3.1 2.1 1.1 0.1) × ($\underline{1.0\ 1.1\ 1.2\ 1.3}$)
2 (3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2\ 1.3}$)
3 (3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2\ 1.3}$)
4 (3.1 2.1 1.2 0.2) × ($\underline{2.0\ 2.1}$ $\underline{1.2\ 1.3}$)
5 (3.1 2.1 1.2 0.3) × (3.0 2.1 $\underline{1.2\ 1.3}$)
6 (3.1 2.1 1.3 0.3) × ($\underline{3.0\ 3.1}$ $\underline{1.2\ 1.3}$)
7 (3.1 2.2 1.2 0.2) × ($\underline{2.0\ 2.1\ 2.2}$ 1.3)
8 (3.1 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 1.3)
9 (3.1 2.2 1.3 0.3) × ($\underline{3.0\ 3.1}$ 2.2 1.3)
10 (3.1 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2}$ 1.3)
11 (3.2 2.2 1.2 0.2) × ($\underline{2.0\ 2.1\ 2.2\ 2.3}$)
12 (3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2\ 2.3}$)
13 (3.2 2.2 1.3 0.3) × ($\underline{3.0\ 3.1}$ $\underline{2.2\ 2.3}$)
14 (3.2 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2}$ 2.3)
15 (3.3 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2\ 3.3}$)

5. As one recognizes, SS15 over $SR_{4,3}$ turns out to be a fibering of SS10 over $SR_{3,3}$:

1	(3.1 2.1 1.1	0.1)	×	(1.0	1.1 1.2 1.3)		
2	(3.1 2.1 1.1	0.2)	×	(2.0	1.1 1.2 1.3)	⟸	(3.1 2.1 1.1)
3	(3.1 2.1 1.1	0.3)	×	(3.0	1.1 1.2 1.3)		
4	(3.1 2.1 1.2	0.2)	×	(2.0	2.1 1.2 1.3)	⟸	
5	(3.1 2.1 1.2	0.3)	×	(3.0	2.1 1.2 1.3)		(3.1 2.1 1.2)
6	(3.1 2.1 1.3	0.3)	×	(3.0	3.1 1.2 1.3)	⟸	(3.1 2.1 1.3)
7	(3.1 2.2 1.2	0.2)	×	(2.0	2.1 2.2 1.3)	⟸	
8	(3.1 2.2 1.2	0.3)	×	(3.0	2.1 2.2 1.3)		(3.1 2.2 1.2)
9	(3.1 2.2 1.3	0.3)	×	(3.0	3.1 2.2 1.3)	⟸	(3.1 2.2 1.3)

10	(3.1 2.3 1.3	0.3)	×	(3.0	3.1 3.2 1.3)	(3.1 2.3 1.3)
11	(3.2 2.2 1.2	0.2)	×	(2.0	2.1 2.2 2.3)	
12	(3.2 2.2 1.2	0.3)	×	(3.0	2.1 2.2 2.3)	(3.2 2.2 1.2)
13	(3.2 2.2 1.3	0.3)	×	(3.0	3.1 2.2 2.3)	
14	(3.2 2.3 1.3	0.3)	×	(3.0	3.1 3.2 2.3)	(3.2 2.3 1.3)
15	(3.3 2.3 1.3	0.3)	×	(3.0	3.1 3.2 3.3)	(3.3 2.3 1.3)

However, at the same time, SS35 over $SR_{4,4}$ is one of the further fiberings both of SS10 over $SR_{3,3}$ and of SS15 over $SR_{4,3}$ (cf. Toth 2007, pp. 214 ss.):

1	(3.0	2.0	1.0	0.0)	×	(0.0	0.1	0.2	0.3)	
2	(3.0	2.0	1.0	0.1)	×	(1.0	0.1	0.2	0.3)	(3.1 2.1 1.1 0.1)
3	(3.0	2.0	1.0	0.2)	×	(2.0	0.1	0.2	0.3)	(3.1 2.1 1.1)
4	(3.0	2.0	1.0	0.3)	×	(3.0	0.1	0.2	0.3)	
5	(3.0	2.0	1.1	0.1)	×	(1.0	1.1	0.2	0.3)	(3.1 2.1 1.1 0.1)
6	(3.0	2.0	1.1	0.2)	×	(2.0	1.1	0.2	0.3)	(3.1 2.1 1.1)
7	(3.0	2.0	1.1	0.3)	×	(3.0	1.1	0.2	0.3)	
8	(3.0	2.0	1.2	0.2)	×	(2.0	2.1	0.2	0.3)	(3.1 2.1 1.2 0.2)
9	(3.0	2.0	1.2	0.3)	×	(3.0	2.1	0.2	0.3)	(3.1 2.1 1.2)
10	(3.0	2.0	1.3	0.3)	×	(3.0	3.1	0.2	0.3)	(3.1 2.1 1.3 0.3)
										(3.1 2.1 1.3)
11	(3.0	2.1	1.1	0.1)	×	(1.0	1.1	1.2	0.3)	
12	(3.0	2.1	1.1	0.2)	×	(2.0	1.1	1.2	0.3)	(3.1 2.1 1.1 0.1)
13	(3.0	2.1	1.1	0.3)	×	(3.0	1.1	1.2	0.3)	(3.1 2.1 1.1)
14	(3.0	2.1	1.2	0.2)	×	(2.0	2.1	1.2	0.3)	(3.1 2.1 1.2 0.2)
15	(3.0	2.1	1.2	0.3)	×	(3.0	2.1	1.2	0.3)	(3.1 2.1 1.2)
16	(3.0	2.1	1.3	0.3)	×	(3.0	3.1	1.2	0.3)	(3.1 2.1 1.3 0.3)
										(3.1 2.1 1.3)
17	(3.0	2.2	1.2	0.2)	×	(2.0	2.1	2.2	0.3)	(3.1 2.2 1.2 0.2)
18	(3.0	2.2	1.2	0.3)	×	(3.0	2.1	2.2	0.3)	(3.1 2.2 1.2)
										(3.1 2.2 1.3 0.3)
19	(3.0	2.2	1.3	0.3)	×	(3.0	3.1	2.2	0.3)	(3.1 2.2 1.3)

20	(3.0 2.3 1.3 0.3) × (3.0 3.1 3.2 0.3)	⇐	(3.1 2.3 1.3 0.3) (3.1 2.3 1.3)
21	(3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)		
22	(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)	⇐	(3.1 2.1 1.1 0.1)
23	(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)	⇐	(3.1 2.1 1.1)
24	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)	⇐	(3.1 2.1 1.2 0.2)
25	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)	⇐	(3.1 2.1 1.2)
26	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)	⇐	(3.1 2.1 1.3 0.3) (3.1 2.1 1.3)
27	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)	⇐	(3.1 2.2 1.2 0.2)
28	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)	⇐	(3.1 2.2 1.2)
29	(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)	⇐	(3.1 2.2 1.3 0.3) (3.1 2.2 1.3)
30	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)	⇐	(3.1 2.3 1.3 0.3) (3.1 2.3 1.3)
31	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)	⇐	(3.2 2.2 1.2 0.2)
32	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)	⇐	(3.2 2.2 1.2)
33	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)	⇐	(3.2 2.2 1.3 0.3) (3.2 2.2 1.3)
34	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)	⇐	(3.2 2.3 1.3 0.3) (3.2 2.3 1.3)
35	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)	⇐	(3.3 2.3 1.3 0.3) (3.3 2.3 1.3)

As one can see, there are two groups:

1. Direct fiberings of $SR_{3,3} \rightarrow (SR_{4,3} = SR_{4,4})$; f.ex:

(3.1 2.1 1.3) ⟶ (3.0 2.0 1.3 0.3)

2. Indirect fiberings of $SR_{3,3} \rightarrow SR_{4,3} \rightarrow SR_{4,4}$:

(3.1 2.1 1.1) → (3.1 2.1 1.1 0.1) → (3.0 2.0 1.0 0.0)
(3.1 2.1 1.1) → (3.1 2.1 1.1 0.1) → (3.0 2.0 1.0 0.1)
(3.1 2.1 1.1) → (3.1 2.1 1.1 0.1) → (3.0 2.0 1.0 0.2)
(3.1 2.1 1.1) → (3.1 2.1 1.1 0.1) → (3.0 2.0 1.0 0.3)

Since the sign classes from $(SR_{3,3} \rightarrow SR_{4,4}) \setminus (SR_{3,3} \rightarrow SR_{4,3})$ are exactly those sign classes that are lacking in SS35, SS15 of $SR_{4,3}$ is a proper subset of SS35 of $SR_{4,4}$ (nos. 21-35).

In order to show the arithmetic connections between the number of sign classes with partial order and the number of sign classes without partial order, i.e. without triadic ($a \leq b \leq c$), tetradic ($a \leq b \leq c \leq d$), pentadic ($a \leq b \leq c \leq d \leq e$), ... order, we give the following little table (cf. Toth 2007, p. 222):

Sign relation	Number of sign classes with partial order	Number of sign classes without partial order
$SR_{3,3}$	10	$3^3 = 27$
$SR_{4,3}$	15	$4^3 = 64$
$SR_{4,4}$	35	$4^4 = 256$
$SR_{5,4}$	70	$5^4 = 625$
$SR_{5,5}$	126	$5^5 = 3’125$
$SR_{6,6}$	462	$6^6 = 46’656$

One recognizes that we are dealing here with 2-, 3-, 4-, 5- and 6dimensional numbers which are part-sets of the set of the figurative numbers (cf. Flachsmeyer 1969, p. 74). They emerge by continuous addition of the members of an arithmetic series. Such multi-dimensional numbers can be displayed easiest by aid of the following Pascal’s Triangle, in which we show the connection of the numbers of the partially ordered sign classes of $SR_{3,3}$, $SR_{4,3}$, $SR_{3,3}$, $SR_{4,4}$, $SR_{5,4}$, and $SR_{5,5}$:

1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	
1	3	6	**10** →	**15**	21	28	36		
				↓					
1	4	10	20	**35**	56	84			
				↓					
1	5	15	35	**70** →	**126**				
1	6	21	56	126					
1	7	28	84						
1	8	36							
1	9								
1									

We also can calculate them by the well-known simple formulas:

Triangular numbers:	1, 3, 6, **10**, **15**, 21, 28, 36, 45, 55, …	½ n (n + 1)
Tetrahedral numbers:	1, 4, 10, 20, **35**, 56, 84, 120, 165, 220, …	1/6 n (n + 1) (n + 2)
4-dimensional numbers:	1, 5, 15, 35, **70**, **126**, 210, 330, 495, 715, …	1/24 n (n + 1) (n + 2) (n + 3)
5-dimensional numbers:	1, 6, 21, 56, 126, 252, **462**, 792, 1287, 2002, …	1/120 n (n + 1) (n + 2) (n + 3) (n + 4), usw.

Therefore, we recognize that the number of the partially ordered sign classes of both $SR_{3,3}$ and $SR_{4,3}$ is a triangular number, while the number of the partially ordered sign classes of the also tetradic sign-relation $SR_{4,4}$ is tetrahedral. The same happens on the next higher semiotic level: The number of the partially ordered sign classes of both $SR_{5,4}$ and $SR_{5,5}$ is a 4-dimensional number, while the number of the partially ordered sign classes of the also pentadic sign-relation $SR_{5,5}$ is a 5-dimensional number. Thus, it gets evident from the arithmetic level of the figurative numbers, too, that a sign-relation $SR_{n,n+1}$ is a fibering of a sign-relation $SR_{n,n}$ and not a simple part-relation of SR_{n+n+1} (cf. also Toth 2003, pp. 54 ss.).

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