Prof. Dr. Alfred Toth

The Droste effect in semiotics

Bim Coiffeur bin i gsässe vor em Spiegel, luege dry
Und gseh dert drinn e Spiegel wo ar Wand isch vis-à-vis
Und dert drin spieglet sech dr Spiegel da vor mir
Und i däm Spiegel widerum dr Spiegel hindefür

Und so geng wyter, s'isch gsy win e länge Korridor
I däm my Chopf gwüss hundertfach vo hinden und vo vor
Isch ufgreit gsy i eir Kolonne, z'hinderscht isch dr Chopf
I ha ne nümme gchennt, so chly gsy win e Gufechnopf

My Chopf, dä het sich dert ir Wyti, stellet öich das vor
Verloren ir Unäntlechkeit vom länge Korridor
I ha mi sälber hinde gseh verschwinde, ha das gseh
Am heiterhälle Vormittag und wi we nüt wär gscheh

Vor Chlupf han i mys Muul ufgscperrt, da sy im Korridor
Grad hundert Müüler mit ufgange win e Männerchor
E Männerchor us mir alei, es cheibe gspässigs Gfüel
Es metaphysischs Grusle het mi packt im Coiffeurgstüel

I ha d'Serviette vo mer grissen, ungschore sofort
Das Coiffeurgschäft verla mit paar entschuldigende Wort
Und wenn dir findet i sött e chly meh zum Coiffeur ga
De chöit dir jitz verstah warum i da e Hemmig ha

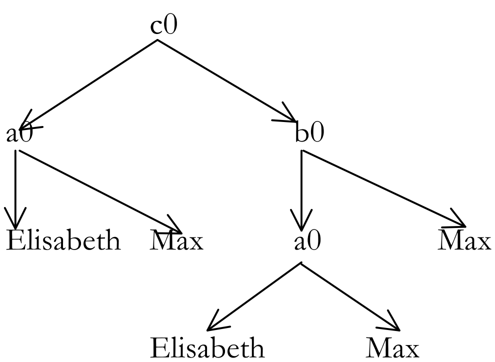
Mani Matter alias Dr. Hans-Peter Matter (1936-1972)*

1. The so-called Droste effect or mise en abyme points to a specific kind of self-containing picture that again contains itself and so on with theoretically infinite recursion. The Droste effect is thus based on a set-theoretic axiom system with anti-foundation axiom that allows self-referentiality: "Soient E un ensemble, E' un de ses éléments, E" un élément quelconque de E', et ainsi de suite. J'appelle *descente* la suite des passages de E à E', de E' à E", etc. Cette descente prend fin lorsqu'on tombe sur un élément indécomposable. Dans ce cas elle est finie, mais elle peut ne pas l'être, ce qui arrive par exemple pour tout ensemble de deuxième sorte E, losqu'on passe de cet ensemble E à l'élément E' qui lui est isomorphe, de E' à son isomorphe E", et ainsi de suite. Je dirai qu'un ensemble est *ordinaire* lorsqu'il ne donne lieu

* Literal, but clumsy translation of this song in Bernese Swiss German dialect: "At the hairdresser's I was sitting, looking in / The mirror that was placed at the wall in the rear of me / And in this mirror's mirroring the mirror in front of me / And in this mirror again the mirror at the back of me // And so' lways further, it was like a long-long corridor / In which my head was lined up hundred-fold from the back and from the front / It was lined up in one column, rearmost there was the head / I could not see him anymore, it was so small like a needle-pin // My head got lost - I really want you to imagine that – in the eternity of this long corridor / I have seen myself vanish, did have seen that / In broad daylight one morning and if nothing would have happened // With fear I gaped my mouth, but look there in the corridor / Just hundred mouths did gape like in a singers' choir / A singers' choir from me alone – a very strange feeling! / A metaphysic creeping caught me in my dresser's stool // Rapidly, I removed my towel, still being unshaved / And left this salon with a few apologizing words / And if you feel that I should go more often to the hairdresser / Then you may now understand why this idea causes me to fear."

qu'à des descentes finies; je dirai qu'il est *extraordinaire* lorsque parmi ses descentes il y en a qui sont infinies" (Mirimanoff 1917, p. 42).

Therefore, a not-well-founded set is an extraordinary set which contains infinitely descending elements that consist of an element of this set, an element of this element, an element of the element of this element and so on ad infinitum. Such sets are also called hypersets. For the sake of illustration I shall give here an example from Barwise and Etchemendy (1987, pp. 35s.): The set c0 = (a0, b0), whereby a0 = {Elizabeth, Max} and b0 = {a0, Max}. Amongst several possibilities to show this set by graphs, there is the following:

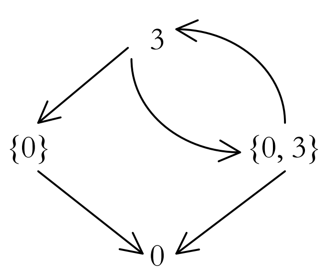


It must be pointed out that an axiom system with foundation axiom excludes such a set and its respective graph.

2. Ordered sets can be defined by unordered ones whose elements are unordered set, i.e. (a, b) = {{a}, {a, b}} (Wiener 1914). Therefore, from the equation x = (0, x), we get the new equation x = {{0}, {0, x}}. According to Aczel (1988, p. 8), this equation with one variable X is equivalent to the following system of four equations with the four variables x = {y, z}, y = {w}, z = {w, x}, w = 0. Hence, in a first step, we can define the three prime-signs like follows:

1 = {y, z}, y = {0}, z = {0, 1}, i.e. 1 = {{0}, {0, 1}}
2 = {y, z}, y = {0}, z = {0, 2}, i.e. 2 = {{0}, {0, 2}}
3 = {y, z}, y = {0}, z = {0, 3}, i.e. 1 = {{0}, {0, 3}}

According to a proposition by Aczel (1988, p. 9), such equations can be visualized by diagrams like the following. As an example, we shall take the category of thirdness, i.e. x = 3:



In a second step, we will represent the 9 sub-signs of the semiotic matrix as follows:

(1.1) = (1, 1) = {{1}, {1, 1}}　　(2.1) = (2, 1) = {{2}, {2, 1}}

(3.1) = (3, 1) = {{3}, {3, 1}}

(1.2) = (1, 2) = {{1}, {1, 2}}　　(2.2) = (2, 2) = {{2}, {2, 2}}

(3.2) = (3, 2) = {{3}, {3, 2}}

(1.3) = (1, 3) = {{1}, {1, 3}}　　(2.3) = (2, 3) = {{2}, {2, 3}}

(3.1) = (3, 3) = {{3}, {3, 3}}

In a third and last step, we will show the 10 sign classes, remembering that triples are represented as pairs of pairs. (The respective reality thematics can be obtained by simple exchanging of left by right bracketing.):

(3.1 2.1 1.1) = ((3.1), (2.1, 1.1)) = {{{{3}, {3, 1}}, {{2}, {2, 1}}, {1}, {1, 1}}}
(3.1 2.1 1.2) = ((3.1), (2.1, 1.2)) = {{{{3}, {3, 1}}, {{2}, {2, 1}}, {1}, {1, 2}}}
(3.1 2.1 1.3) = ((3.1), (2.1, 1.3)) = {{{{3}, {3, 1}}, {{2}, {2, 1}}, {1}, {1, 3}}}
(3.1 2.2 1.2) = ((3.1), (2.2, 1.2)) = {{{{3}, {3, 1}}, {{2}, {2, 2}}, {1}, {1, 2}}}
(3.1 2.2 1.3) = ((3.1), (2.2, 1.3)) = {{{{3}, {3, 1}}, {{2}, {2, 2}}, {1}, {1, 3}}}
(3.1 2.3 1.3) = ((3.1), (2.3, 1.3)) = {{{{3}, {3, 1}}, {{2}, {2, 3}}, {1}, {1, 3}}}
(3.2 2.2 1.2) = ((3.2), (2.2, 1.2)) = {{{{3}, {3, 2}}, {{2}, {2, 2}}, {1}, {1, 2}}}
(3.2 2.2 1.3) = ((3.2), (2.2, 1.3)) = {{{{3}, {3, 2}}, {{2}, {2, 2}}, {1}, {1, 3}}}
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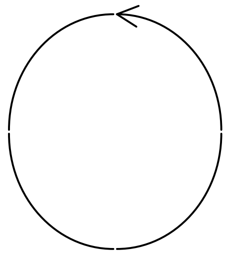
(3.3 2.3 1.3) = ((3.3), (2.3, 1.3)) = {{{{3}, {3, 3}}, {{2}, {2, 3}}, {1}, {1, 3}}}

3. Now let us further investigate the introduction of the prime-sings. Let be x = (0, x) = {{0}, {0, x}}. If the above system of the four equations with the four variables holds generally, i.e. if x = {y, z}, y = {w}, z = {w, x}, w = 0 the we obtain the following stream of hyper-sets:

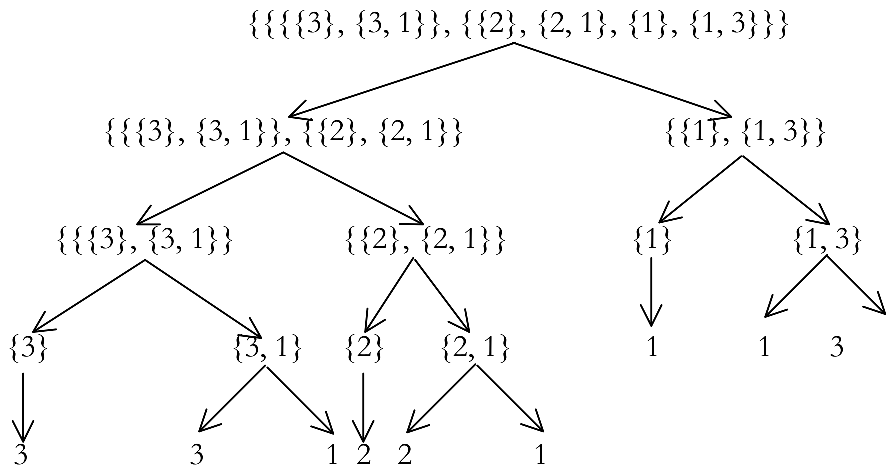
1.　x = {{w}, {w, x}}
1'.　x = {{w}, {w, {{w}, {w, x}}}}
1''.　x = {{w}, {w, {{w}, {w, {{w}, {w, x}}}}}}
1'''.　x = {{w}, {w, {{w}, {w, {{w}, {w, {{w}, {w, x}}}}}}}}
1''''.　x = {{w}, {w, {{w}, {w, {{w}, {w, {{w}, {w}, {{w}, {w, x}}}}}}}}}}, etc.,

where x = {.1., .2., .3.}. If we let grow, in such streams, $1^n \rightarrow \infty$, we get streams of the shape x = (0, (0, (0, …))). Thus, this is Aczel's so-called (infinite) "unfolding" of the original pair-set equation x = (0, x). However, the most important issue is, that in such a Zermelo-Fränkel axiom system with anti-foundation axiom, it is apparently **Ω** = {**Ω**} and thus also **Ω** = {**Ω**} = {**Ω**, **Ω**}. In order to show the semiotic relevance of that, we just have to look at the directed graphs of the genuine sub-signs; in the following, **Ω** can stand for (.1.), (.2.) or (.3.):

$\Omega = \{\Omega\}$



We now show the stem-diagram of the sign class (3.1 2.1 1.3) in the "unfolded" notation of unordered sets in the frame of the Zermelo-Fränkel axiom system with anti-foundation axiom:



4. Now, it was shown in Toth (2008a) that the 6 transpositions of a sign class can be ordered pairwise in 3 groups of orthogonal transpositions in which thus always 2 transpositions stand in a semiotic mirror-function (M) to one another:

1 (3.1 2.1 1.3)	3 (1.3 3.1 2.1)	5 (2.1 1.3 3.1)
2 (1.3 2.1 3.1)	4 (2.1 3.1 1.3)	6 (3.1 1.3 2.1)

Thus,

M(3.1 2.1 1.3) = (1.3 2.1 3.1);
M(1.3 3.1 2.1) = (2.1 3.1 1.3);
M(2.1 1.3 3.1) = (3.1 1.3 2.1).

Together with the above introduction of Mirimanoff-Aczel's anti-foundation axiom into semiotics, it is therefore possible to show exactly the infinite regress of self-referentiality in semiotics, or what we call here the "semiotic Droste effect":

http://www.josleys.com/show_gallery.php?galid=291

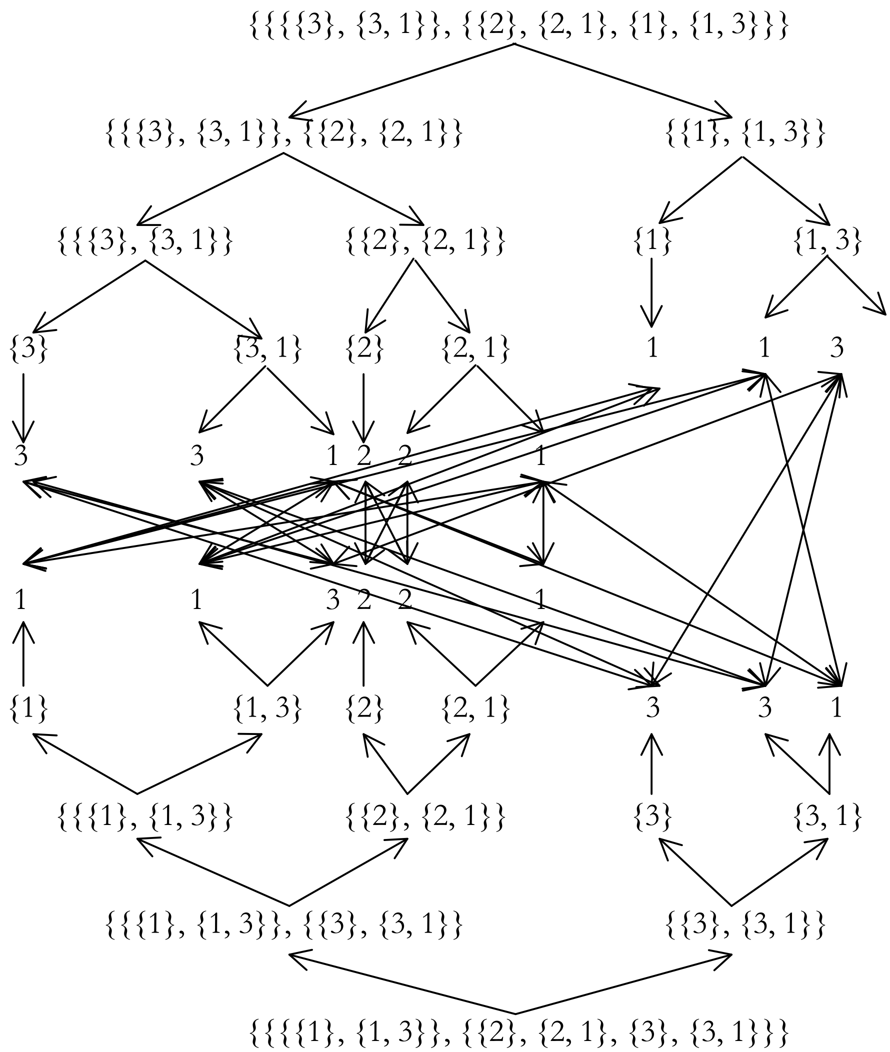
Using the above sign class (3.1 2.1 1.3), we thus get

(3.1 2.1 1.3) = ((3.1), (2.1, 1.3)) = {{{{3}, {3, 1}}, {{2}, {2, 1}}, {1}, {1, 3}}}
(1.3 2.1 3.1) = ((3.1), (2.1, 1.3)) = {{{{1}, {1, 3}}, {{2}, {2, 1}}, {3}, {3, 1}}}

(1.3 3.1 2.1) = ((3.1), (2.1, 1.3)) = {{{{1}, {1, 3}}, {{3}, {3, 1}}, {2}, {2, 1}}}
(2.1 3.1 1.3) = ((3.1), (2.1, 1.3)) = {{{{2}, {2, 1}}, {{3}, {3, 1}}, {1}, {1, 3}}}

(2.1 1.3 3.1) = ((3.1), (2.1, 1.3)) = {{{{2}, {2, 1}}, {{1}, {1, 3}}, {3}, {3, 1}}}
(3.1 1.3 2.1) = ((3.1), (2.1, 1.3)) = {{{{3}, {3, 1}}, {{1}, {1, 3}}, {2}, {2, 1}}}

We will now compare the first pair of mirroring transpositions and show them in the form of the above stem diagram. It is easy to imagine what complex graphs would arise, if all three pairs of mirroring transpositions would be depicted.



5. Theoretical semiotics can be built upon any set theoretic axiom system with or without foundation axiom, simply because prime-signs make use only of the first three natural numbers, and therefore, the well-known paradoxes do not appear on trivial reasons. However, the circularities which arise from the definition of the sign as an ordered relation over relations (cf. Toth 2008b) point towards the fact that for a set theoretic foundation of semiotics one best chooses the Zermelo-Fränkel axiom system with Mirimanoff-Aczel's anti-foundation axiom (cf. Toth 2007, pp. 17 ss.). With this choice, alleged semiotic paradoxes like the Droste or "Laughing Cow" effect can be handled in a mathematical-semiotic framework that does not exclude self-referentiality. As a matter of fact, self-referentiality turns out to be not only a logical but a general semiotic feature that includes all three dimensions and all the six transpositions of a sign relation in the form of "strange loops" in tangled hierarchies.

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