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Discrete Subgroups of the semiotic Euclidean group

Mit seiner Dampfmaschine treibt
er Hut um Hut aus seinem Hut
und stellt sie auf in Ringelreihn
wie man es mit Soldaten tut.

Dann grüßt er sie mit seinem Hut
der dreimal grüßt mit einem du.
Das traute sie vom Kakasie
ersetzt er durch das Kakadu.

Er sieht sie nicht und grüßt sie doch
er sie mit sich und läuft um sich.
Die Hüte inbegriffen sind
und deckt den Deckel ab vom Ich.

Hans Arp (1963, p. 83)[1]

1. In the R^2-model of the Euclidean plane, the set of all isometries is the Euclidean group E_2 with the composition of transformations as the binary operation. There are four types of isometries: translations, rotations, reflections, and glide reflections. The set of all translations T_2 is the translational subgroup T_2 of E_2, $T_2 < E_2$. The set of rotations with the origin as a center, and reflections in lines containing the origin, represents the subgroup O_2 of E_2, $O_2 < E_2$. This is the orthogonal subgroup of the Euclidean group, denoted as O_2. Every element of E_2 can be represented as a composition of one rotation with the origin as a center or one reflection in a line passing through the origin, and one translation. Thus, every element $\varepsilon \in E_2$ can be represented as $\varepsilon = \sigma\tau$, where $\sigma \in O_2$ and $\tau \in T_2$. From this relationship follows that E_2 is the semidirect product of its subgroups O_2 and T_2. We see that $O_2 \cap T_2 = \varepsilon$, where ε is the identity transformation. Hence, every element of E_2 we can decompose in the product of elements of O_2 and T_2 in a unique way (cit. Kozomara 1998)[2].

The isometries are represented as follows:

(a) Translation by a vector v as (v, I), where I is the unit 2×2 matrix;
(b) Rotation counterclockwise, through the angle θ about x, as $(x - xM^{\tau}, M)$, where

$$M = \begin{pmatrix} \cos\theta - \sin\theta \\ \\ \sin\theta - \cos\theta \end{pmatrix}$$

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1 Literal (but bumpy) translation: "With his steam engine he pushes / hat and hat out of his hat / and lines them up in a merry-go-round /as one does it then with soldiers. // Then he greets them with his hat / who three times does greet by a thou / The familiar "you" of "cocka-you" / He replaces by a "cocka-thou".// He does not see, but greets them though / he them with himself and runs about himself. / The hats, they are included then / and so he uncovers the lid of the I".
2 The further definitions are taken partly literally from Armstrong (1988) and Kozomara (1998).

(c) Reflection in a line pm as (2a, N), where the image of p derived by the translation a is the line p‘ that contains the origin:

$$N = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & -\cos\psi \end{pmatrix}$$

and ψ is the slope of p;

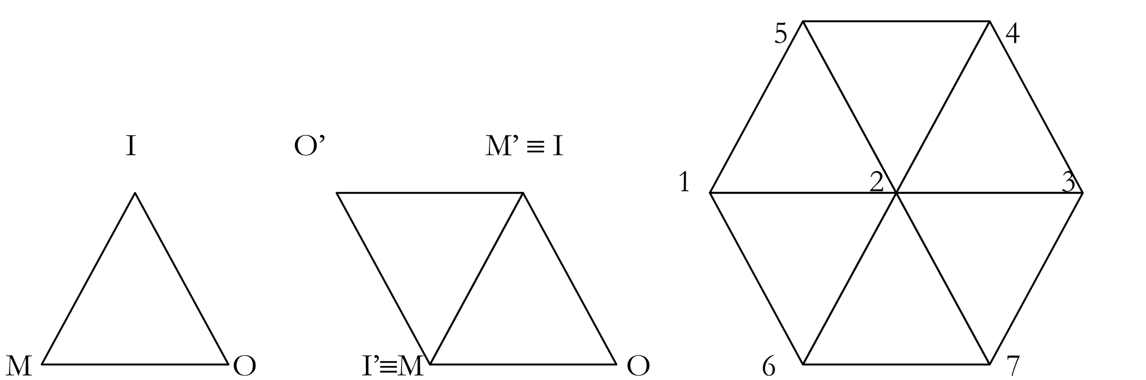
d) Glide reflection by a vector b, in a line that translated by b contains the origin, is represented as (2a+b, N), with N defined as in (c).

2. That there is a semiotic Euclidean group follows from previous studies (cf. Toth 2002, 2007, pp. 37 ss., 52 ss.; 2008a, pp. 57 ss). In this study, we will restrict ourselves to discrete subgroups of the semiotic Euclidean group. According to the number of independent translations contained in a particular group, there are three classes of discrete subgroups of Euclidean group E_2. The first is the class of discrete subgroups of E_2 without translations – the symmetry group of rosettes. This class is infinite. The second class contains the groups with a translation subgroup generated by one single translation – the symmetry group of friezes. That class contains 7 non-isomorphic symmetry groups. The third class is the wallpaper groups. Their translation subgroup is generated by two independent translations, and this class contains 17 non-isomorphic groups.

2.1. The semiotic symmetry groups of rosettes

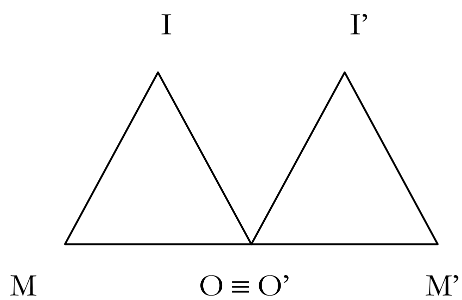
A subgroup D of D_{E2} is the symmetry group of rosettes if it does not contain translations. The elements of a rosette group are the symmetries of a rosette. The rotational subgroup of a rosette group R is generated by a rotation R = $\langle M_0 \rangle$, $\theta \in [0, 2\pi]$. In the case that O_R contains only direct transformations, the unique possibility is R = $\langle M_{[(2\pi/m]} \rangle$. Such a rosette group is isomorphic to a cyclic group C_m.

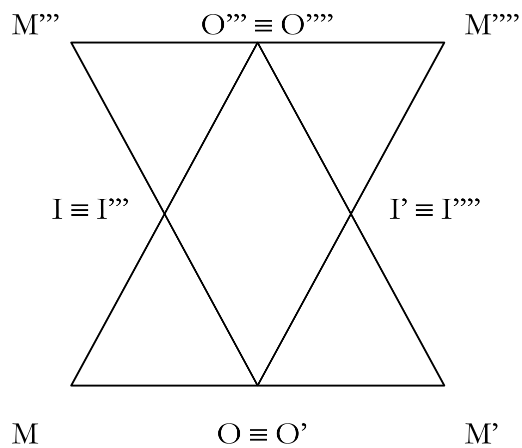
In order to show the different symmetries, I use the framework of my “General Sign Grammar” (Toth 2008b) on the one side and the theory of semiotic transpositions (Toth 2008a, pp. 159 ss.) on the other side. Here are some examples for semiotic rosettes:



1 = M ≡ M”’ 5 = I ≡ M”
2 = O ≡ I” ≡ O”’ ≡ I’ ≡ I””’ 6 = O”’ ≡ O””
3 = M’ ≡ O””’ 7 = M”” ≡ O””’
4 = I’ ≡ O”

Let O_R contain n indirect transformations and a rotation M of the order m, so the number of rotations in O_R is m. Hence, for an indirect transformation S, the compositions SM, SM^2, ..., SM^m are mutually different indirect transformations from O_R, so $m \leq n$. On the other hand, compositions SM, SM^2, ..., SM^m are mutually different direct transformations from O_R, and we have $n \leq m$. Therefore, m = n. We see that all indirect transformations from R have the form $(0, M_{[l(2\pi/m]})$ (0, S), where $l \in$ N and S is an indirect transformation from O_R. Hence, R = <(0, $M_{[(2\pi)/m]}$),(0, S)>. Such a group R is isomorphic to a dihedral group D_m, and we will illustrate it again by the following semiotic symmetries:



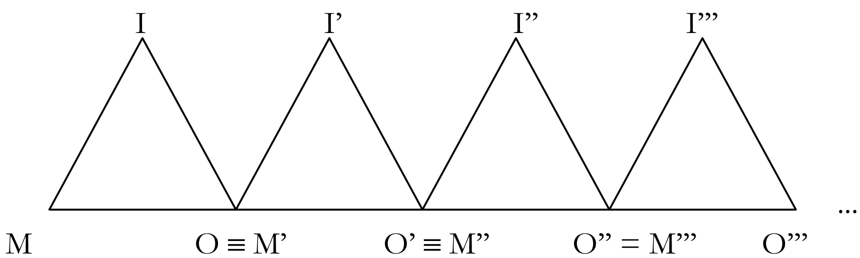


2.2. The semiotic symmetry group of friezes

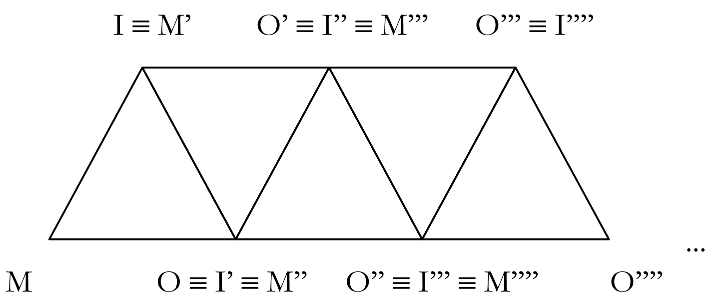
A subgroup B of D_{E2} is the symmetry group of friezes if its translational subgroup is generated by one translation. If the translation by a vector a generates T_B then the lattice is the set of points R = {na | n $\in$ Z}. By θ will be denoted the matrix of rotation about the origin through the angle θ, and by S_φ the reflection in the line passing through the origin of slope $\varphi/2$. Since in the classification of the frieze groups, the vector a will be considered as collinear to the x-axis and since every point of the lattice belongs to the x-axis, we find that the point group of every frieze group B must leave the x-axis invariant, and the only possible transformations contained in the orthogonal group O_B are: I, -I, S_0, and $S_{[(\pi/2]}$. By choosing possible orthogonal groups for the frieze groups, respecting the condition that it must leave the lattice invariant, there are 7 non-isomorphic frieze groups.

With regard to O_B, we have the following possibilities:

1. $O_B = \{I\}$. $B = T_B$.

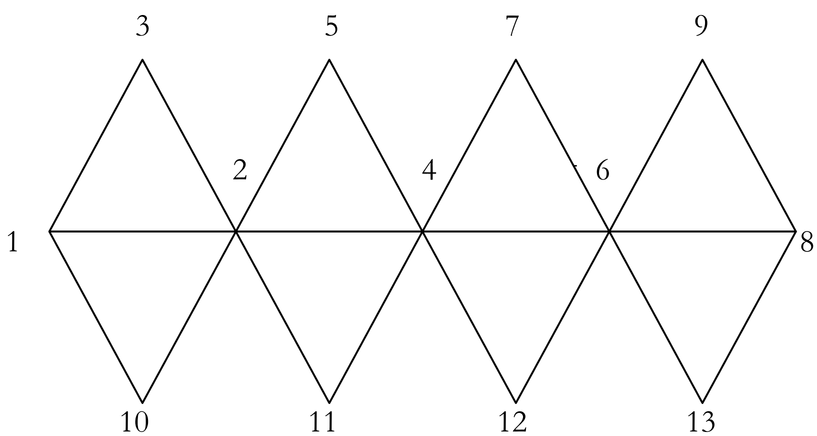


2. $O_B = \langle -I \rangle \{I, -I\}$. $B = T_B \cup T_B\,(0, -I)$.



3. $O_B = \langle S_0 \rangle \{I, S_0\}$. Let S_0 be realized as $(\alpha a, S_0)$.

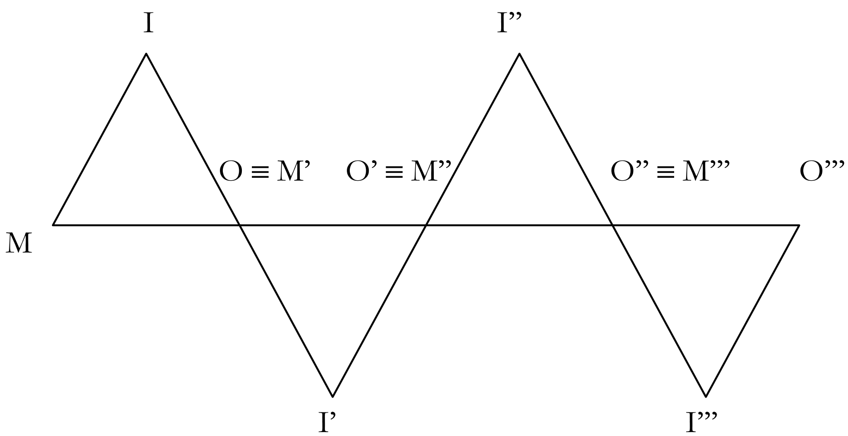
a) In the case that S_0 is realized as a reflection: $B = T_B \cup T_B\,(0, S_0)$



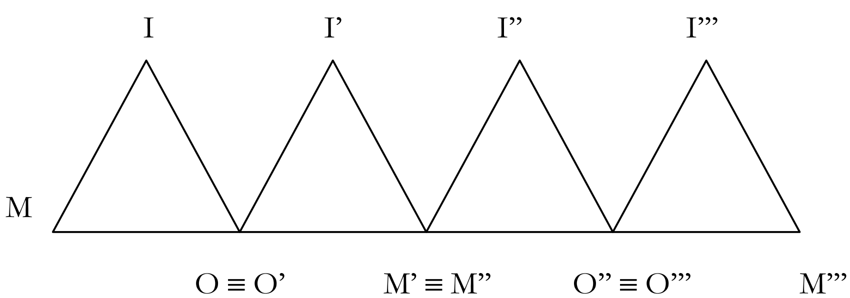
$1 = M \equiv M'$
$2 = O \equiv O' \equiv M'' = M'''$
$3 = I$
$4 = O'' \equiv O''' \equiv M'''' \equiv M'''''$
$5 = I''$
$6 = O'''' \equiv O''''' \equiv M'''''' \equiv M'''''''$
$8 = O'''''' \equiv O'''''''$
$9 = I''''''$
$10 = I'$
$11 = I'''$
$12 = I''''$
$13 = I'''''''$

7 = I’”’

b) The other possibility is that the frieze group does not contain S_0, $(0, S_0) \notin B$. Then $\alpha \notin Z$. Since $(\alpha a, S_0)^2 = (2\alpha, a, I)$, we have that $\alpha = n + ½$, $n \in Z$. So, S_0 is realized as a glide reflection: $B = T_B \cup T_B (½ a, S_0)$.

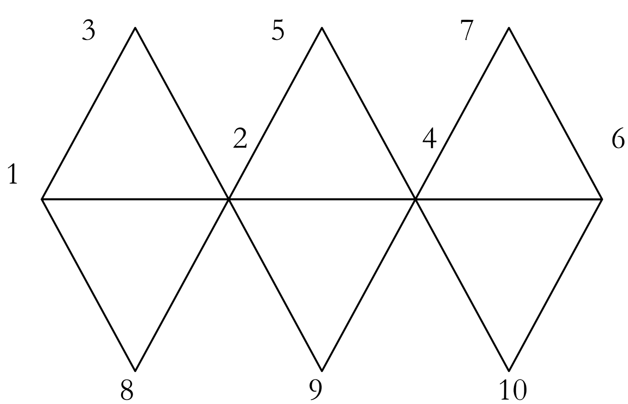


4. $O_B = <S_{[\pi/2]}>\{I, S_{[\pi/2]}\}$. The $S_{[\pi/2]}$ in B must be realized as a reflection. In B there is no translation by a vector normal to x-axis. Hence: $B = T_B \cup T_B (0, S_{[\pi/2]})$



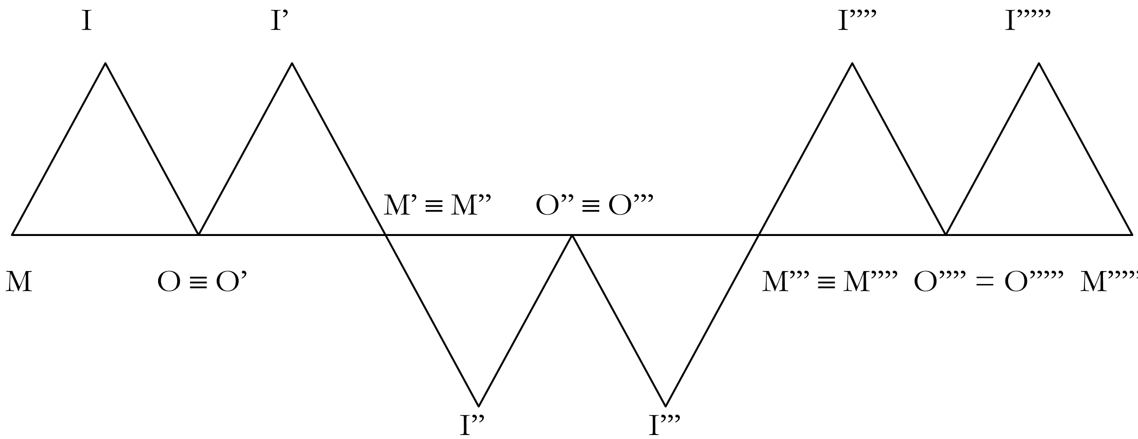
5. $O_B = <S_0, S_{[\pi/2]}>\{I, -I, S_0, S_{[\pi/2]}\}$. There are two possibilities:

a) S_0 is realized as a reflection: $B = T_B \cup T_B (0, -I) \cup T_B (0, S_0) \cup T_B (0, S_{[\pi/2]})$



1 = M ≡ M’	6 = O”” ≡ O””’
2 = O ≡ O’ ≡ O” ≡ O”’	7 = I””
3 = I	8 = I’
4 = M” ≡ M”’ ≡ M”” = M””’	9 = I”’
5 = I”	10 = I””’

b) S_0 is not realized as a reflection. Then, S_0 in B is realized as ((n + ½)a, S_0) and we have that –I is realized as ((n + ½)a, S_0)(0, $S_{\pi/2}$) = ((n + ½)a, -I). B = $T_B \cup T_B$ (0, -I) $\cup$ T_B (½a, S_0) $\cup$ T_B (0, $S_{|\pi/2}$)



The above types of symmetry correspond to Bense’s sign operation of “adjunction” (Bense 1971, p. 52; Toth 2008b, pp. 20 ss.). In the next chapter, we will find several groups that correspond also to semiotic “iteration” and superization” (Bense 1971, pp. 54 ss.; Toth 2008b, pp. 20 ss.)

2.3. The semiotic wallpaper groups

A subgroup K of D_{E2} is the wallpaper group if its transitional subgroup is generated by two translations. The lattice consists of points ma + nb, m, n ∈ Z, where translations by independent vectors a and b generate T_K. Without loss of generality, let:

1. |a| ≤ |b| (otherwise a ↔ b)
2. |a – b| ≤ |a + b| (otherwise a → -b)

Thus, we have the following possible relations between |a|, |b|, |a - b|, |a + b|:

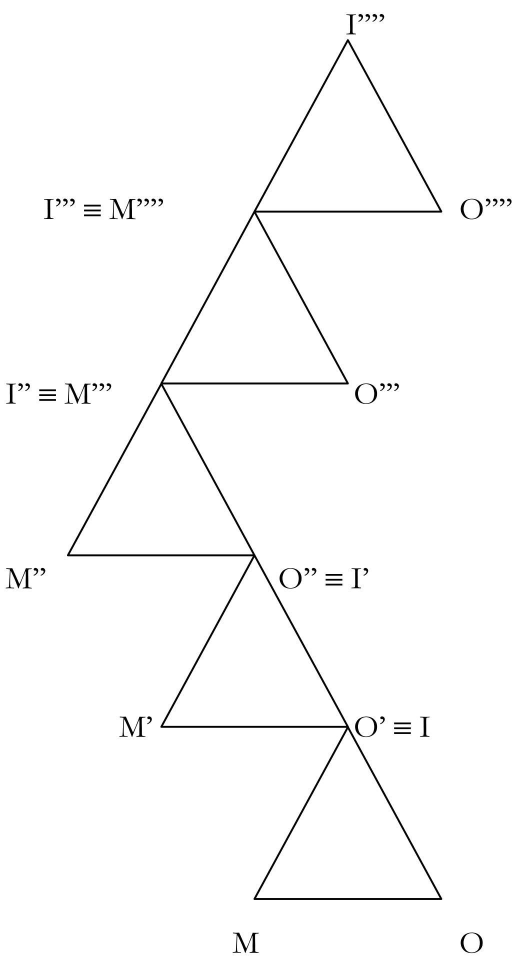
1. |a| < |b| < |a - b| < |a + b| (oblique lattice)
2. |a| < |b| < |a - b| = |a + b| (rectangular lattice)
3.1. |a| < |b| = |a - b| < |a + b| (centered rectangular lattice)
3.2. |a| = |b| < |a - b| < |a + b| (centered rectangular lattice)
4. |a| = |b| < |a - b| = |a + b| (square lattice)

5. $|a| = |b| = |a - b| < |a + b|$ (hexagonal lattice)

There are only four non-trivial rotations through the angles $[2\pi/6]$, $[2\pi/4]$, $[2\pi/3]$, $[2\pi/2]$, i.e. the rotations of the order 6, 4, 3, 2, respectively. Every orthogonal group of a wallpaper group is finite, because the wallpaper group is discrete, so it contains the rotations through the angles $[2\pi/k]$, $k \in Z$. Since the vectors a and b that generate T_K are independent, they represent the basis of R^2. If we combine each lattice with an orthogonal group, there are 17 non-isomorphic wallpaper groups. For every T_K and its orthogonal group O_K, the wallpaper group will be $T_K \cup X_i \in O_K (\tau, X)$, where $\tau \in T_K$ and (τ, X_i) represents the realization of X_i from O_K in K.

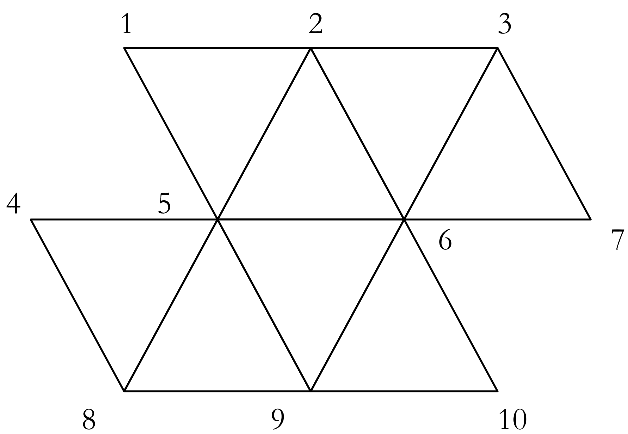
1. As for the oblique lattice, R, $|a| < |b| < |a - b| < |a + b|$, the only element of O_D that preserves R is the rotation through φ about the origin, so $O_K \subset \{\pm I\}$.

1.1. If the only rotation in K is the identity I, we get the simplest case: the wallpaper group generated by translations, $K = \{(ma + nb, I) \mid m, n \in Z\}$.



This symmetry group corresponds to Bense’s sign-operation of superization (cf. Bense 1971, pp. 53 ss.) with at the same time rising and falling cascades (cf. Toth 2008b, pp. 62 ss.).

1.2. O_K contains –I. We get K = T_K {(0, I), (0, -I)}, that is the union of two neighboring classes of T_K. The elements of K, not belonging to T_K, are (ma + nb, I) (0, -I) = (ma + nb, I), m, n $\in$ Z, which means that the elements are translations and half-turns about the points (½ a + ½ nb).



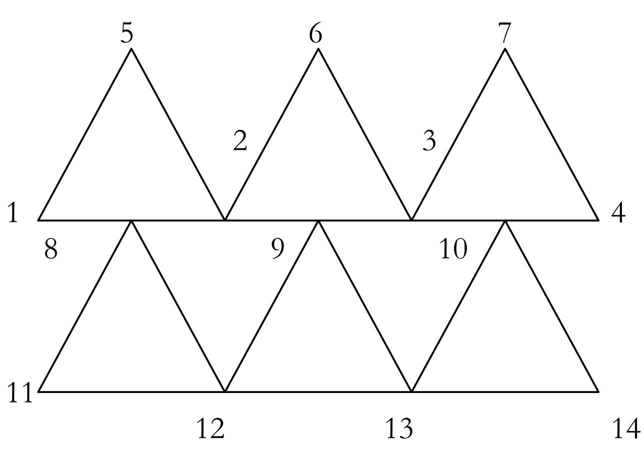
1 = O'	6 = $O \equiv I'' \equiv M''' \equiv M'''''' \equiv I'''''''$
2 = $M' \equiv I \equiv O''$	7 = O'''
3 = $M'' \equiv I'''$	8 = $I'''' \equiv M'''''$
4 = O''''	9 = $O''''' \equiv I'''''' \equiv M'''''''$
5 = $M \equiv I' \equiv M'''' \equiv I''''' \equiv O''''''$	10 = O'''''''

2. $|a| < |b| < |a - b| = |a + b|$

Here, except the coincidence, we have the transformations –I, S_0 and S_π preserving the lattice.

2.1. For $O_K = \{I, S_0\}$ there are two possibilities.

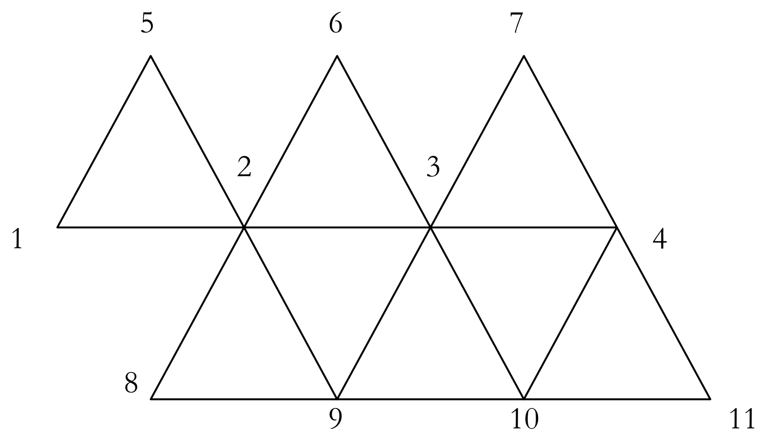
2.1.1. S_0 is realized in K as a reflection (0, S_0). K = $T_K \cup T_K$ (S_0).



1 = M
2 = O ≡ O’
3 = M’ ≡ M’’
4 = O’’
5 = I
6 = I’
7 = I’’

8 = (I’’’) ↔ (M → O)
9 = (I’’’’) ↔ (M’ → O’)
10 = (I’’’’’) ↔ (M’’ → O’’)
11 = M’’’
12 = O’’’ ≡ O’’’’
13 = M’’’’ ≡ M’’’’’
14 = O’’’’’

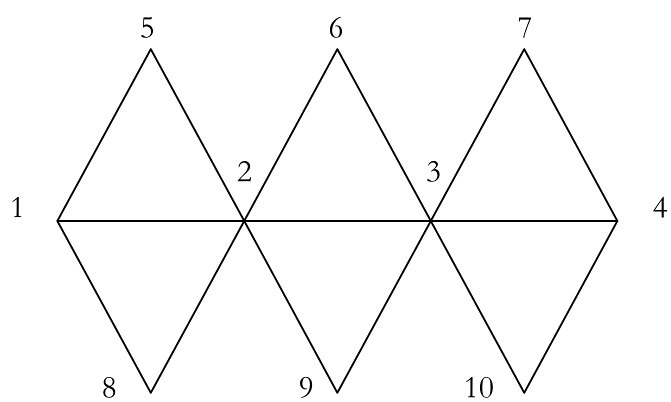
2.1.2. S_0 is realized in K as a glide reflection, so K contains (0, S_0). This is the glide reflection (½a, S_0).



1 = M
2 = O ≡ I’ ≡ M’’
3 = O’’ ≡ I’’’ ≡ M’’’’
4 = O’’’’ ≡ I’’’’’
5 = I
6 = I’’

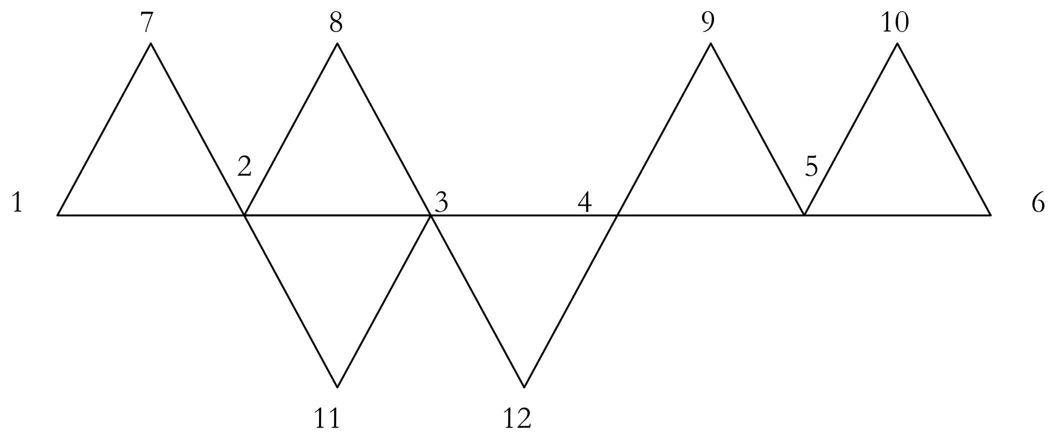
7 = I’’’’
8 = O’
9 = M’ ≡ O’’’
10 ≡ M’’’ ≡ O’’’’’
11 = M’’’’’

2.1.3. O_K is {I, -I, S_0, S_π}. If S_0 and S_π are realized as reflections, K = $T_K \cup X \in O_K$ (O, X).



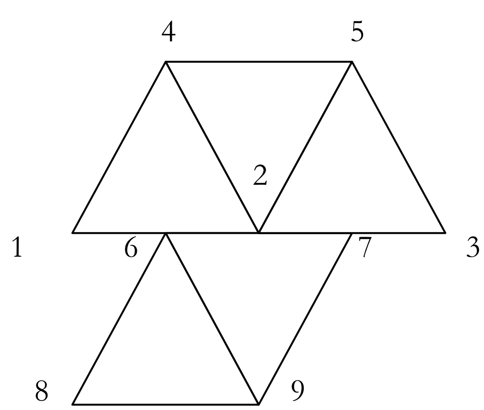
1 = O ≡ O’	6 = I”
2 = M ≡ M’ ≡ M” ≡ O’”	7 = I””
3 = O” ≡ O”” ≡ M’” ≡ O”’”	8 = I’
4 = M”” ≡ M”’”	9 = I’”
5 = I	10 = I”’”

2.1.4. The only element of orthogonal group realized as a reflection is $(0, S_\pi)$. The transformation $-I$ from O_K is realized as $(½, S_0)\ (0, S_\pi) = (½\ a, -I)$. $K = T_K \cup T_K\ (0, S_0) \cup T_K\ (0, S_\pi) \cup T_K\ (½\ a, -I)$.



1 = O	7 = I
2 = M ≡ M’ ≡ O”	8 = I’
3 = O’ ≡ M” ≡ M’”	9 = I””
4 = O’” ≡ O””	10 = I”’”
5 = M”” ≡ M”’”	11 = I”
6 = O”’”	12 = I’”

2.1.5. The third case occurs when K does not contain the reflections. Thus, S_0 is realized as $(½\ a, S_0)$, and S_π as $(½\ b, S_\pi)$. $-I$ is realized as $(½\ a, S_0)\ (½\ b, S_\pi) = (½\ (a – b), -I)$. $K = T_K \cup T_K\ (½\ a, S_0) \cup T_K\ (½\ b, S_\pi) \cup T_K\ (½\ (a – b), -I)$.

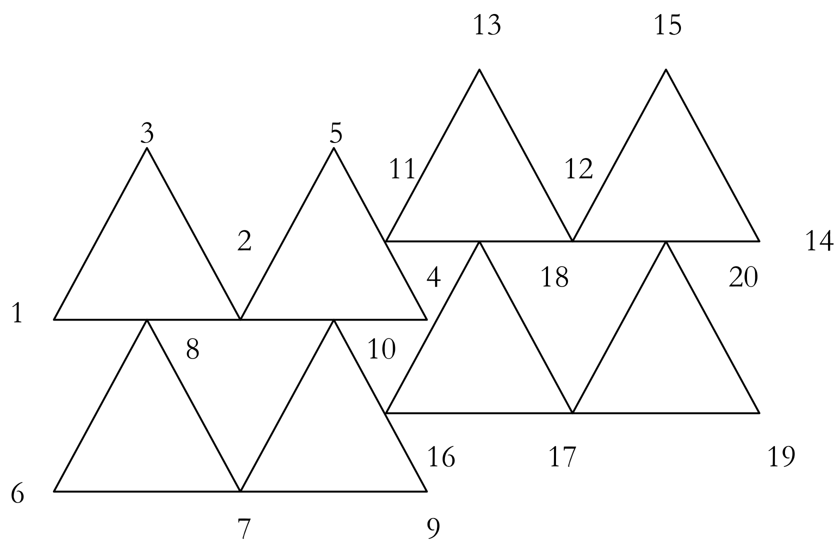


1 = M	6 = I’ ↔ (M → O)
2 = O ≡ I”’ ≡ M””	7 = (M”” → O””) ↔ (M” → O”)
3 = O””	8 = O’
4 = I ≡ O”’	9 = M’ ≡ I”
5 = M”’ ≡ I””	

3. $|a| < |b| = |a - b| < |a + b|$

The elements of O_K are I, -I, S_0, S_π. In order to avoid isomorphic groups, we take that K, where the reflections from O_K are realized as both reflections and glide reflections.

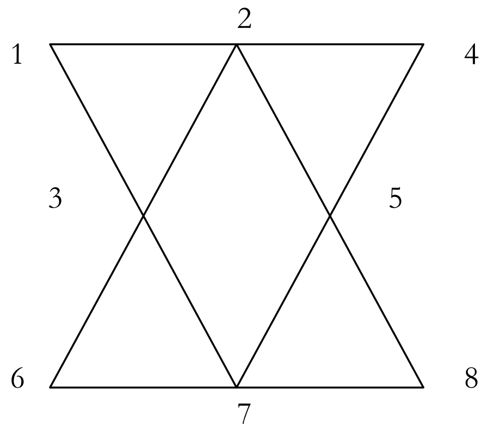
3.1. In the case that $O_K = \{I, S_0\}$, S_0 is realized as a reflection $(2(b – a), S_0)$, and glide reflection as $(½(2(b – a) + a/2, S_0) = (b, S_0)$, we have $K = T_K \cup T_K (b, S_0)$.



1 = M	6 = M”	11 = M”” ↔ (M’ → I’)
2 = O ≡ O’	7 = O” ≡ O”’	12 = O”” ≡ O””’
3 = I	8 = I” ↔ (M → O)	13 = I””
4 = M”	9 = M”’	14 = M””’
5 = I”	10 = I”’ ↔ (O’ ↔ M’)	15 = I””’

16 = M”””’ ↔ (M”’ → I”’)
17 = O”””’ ≡ O”””
18 = I”””’ ↔ (M”” → O””)
19 = M”””
20 = I”””’ ↔ (O””’ → M””’)

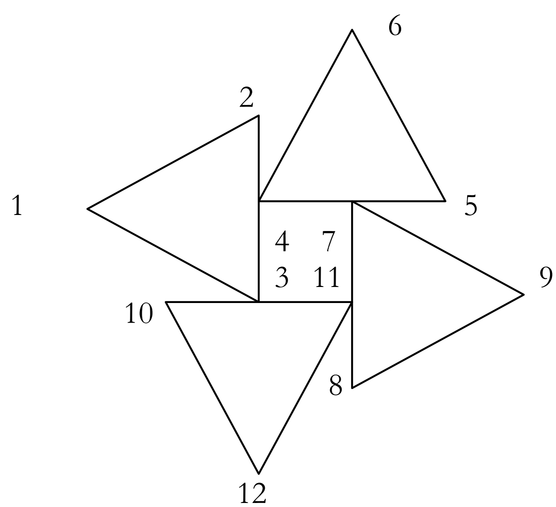
3.2. In the case that O_K = {I, -I, S_0, S_π), we have K = $T_K \cup T_K$ (b, S_0) $\cup$ T_K (a, S_π).



1 = O
2 = M ≡ M"
3 = I ≡ I'
4 = O"
5 = I" ≡ I'"
6 = O'
7 = M' ≡ M'"
8 = O'"

4. |a| = |b| < |a - b| = |a + b|

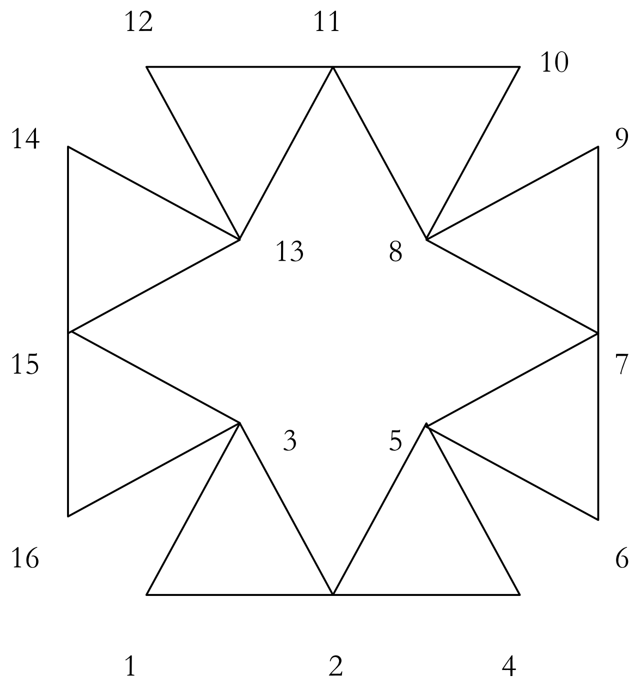
4.1. In the case O_K = <$M_{[\pi/2]}$>, we have K = $T_K \cup T_K$ (0, $M_{\pi/2}$) $\cup$ T_K (0, M_π) $\cup$ T_K (0, $M_{3\pi/2}$).



1 = I
2 = O
3 = M ↔ (O'" → M'")
4 = M' ↔ (M → O)
5 = O'
6 = I'
7 = M" ↔ (M' → O')
8 = O"
9 = I"
10 = O'"
11 = M'" ↔ (M" ↔ O")
12 = I'"

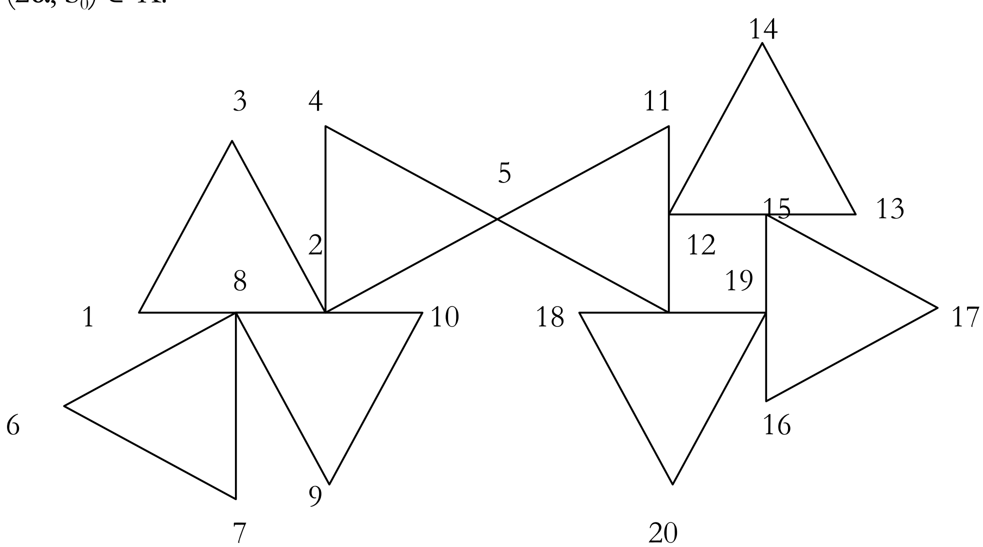
4.2. In the case $S_0 \in O_K$, $O_K = \{I, M_{[\pi/2]}, -I, M_{[3\pi/2]}, S_0, S_{[\pi/2]}, S_\pi, S_{[[\pi/2]}\}$

4.2.1. S_0 is realized as a reflection



$1 = M$
$2 = O \equiv O'$
$3 = I \equiv I''''''$
$4 = M'$
$5 = I' \equiv I''$
$6 = M''$
$7 = O'' = O'''$
$8 = I''' \equiv I''''$
$9 = M'''$
$10 = M''''$
$11 = O'''' \equiv O'''''$
$12 = M''''$
$13 = I'''' \equiv I'''''$
$14 = M'''''$
$15 = O''''' \equiv O''''''$
$16 = M''''''$

4.2.2. S_0 is not realized as a reflection. Let S_0 be realized as $(\alpha a + \beta b, S_0)$, so $(\alpha a + \beta b, S_0)^2 = (2\alpha, S_0) \in K$.

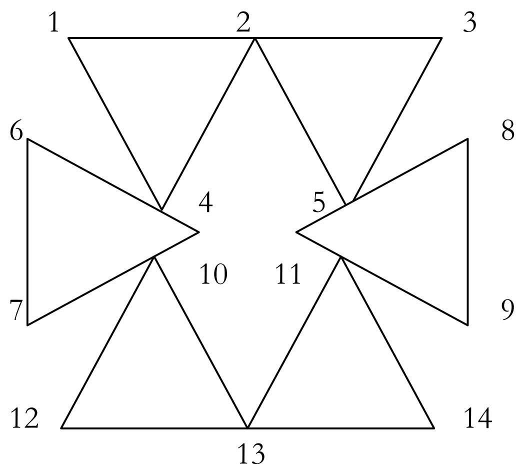


$1 = O$	$11 = M''''$
$2 = M \leftrightarrow (M''' \rightarrow O''') \equiv M'$	$12 = M'''''' \leftrightarrow (O'''' \rightarrow M'''')$
$3 = I$	$13 = O''''''$
$4 = O'$	$14 = I''''''$
$5 = I' \equiv I''''$	$15 = M''''''' \leftrightarrow (M'''''' \rightarrow O'''''')$

6 = I”
7 = O”
8 = M” ↔ (O → M) ≡ M”’
9 = I”’
10 = O”’
16 = O”””’
17 = I”””’
18 = O””’
19 = M””’ ↔ (M”””’ → O”””’)
20 = I””’

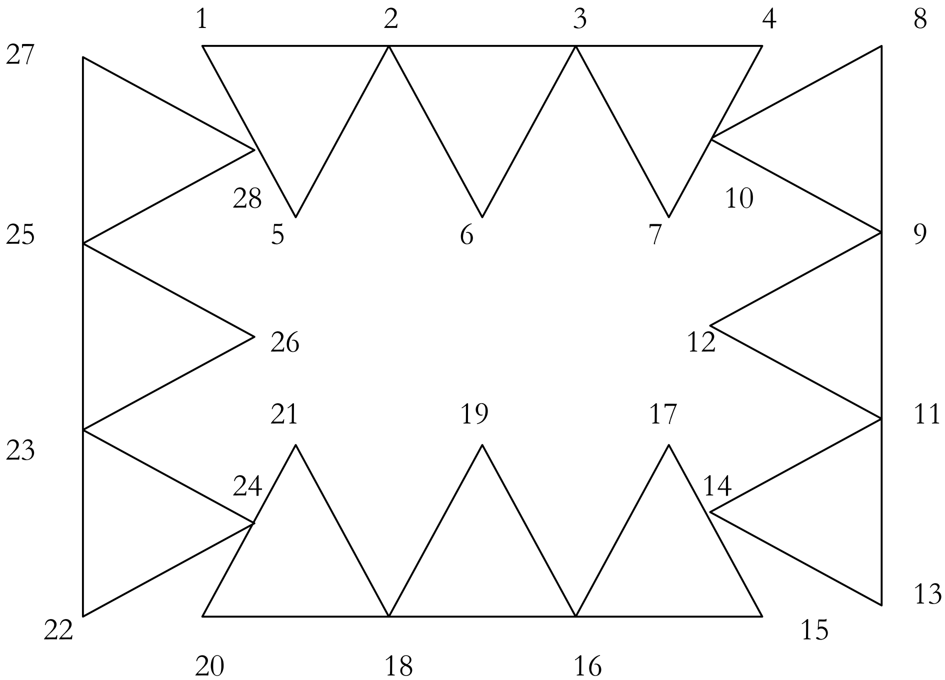
5. |a| = |b| = |a - b| < |a + b|. Since S_0 $M_{[k\pi/3]}$ = $S_{[k\pi/3]}$, and all of them are realized as reflections in K.

5.1. O_K is generated by $M_{[\pi/3]}$. Here K = $T_K \cup_{k=1,\ldots5}$ (0, $M_{[k\pi/3]}$).



1 = O
2 = M ≡ O’
3 = M’
4 = I ↔ (M””’ → I””’)
5 = I’ ↔ (I” → O”)
6 = M””’
7 = O””’
8 = O”
9 = M”
10 = I””
11 = I”’
12 = M””
13 = O”” ≡ M”’
14 = O”’

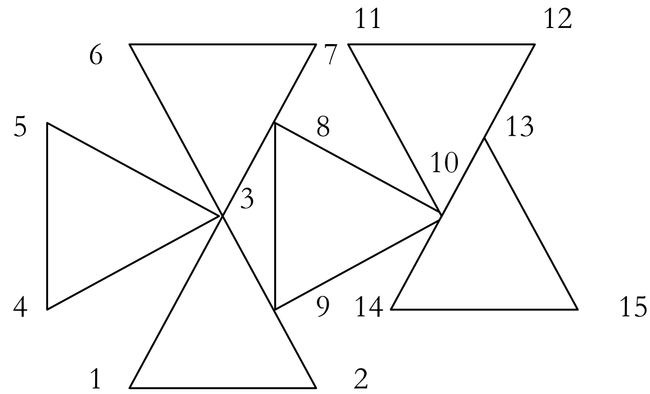
5.2. $O_K = <M_{[\pi/3]}, S_0>, K = T_K \cup_{k=1,...,5} (0, S_{[k\pi/3]})$



1 = O’
2 = M’ ≡ M
3 = O ≡ O’’’’’’’’’’’’
4 = M’’’’’’’’’’’’
5 = I’
6 = I
7 = I’’’’’’’’’’’’
8 = M’’’’’’’’’’’
9 = O’’’’’’’’’’ ≡ O’’’’’’’’’’
10 = I’’’’’’’’’’’ ↔(I’’’’’’’’’’’’ → M’’’’’’’’’’’’)
11 = M’’’’’’’’’’ ≡ M’’’’’’’’’
12 = O’’’’’’’’’’
13 = O’’’’’’’’’
14 = I’’’’’’’’’ ↔ (I’’’’’’’’ → O’’’’’’’’)

15 = O’’’’’’’
16 = M’’’’’’’ ≡ M’’’’’’
17 = I’’’’’’’
18 = O’’’’’’ ≡ O’’’’’
19 = I’’’’’’
20 = M’’’’’
21 = I’’’’’
22 = M’’’’
23 = O’’’’ ≡ O’’’
24 = I’’’’ ↔ (I’’’’’ → M’’’’’)
25 = M’’’ ≡ M’’
26 = I’’’
27 = O’’
28 = I’’ ↔ (I’ → O’)

5.3. O_K is generated by $M_{[2\pi/3]}$. $K = T_K \cup_{k=1,...,3} (0, M_{[2k\pi/3]})$.



$1 = M''$

$2 = O''$

$3 = I'' \equiv I \equiv I$

$4 = O$

$5 = M$

$6 = O'$

$7 = M'$

$8 = M''' \leftrightarrow (I' \rightarrow M')$

$9 = O''' \leftrightarrow (O'' \rightarrow I'')$

$10 = I''' \equiv I'''' \leftrightarrow (I''''' \rightarrow M''''')$

$11 = O''''$

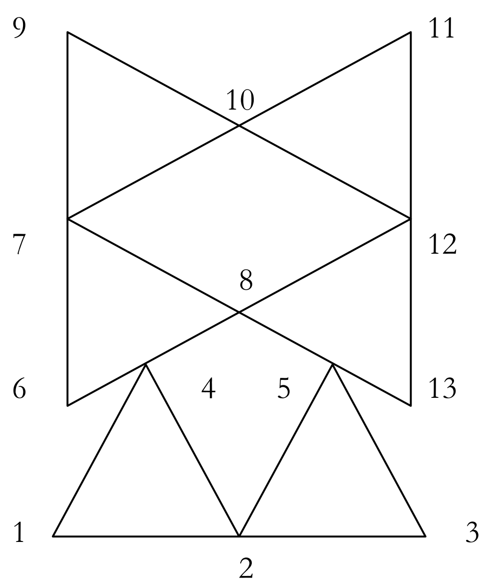
$12 = M''''$

$13 = (M''''' \rightarrow I''''') \cap ((I'''' \rightarrow M'''') \cap (I''' \equiv I''''))$

$14 = M'''''$

$15 = O'''''$

5.4. When O_K is generated by M_K $[2\pi/3]$ and S_0: $K = T_K \cup_{k=1,...,3} (0, M_{[2k\pi/3]}) \cup_{k=1,...,3} (0, S_{[2k\pi/3]})$



$1 = M$

$2 = O \equiv O''''''$

$3 = M'''''$

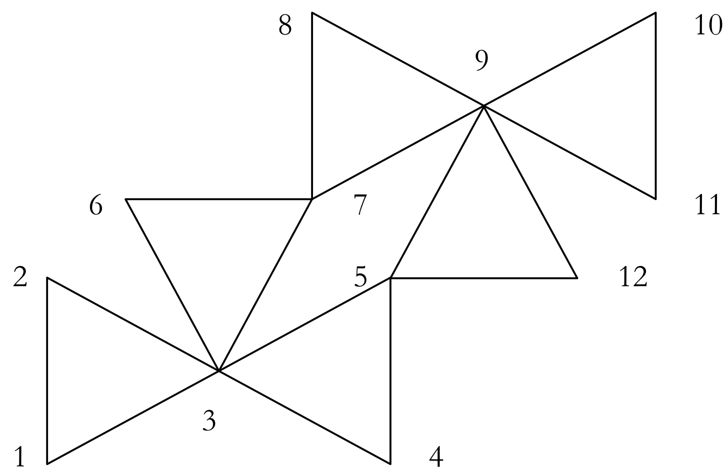
$8 = I' \equiv I''''$

$9 = M''$

$10 = I'' \equiv I'''$

4 = I ↔ (M’ → I’)	11 = M’’’
5 = I’’’’’ ↔ (I’’’’ → M’’’’)	12 = O’’’ ≡ O’’’’
6 = M’	13 = M’’’’
7 = O’ ≡ O’’	

5.5. $O_K = <M_{[2\pi/3]}, S_{[\pi/3]}>$. $K = T_K \cup (0, M_{-[2\pi/3]}) \cup T_K (0, M_{-[2\pi/3]}) \cup T_K (0, S_{[\pi/3]}) \cup T_K (0, S_{[5\pi/3]}) \cup T_K (0, S_{\pi})$.



1 = O	7 = M’ ≡ M’’’
2 = M	8 = O’’’
3 = I ≡ M’’ ≡ I’	9 = I’’’ ≡ I’’’’ ≡ I’’’’’
4 = O’’	10 = M’’’’
5 = I’’ ≡ O’’’’’	11 = O’’’’
6 =O’	12 = M’’’’’

Thus, not only all the rosette and the friezes groups, but also all 17 different cases of the wallpaper group can be found in semiotic structures generated by aid of the framework of “General Sign Grammar” (Toth 2008b), i.e. 2 for the oblique, 6 for rectangular, 7 for centered, 11 for square, and 11 for hexagonal lattices. Thus, further investigations in semiotic group and crystallography theory will be proven to be extremely useful.

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