Prof. Dr. Alfred Toth

Semiotic Derangements

1. In combinatorial mathematics, a derangement is a permutation that leaves no element of the set in its original position. Thus, a derangement is a bijection φ from a set S into itself with no fixed points, i.e. for all x in S, $\varphi(x) \neq x$. The number of derangements of a set are called subfactorials and are a special case of the rencontre numbers (Riordan 1958, pp. 57 ss.):

1							
0	1						
1	0	1					
2	3	0	1				
9	8	6	0	1			
44	45	20	10	0	1		
265	264	135	40	15	0	1	...

The numbers in the leftmost column are the derangements. They are calculated recursively by the formula

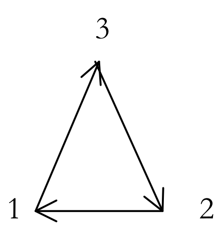
$$D_{n+1} = n(D_n + D_{n-1})$$

Using this principle of inclusion and exclusion, we also get (Hassani 2003):

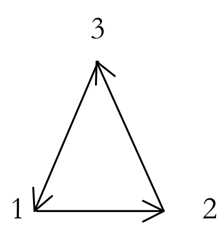
$$D_n = n!\left(\frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!}\right), n \geq 2$$

2. A set of 3 elements has 2 derangements. As a semiotic example, we take $SR_{3,3}$ = (.1., .2., .3.), i.e. the set of the triadic-trichotomic prime-signs. The derangements of $SR_{3,3}$ are:

(.3., .1., .2.):

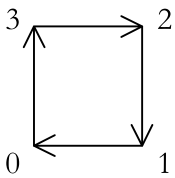


(.2., .3., .1.):

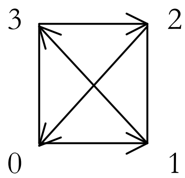


A set of 4 elements has 9 derangements. Therefore we get for $SR_{4,4}$ = (.0., .1., .2., .3.):

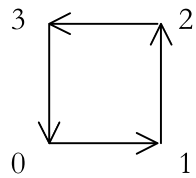
(.3., .2., .1., .0.):



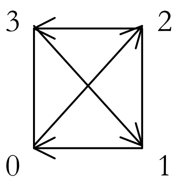
(.3., .2., .0., .1.):



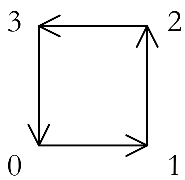
(.3., .0., .1., .2.):



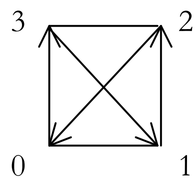
(.2., .3., .1., .0.):



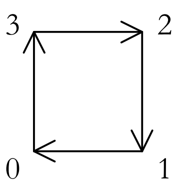
(.2., .3., .0., .1.):



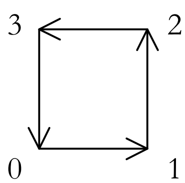
(.2., .0., .3., .1.):



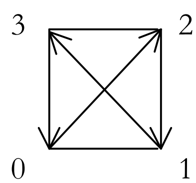
(.1., .0., .3., .2.):



(.1., .2., .3., .0.):

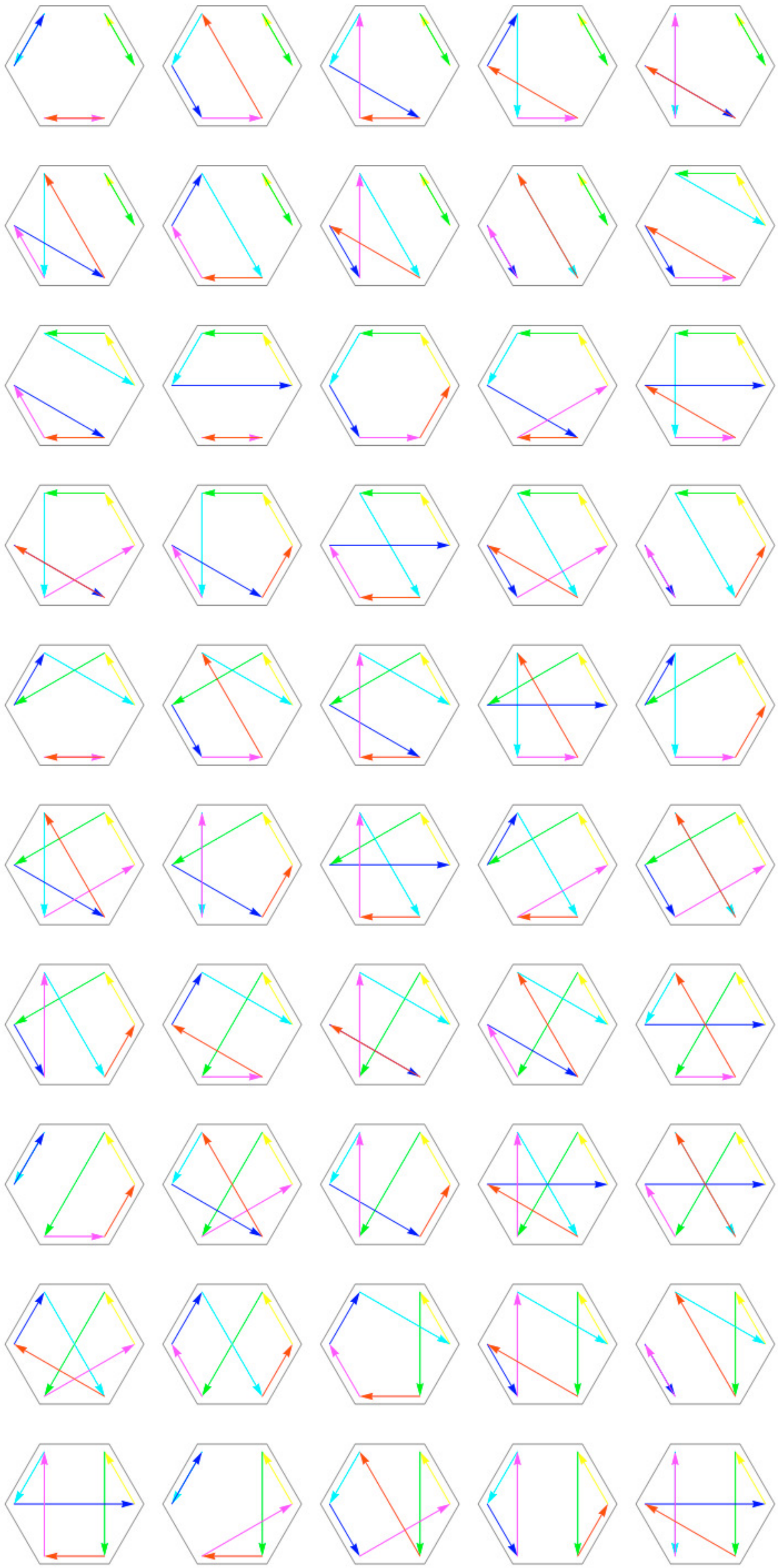


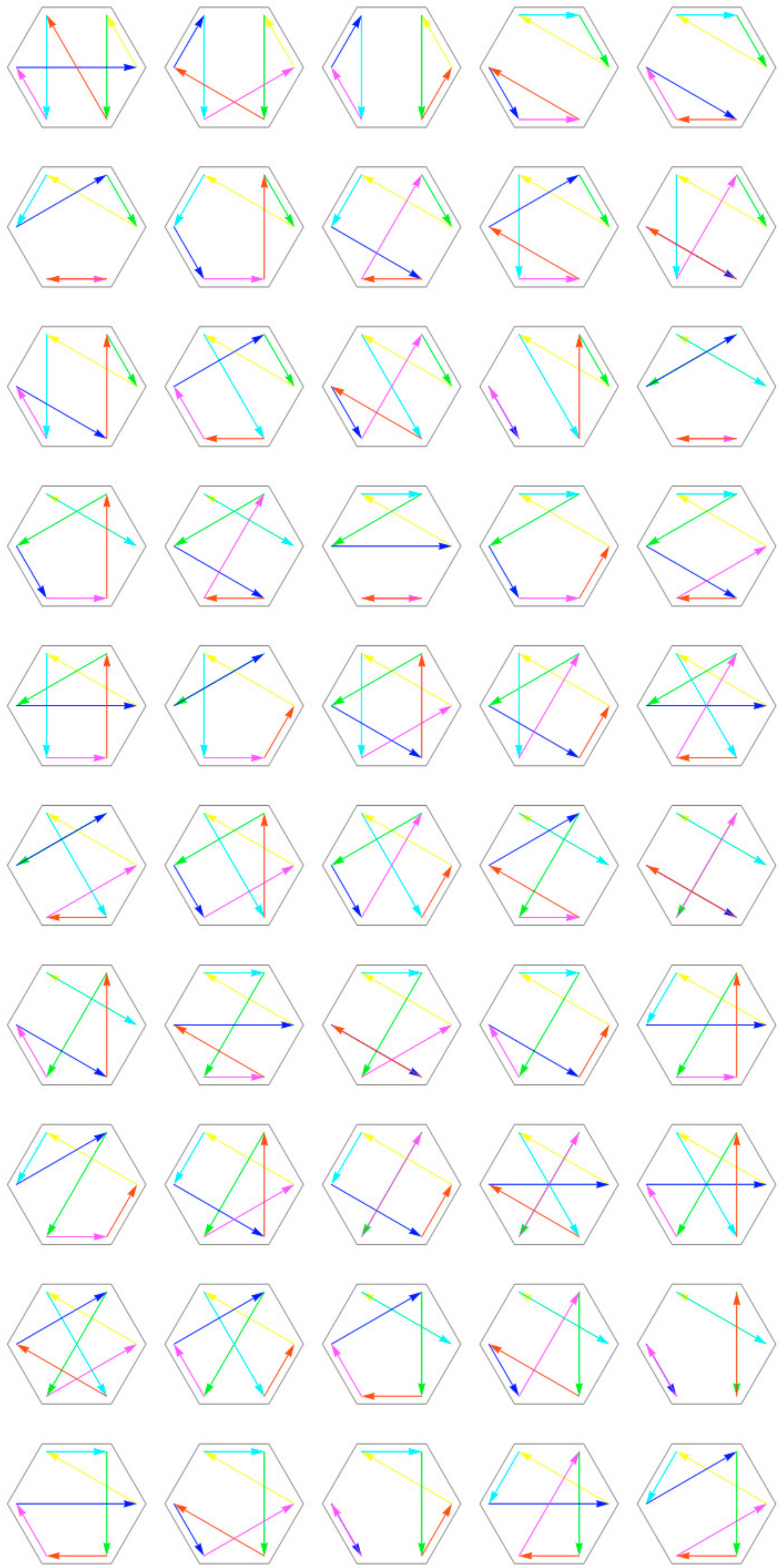
(.1., .3., .0., .2.):

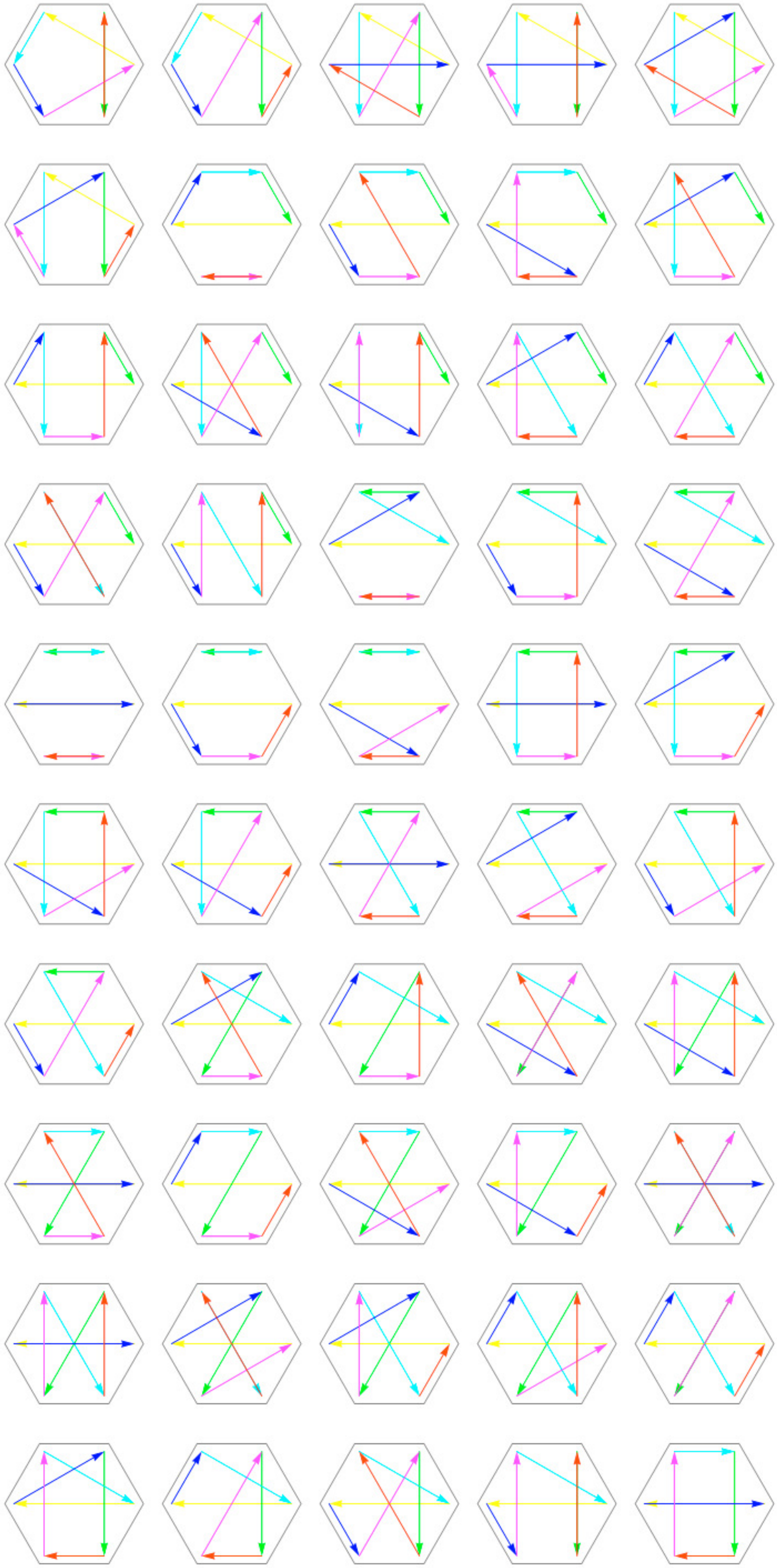


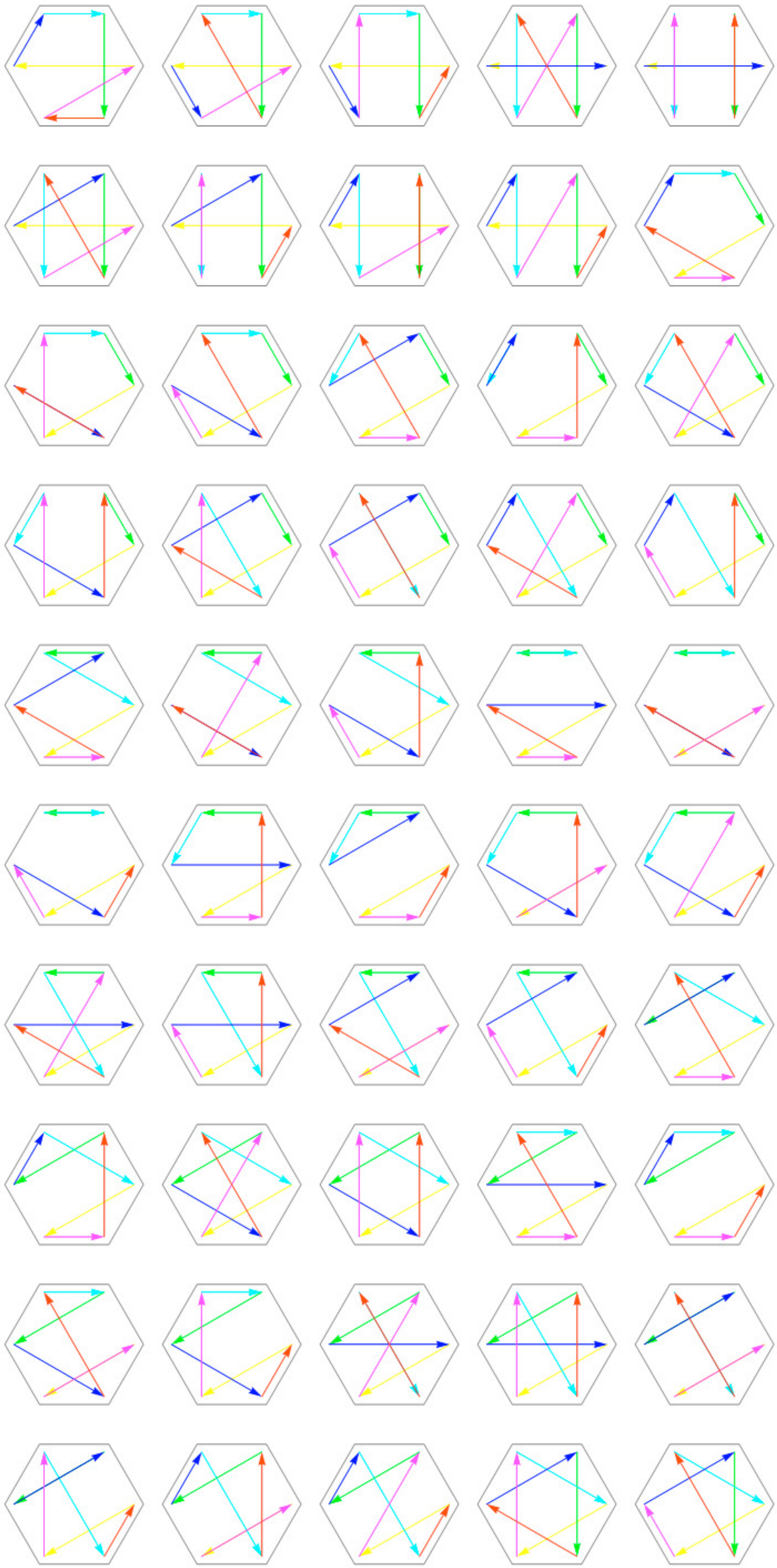
For $SR_{4,3}$ = (0., .1., .2., .3.), we get the same number of derangement, but only for its tetradic semiotic values. For the trichotomic semiotic values, we get the same number of derangements as for $SR_{3,3}$!

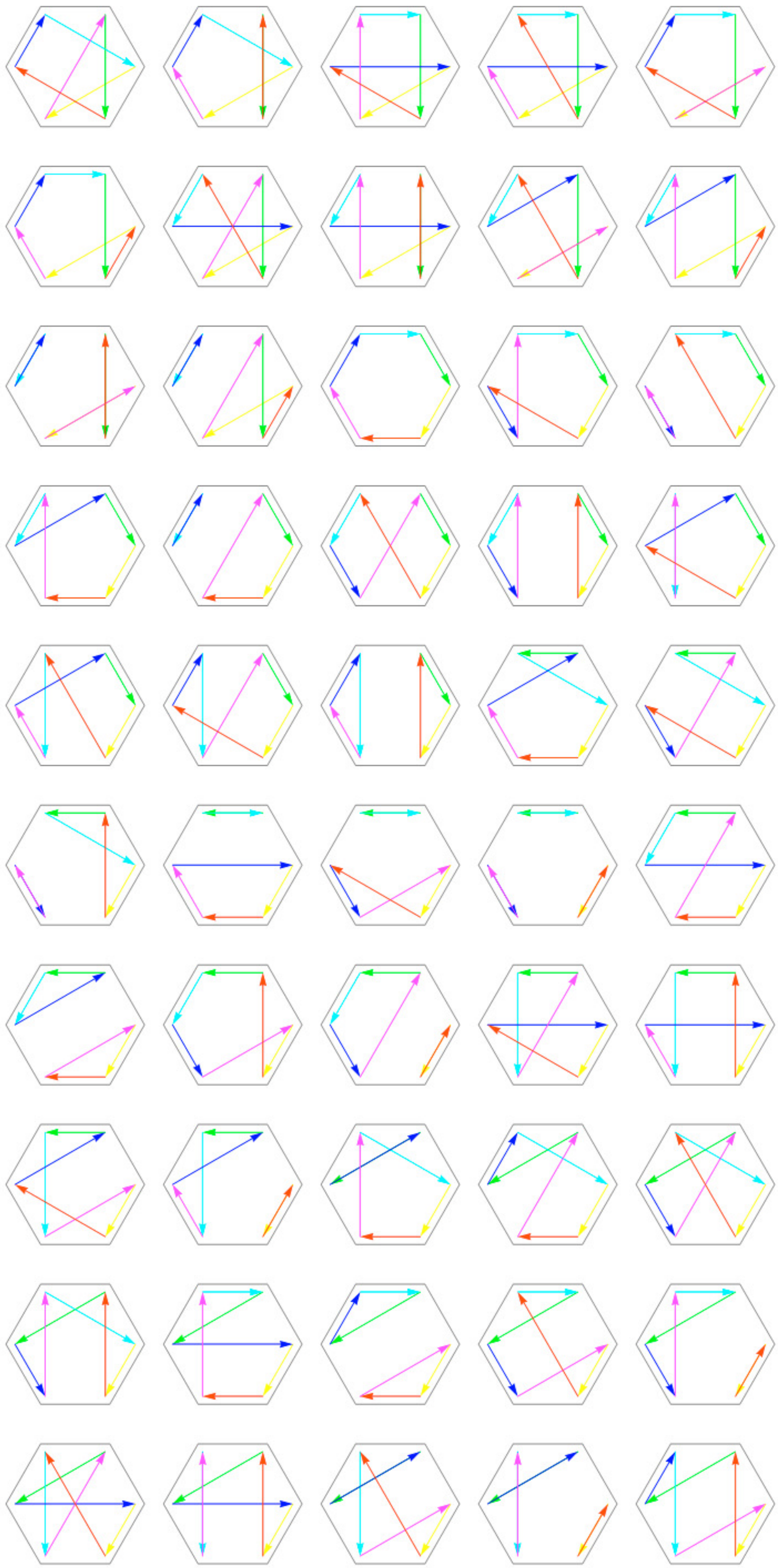
A set of 6 elements has 265 derangements. From semiotics, we can take either the set of the 6 permutations of each sign class (P(SCl)) or the set of the 6 permutations of each reality thematic (P(RTh)) and thus get the total amount of 530 semiotic derangements. The following graphical representation of the 265 derangements of a hexagonal structure I owe to Robert M. Dickau (Chicago, Ill.) and Pascal Steiner (Langenbruck, Switzerland). *Heartiest thanks to Robert and Pascal for your deeply appreciated work!*

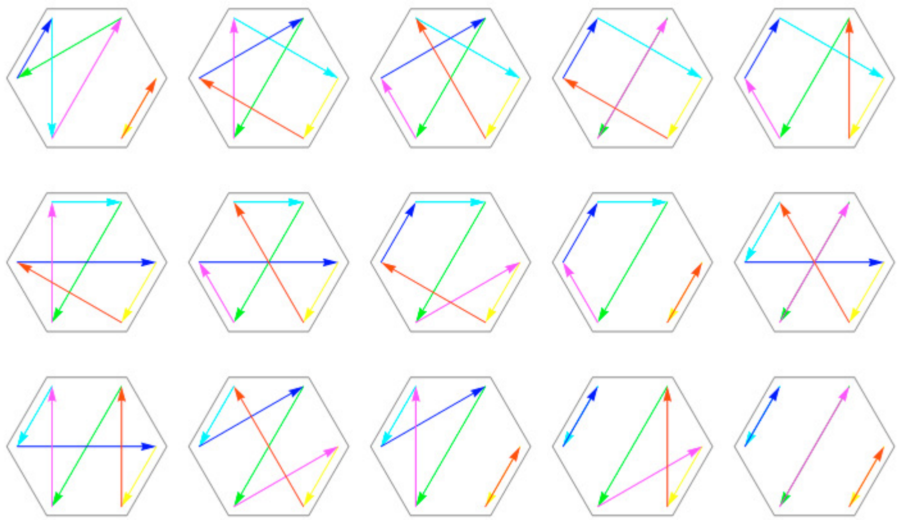












3. If we compare the number of permutations with the number of derangements of a semiotic set with n elements, we have:

n	semiotic system	number of permutations	number of derangements
3	$SR_{3,3}$	6	2
4	$SR_{4,3}$; $SR_{4,4}$	24	9
6	P(SCl); P(RTh)	720	265
10	SCl_{10}; RTh_{10}	3'628'800	1'334'961

As we recall (cf. Toth 2008, pp. 159 ss.), dynamic semiotic morphisms are defined pairwise and crosswise over n-tupels of sub-signs[1]:

M((a.b), (c.d), (e.f)) := [[[a.c], [b.d], [[c.e], [d.f]]]

Therefore, M is by itself a permutation of the constitutive prime-signs of an n-adic sign relation, letting ½ n of the prime-signs at their original places. If we now permute the order of the three elements of triadic-trichotomic sign relation $SR_{3,3}$, we get

(a.b), (c.d), (e.f) (c.d), (e.f), (a.b)
(a.b), (e.f), (c.d) (e.f), (a.b), (c.d)
(c.d), (a.b), (e.f) (e.f), (c.d), (a.b)

out of which

(c.d), (e.f), (a.b) and
(e.f), (a.b), (c.d)

are the only derangements. Therefore, the set of derangements of a sign relation does not contain any of the pairs of its permuted prime-signs defined as semiotic morphisms, while the permutations do.

It goes without special mentioning that both permutations and derangements increase impressively the usual semiotic structures defined over an n × n matrix from a n-adic n-otomic sign relation $SR_{4,4}$. Derangements will be of special interest in those cases, where one is interested in the sets of permutations of sign relations, which do not share the semiotic morphisms present in the original sign relations.

Bibliography

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http://mathforum.org/advanced/robertd/derangements.html

Hassani, Mehdi, Derangements and applications. In: Journal of Integer Sequences 6/1, 2003, article 03.1.2

Riordan, John, An Introduction to Combinatorial Analysis. New York 1958

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1 Generally, a 3-adic sign relation has 3 pairs of sub-signs; a 4-adic sign relation has 4 +2 = 6 pairs of sub-signs; a 5-adic sign relation has 5 +3 = 8 pairs of sub-signs, etc.