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Crossing and noncrossing semiotic partitions

1. A partition of a set S is a pairwise disjoint set of non-empty subsets, called "parts" or "blocks", whose union is all of S. Consider a finite set that is linearly ordered, or arranged in a cyclic order like the vertices of a regular n-gon. Let be S = {1, ..., n}. A noncrossing partition of S is a partition in which no blocks cross each other, i.e., if a and b belong to one block and x and y to another, they are not arranged in the order a x b y. If one draws an arch based at a and b, and another arch based at x and y, then the two arches cross each other if the order is a x b y, but not if it is a x y b or a b x y. In the latter two orders the partition {{a, b}, {x, y} is noncrossing (Simion 2000).

Equivalently, if we label the vertices of a regular n-gon with the numbers 1 through n, the convex hulls of different blocks of the partition are disjoint from each other, i.e., they also do not cross each other (Kreweras 1972):

http://en.wikipedia.org/wiki/Noncrossing_partition

2. We will demonstrate the semiotic relevance of noncrossing and crossing partitions at the hand of the sets of the 6 permutations of each triadic-trichotomic sign class and reality thematic (cf. Toth 2008, pp. 177 ss.) defined over the sign relation $SR_{3,3}$.

Generally, each sign class (3.a 2.b 1.c) has the following permutations:

(3.a 2.b 1.c) (2.b 1.c 3.a)
(3.a 1.c 2.b) (1.c 3.a 2.b)
(2.b 3.a 1.c) (1.c 2.b 3.a)

and each reality thematic (c.1 b.2 a.3) has the following permutations:

(c.1 b.2 a.3) (a.3 c.1 b.2)
(b.2 c.1 a.3) (b.2 a.3 c.1)
(c.1 a.3 b.2) (a.3 b.2 c.1)

Therefore, we get 21 possible combinations of pairs of partitions of sign classes:

(3.a 2.b 1.c) (3.a 2.b 1.c) (3.a 2.b 1.c) (3.a 2.b 1.c) (3.a 2.b 1.c) (3.a 2.b 1.c)

(3.a 2.b 1.c) (3.a 1.c 2.b) (2.b 3.a 1.c) (2.b 1.c 3.a) (1.c 3.a 2.b) (1.c 2.b 3.a)

(3.a 1.c 2.b) (3.a 1.c 2.b) (3.a 1.c 2.b) (3.a 1.c 2.b) (3.a 1.c 2.b)

(3.a 1.c 2.b) (2.b 3.a 1.c) (2.b 1.c 3.a) (1.c 3.a 2.b) (1.c 2.b 3.a)

(2.b 3.a 1.c) (2.b 3.a 1.c) (2.b 3.a 1.c) (2.b 3.a 1.c)

(2.b 3.a 1.c) (2.b 1.c 3.a) (1.c 3.a 2.b) (1.c 2.b 3.a)

(2.b 1.c 3.a) (2.b 1.c 3.a) (2.b 1.c 3.a)

(2.b 1.c 3.a) (1.c 3.a 2.b) (1.c 2.b 3.a)

(1.c 3.a 2.b) (1.c 3.a 2.b)

(1.c 3.a 2.b) (1.c 2.b 3.a)

(1.c 2.b 3.a)

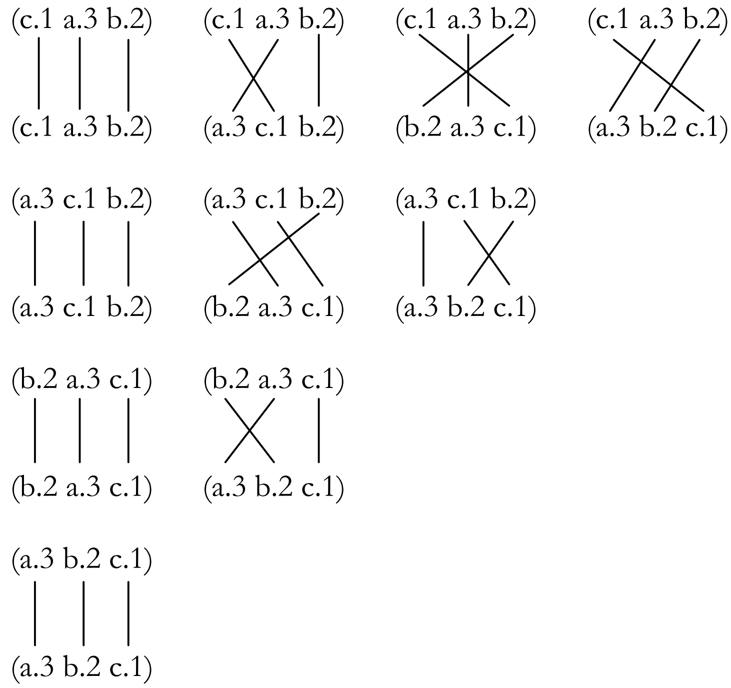
(1.c 2.b 3.a)

and 21 possible combinations of pairs of partitions of reality thematics:

(c.1 b.2 a.3) (c.1 b.2 a.3) (c.1 b.2 a.3) (c.1 b.2 a.3) (c.1 b.2 a.3) (c.1 b.2 a.3)

(c.1 b.2 a.3) (b.2 c.1 a.3) (c.1 a.3 b.2) (a.3 c.1 b.2) (b.2 a.3 c.1) (a.3 b.2 c.1)

(b.2 c.1 a.3) (b.2 c.1 a.3) (b.2 c.1 a.3) (b.2 c.1 a.3) (b.2 c.1 a.3) (b.2 c.1 a.3) (c.1 a.3 b.2) (a.3 c.1 b.2) (b.2 a.3 c.1) (a.3 b.2 c.1)



3. As we expected, we first see that the orderings of the partitions of the reality thematics are dual to the orderings of the sign classes (and of course vice versa), and second, that only the following partitions are **noncrossing**:

aa bb cc, aa cc bb, bb aa cc, bb cc aa, cc aa bb, cc bb aa,

i.e. partitions with the category theoretic structure $[id_x, id_y, id_z]$ with $x, y, z \in \{a, b, c\}$.

The following partitions contain exactly **two pairs** of sub-signs that are noncrossing:

ab ca bc, ac cb ba, bc ab ca, bc ca ab,

i.e. those partitions whose category theoretic structure does neither contain identitive morphisms nor to any morphism also its dual morphism.

The following partitions contain exactly **one pair** of sub-signs that is noncrossing:

aa bc cb, ab ba cc, ac ca bb, bb ac ca, bc cb aa, cc ab ba,

i.e. those pairs which contain one identitive morphism and for the other sub-sign also its dual morphism, i.e. $(id_x, (b \rightarrow c), (c \rightarrow b))$ with $x \in \{x, y, z\}$ and $(b \rightarrow c) \in \{\alpha, \beta\}$. Since all three morphisms can take all three places, the set of morphisms is unordered.

The following partitions contain three pairs of sub-signs that are **crossing**:

ac bb ca, ab cc ba, bc aa cb,

i.e. those partitions with the category theoretic structure [(b→c), id_a, (c→b)] with a ∈ {a, b, c} and (b→c) ∈ {α, β}. Thus, the partitions with 2 pairs and with 3 pairs, that are crossing, differ only in the order of the morphisms in their category theoretic structure.

Bibliography

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