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Complete and incomplete fuzzy structural realities

1. In the present study, I want to demonstrate by aid of semiotic fuzzy sets, introduced in Toth (2008a, b) that the system of the 10 sign classes is highly fragmentary both from the standpoint of representation and from presentation. The latter can be shown best by comparing the structural realities presented in the reality thematics of the semiotic systems of the 10 and 27 sign classes (SS10; SS27), respectively. The graphs of the respective fuzzy sets, however, do not point to a simple inclusion relation (SS10) $\subset$ (SS27), but towards a type of polycontextural semiotic "inclusion" sketched already in Toth (2003, pp. 54 ss.).

2. If we have a look at SS10, we recognize that it is impossible to order its sign classes according to both increasing ("generative" or "semiosic") interpretant and object relation, but only according to either one. Thus, we get

either	or
(**3.1** 2.1 1.1)	(3.1 **2.1** 1.1)
(**3.1** 2.1 1.2)	(3.1 **2.1** 1.2)
(**3.1** 2.1 1.3)	(3.1 **2.1** 1.3)
(**3.1** 2.2 1.2)	(3.1 **2.2** 1.2)
(**3.1** 2.2 1.3)	(3.1 **2.2** 1.3)
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(**3.1** 2.3 1.3)	(3.2 **2.2** 1.2)
(**3.2** 2.2 1.2)	(3.2 **2.2** 1.3)
(**3.2** 2.2 1.3)	(3.1 **2.3** 1.3)
(**3.2** 2.3 1.3)	(3.2 **2.3** 1.3)
(**3.3** 2.3 1.3)	(3.2 **2.3** 1.3)

As one sees, the first 5 sign classes are the same in both orderings, but starting with the 6th sign class, one has to decide to order the sign classes either according to their interpretant (left) or their object relation (right). Generally, the same holds true for any ordering of SS10 according to two sign relations (I-O/O-I; I-M/M-I; O-M/M-O). We will formulate this in the form of a semiotic theorem:

Theorem: It is impossible to order all sign classes of SS10 according to more than one sign relation in strictly increasing (generative; semiosic) or strictly decreasing (degenerative; retro-semiosic) order.

However, one also recognizes that the first three sign classes above the dashed line form a Trichotomic Triad (cf. Walther 1981, p. 36), while the other seven sign classes do of course not. Nevertheless, Walther (1982) has shown that SS10 can still be ordered in a system of three Trichotomic Triads, if the following two conditions are fulfilled:

1. The eigenreal sign class (3.1 2.2 1.3) must not be a part of any of the three Trichotomic Triads.

2. The three Trichotomic triads must consist of reality thematics whose dual sign classes are ordered not according to the I- or M-relation, but to the O-relation, whereby the first Trichotomic Triad comprises only sign classes whose object relation is (2.1), the second Trichotomic Triad only sign classes whose object relation is (2.2), and the third Trichotomic Triad only sign classes whose object relation is (2.3).

Now, since SS10 comprises three sign classes with (2.1) and three sign classes with (2.3), but four sign classes with (2.2), the eigenreal sign class (3.1 2.2 1.3) must non be a part of any of the three Trichotomic Triads, which gives us again condition 1. However, since the eigenreal sign class is connected with any other sign class of SS10 by at least one sub-sign, it is therefore connected with all three Trichotomic Triads. In other words, the drawback that (3.1 2.2 1.3) cannot be part of the three Trichotomic Triads is turned into the benefit that only its position outside of the system of the three Trichotomic Triads enables it to "determine" (Walther 1982) the semiotic "duality system" built up by SS10 and their dual reality thematics.

However, this "benefit" is based solely on the fact that SS10 and the three Trichotomic Triads constructed from it are highly fragmentary. This can be seen best, if we have a look at the first Trichotomic Triad above the dashed line in the above table:

(1.1 1.2 1.3) M-them. M
(2.1 1.2 1.3) M-them. O
(3.1 1.2 1.3) M-them. I

In order to get a complete system of both thematzing and thematized realities, one would await all 27 possible combinations from the following general scheme of semiotic thematization:

({M, O, I})-thematized ({M, O, I}),

hence, f. ex., also structural realities like

*(1.1 2.2 2.3) × *(3.2 2.2 1.1)
(2.1 2.2 2.3) × (3.2 2.2 1.2)
(3.1 2.2 2.3) × (3.2 2.2 1.3),

where the first dual system does not belong to SS10 (marked by asterisk)

or

*(1.1 3.2 3.3) × *(3.3 2.3 1.1)
*(2.1 3.2 3.3) × *(3.3 2.3 1.2)
(3.1 3.2 3.3) × (3.3 2.3 1.3),

where the first two dual systems do not belong either to SS10. Thus, the three Trichotomic Triads constructed from SS10 are not symmetric, and the second two Trichotomic Triads of SS10 do not obey the constructional system of the first one, which reasons thus point out that SS10 is highly fragmentary.

Moreover, if we calculate all 27 possible combinations, it also would turn out that the eigenreal sign class which differs from all other sign classes from SS10 in having a triadic structural reality and thus allowing three and not only one type of thematization:

(1.3, 2.2)-them. (3.1)
(1.3, 3.1)-them. (2.2)
(2.2, 3.1)-them. (1.3)

must be combined with the reality thematics of each of the three Trichotomic triads of SS10, which would result in nine Trichotomic Triads and thus again in 27 sign classes. To be brief, if one takes into account that SS10 is fragmentary from above mentioned reasons, we have no other choice than to substitute SS10 by SS27.

As we will see in next chapter, SS27, in addition, also displays structures of presented realities that are only shown, in SS10, by the structural realities of the eigenreal sign class and by transpositions of sign classes, namely the differentiation between left and right thematization (a, b) as well as "sandwich thematization" (c) and their respective reality structures with inverted order of the thematizing sub-signs (d, e, f), f. ex.

a. (3.1 2.2 1.2) × (2.1 2.2 1.3)	d. (2.2 2.1 1.3) × (3.1 1.2 2.2)
b. *(3.3 2.3 1.1) × *(1.1 3.2 3.3)	e. *(1.1 3.3 3.2) × *(2.3 3.3 1.1)
c. *(3.2 2.1 1.2) × *(2.1 1.2 2.3)	f. *(2.3 1.2 2.1) × *(1.2 2.1 3.2)
Dual systems of SS10/27	Transpositional Dual systems of SS10/27

Furthermore, in SS 27, there are several cases of triadic structural realities outside of the context of eigenreality, f. ex.

*(3.1 2.2 1.1) × *(1.1 2.2 1.3), i.e. { (1.1, 2.2)-them. (1.3); (1.1, 1.3)-them. (2.2); (1.3, 2.2)-them. (1.1),

generally in all sign classes in SS27 whose trichotomic values are pairwise different, i.e. in all (3.a 2.b 1.c) with a ≠ b ≠ c.

Because of the mentioned structures of presented realities that show types that do not occur in the usual display of SS10, for the reality thematics and thus for the structural realities of SS27, we find

SS 10 ⊄ SS27,

although for the dual sign classes, SS10 ⊂ SS27 holds true. This "paradox" situation shows that purely formal duality does not hold true for reality thematics, which thus apparently transcend purely syntactic logic. As we already pointed out, SS10 is not a sub-set of SS27, but a morphogrammatic fragment (cf. Toth 2003, pp. 54 ss.), which proves that we have to deal here with a polycontextural feature of theoretical semiotics and thus of qualitative-mathematical semiotics.

3. Before the background of the above statements, we can now order the sign classes and reality thematics of SS27 in Trichotomic Triads according to both increasing I- and O- sign relation:

1. (3.1 2.1 1.1) × (1.1 1.2 1.3) 2. (3.1 2.1 1.2) × (2.1 1.2 1.3) 3. (3.1 2.1 1.3) × (3.1 1.2 1.3)	TrTr1	16. *(3.2 2.3 1.1) × *(1.1 3.2 2.3) 17. *(3.2 2.3 1.2) × *(2.1 3.2 2.3) 18. (3.2 2.3 1.3) × (3.1 3.2 2.3)	TrTr6
4. *(3.1 2.2 1.1) × *(1.1 2.2 1.3) 5. (3.1 2.2 1.2) × (2.1 2.2 1.3) 6. (3.1 2.2 1.3) × (3.1 2.2 1.3)	TrTr2	19. *(3.3 2.1 1.1) × *(1.1 1.2 3.3) 20. *(3.3 2.1 1.2) × *(2.1 1.2 3.3) 21. *(3.3 2.1 1.3) × *(3.1 1.2 3.3)	TrtR7
7. *(3.1. 2.3 1.1) × *(1.1 3.2 1.3) 8. *(3.1 2.3 1.2) × *(2.1 3.2 1.3) 9. (3.1 2.3 1.3) × (3.1 3.2 1.3)	TrTr3	22. *(3.3 2.2 1.1) × *(1.1 2.2 3.3) 23. *(3.3 2.2 1.2) × *(2.1 2.2 3.3) 24. *(3.3 2.2 1.3) × *(3.1 2.2 3.3)	TrTr8
10. *(3.2 2.1 1.1) × *(1.1 1.2 2.3) 11. *(3.2 2.1 1.2) × *(2.1 1.2 2.3) 12. *(3.2 2.1 1.3) × *(3.1 1.2 2.3)	TrTr4	25. *(3.3 2.3 1.1) × *(1.1 3.2 3.3) 26. *(3.3 2.3 1.2) × *(2.1 3.2 3.3) 27. (3.3 2.3 1.3) × (3.1 3.2 3.3)	TrTr9
13. *(3.2 2.2 1.1) × *(1.1 2.2 2.3) 14. (3.2 2.2 1.2) × (2.1 2.2 2.3) 15. (3.2 2.2 1.3) × (3.1 2.2 2.3)	TrTr5		

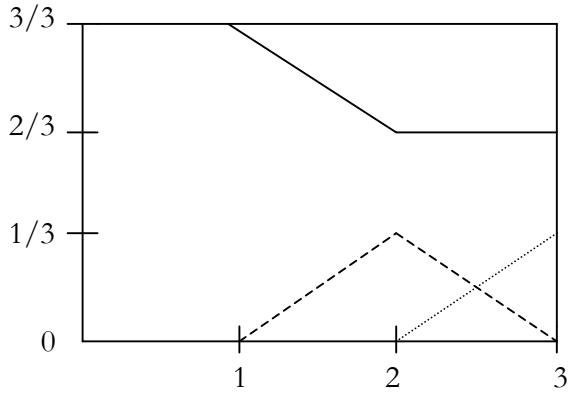
As one easily sees, it is possible to order the dual systems of SS27 according to any pair of sign relations.

4. In this last chapter, we can now finally demonstrate the fragmentarism of SS10 compared to SS27 by aid of semiotic fuzzy sets. We will draw the graphs for all reality thematics of SS27 and mark the dual-systems that do not belong to SS10 again by asterisk. As one sees without any further comment, the main result is that most of the following fuzzy graphs could not even been drawn, since most of the respective trichotomic triads do simply not exist in SS10.

4.1. TrTr1

1. (3.1 2.1 1.1) × (1.1 1.2 1.3)
2. (3.1 2.1 1.2) × (2.1 1.2 1.3)
3. (3.1 2.1 1.3) × (3.1 1.2 1.3)

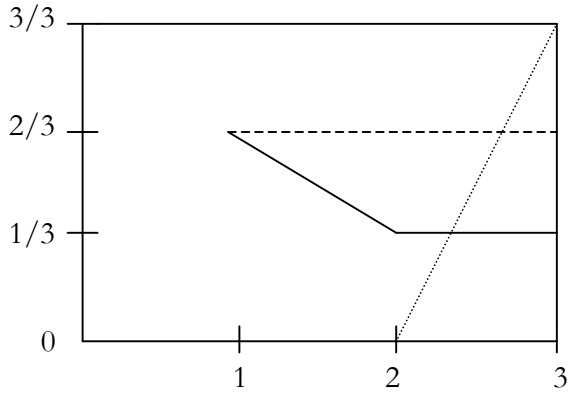
Semiotic fuzzy set for TrTr1



4.2. TrTr2

4. *(3.1 2.2 1.1) × *(1.1 2.2 1.3)
5. (3.1 2.2 1.2) × (2.1 2.2 1.3)
6. (3.1 2.2 1.3) × (3.1 2.2 1.3)

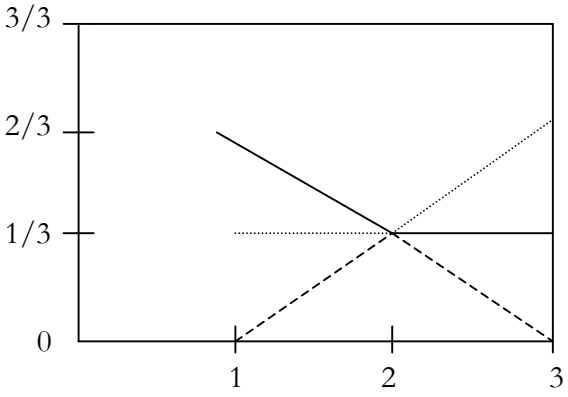
Semiotic fuzzy set for TrTr2



4.3. TrTr3

7. *(3.1. 2.3 1.1) × *(1.1 3.2 1.3)
8. *(3.1 2.3 1.2) × *(2.1 3.2 1.3)
9. (3.1 2.3 1.3) × (3.1 3.2 1.3)

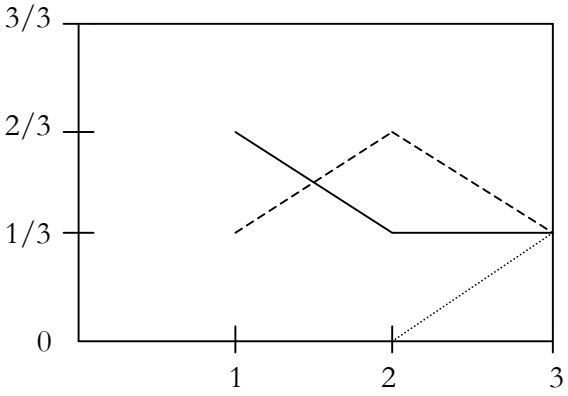
Semiotic fuzzy set for TrTr3



4.4. TrTr4

10. *(3.2 2.1 1.1) × *(1.1 1.2 2.3)
11. *(3.2 2.1 1.2) × *(2.1 1.2 2.3)
12. *(3.2 2.1 1.3) × *(3.1 1.2 2.3)

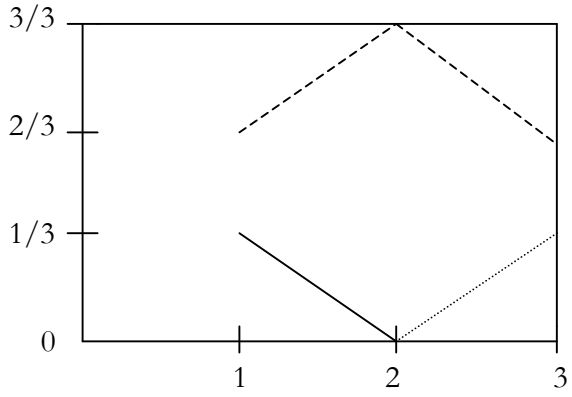
Semiotic fuzzy set for TrTr4



4.5. TrTr5

13. *(3.2 2.2 1.1) × *(1.1 2.2 2.3)
14. (3.2 2.2 1.2) × (2.1 2.2 2.3)
15. (3.2 2.2 1.3) × (3.1 2.2 2.3)

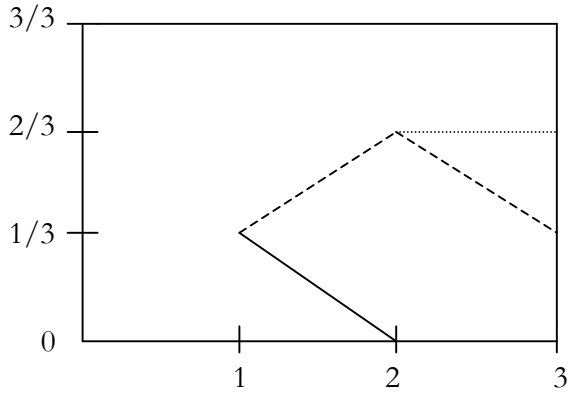
Semiotic fuzzy set for TrTr5



4.6. TrTr6

16. *(3.2 2.3 1.1) × *(1.1 3.2 2.3)
17. *(3.2 2.3 1.2) × *(2.1 3.2 2.3)
18. (3.2 2.3 1.3) × (3.1 3.2 2.3)

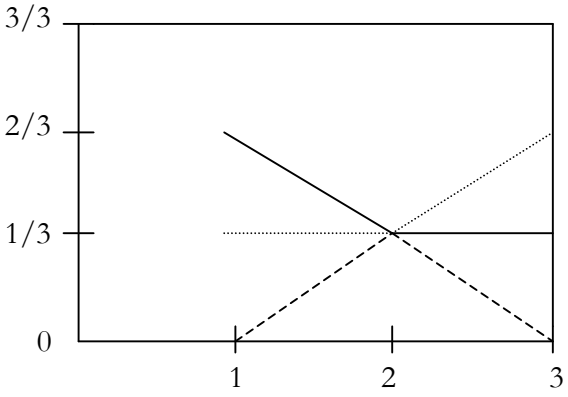
Semiotic fuzzy set for TrTr6



4.7. TrTr7

19. *(3.3 2.1 1.1) × *(1.1 1.2 3.3)
20. *(3.3 2.1 1.2) × *(2.1 1.2 3.3)
21. *(3.3 2.1 1.3) × *(3.1 1.2 3.3)

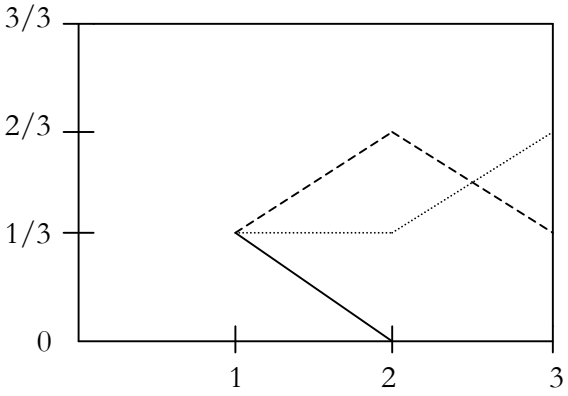
Semiotic fuzzy set for TrTr7



4.8. TrTr8

22. *(3.3 2.2 1.1) × *(1.1 2.2 3.3)
23. *(3.3 2.2 1.2) × *(2.1 2.2 3.3)
24. *(3.3 2.2 1.3) × *(3.1 2.2 3.3)

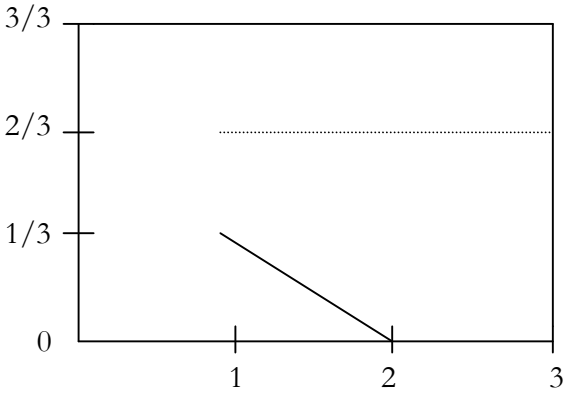
Semiotic fuzzy set for TrTr8



4.9. TrTr9

25. *(3.3 2.3 1.1) × *(1.1 3.2 3.3)
26. *(3.3 2.3 1.2) × *(2.1 3.2 3.3)
27. (3.3 2.3 1.3) × (3.1 3.2 3.3)

Semiotic fuzzy set for TrTr9



The above displayed graphs for semiotic fuzzy sets show that the semiotic reality of SS10 is only a small morphogrammatic fragment of the complete semiotic reality of SS27. Therefore, the above graphs also show that the range of the membership functions of semiotic realities in SS27 is much wider than in SS10 and thus "scoops out" maximally the semiotic continuum of **possible** realities most of which are, however, unrealized in SS10.

Bibliography

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003
Toth, Alfred, Fuzzy semiotic sets. Ch. 27 (2008a)
Toth, Alfred, Trichotomic Triads as fuzzy sets. Ch. 28 (2008b)
Walther, Elisabeth, Vorläufige Bemerkungen zu Trichotomischen Triaden. In: Semiosis 21, 1981, pp. 29-39
Walther, Elisabeth, Nachtrag zu Trichotomischen Triaden. In: Semosis 27, 1982, pp. 15-20