Prof. Dr. Alfred Toth

A class of semiotic graphs from transversals

1. A transversal is a set containing exactly one element from each set of a collection of sets. Thus it is a part-set of the quotient map induced by the family of sets. Recently, transversals have been discussed especially in connection with matroid theory, where they form "systems of different representatives" (Läuchli 1998, pp. 81 ss.). A matroid $\underline{M}$ is a finite set E along with a collection $\underline{I}$ of subsets of E called independent sets which satisfy the following conditions:

1. The empty set $\varnothing$ is in $\underline{I}$.
2. If $I_1 \in \underline{I}$ and $I_2 \in I_1$, then $I_2 \in \underline{I}$.
3. If $I_1, I_2 \in \underline{I}$ and $|I_2| > |I_1|$, then there exists $e \in I_2 \backslash I_1$ such that $I_1 \cup \{e\} \in \underline{I}$.

Let S be the semiotic power set, i.e. S = {{1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}, ∅} (cf. Toth 2007, pp. 99 ss.), then all conditions are fulfilled. The bases $\underline{B}$ of a matroid $\underline{M}$ are the maximal independent sets and satisfy the conditions:

1. $\underline{B}$ is non-empty.
2. If $B_1, B_2 \in \underline{B}$ and $e \in B_1 \backslash B_2$, then there exists $e' \in B_2 \backslash B_1$ with $(B_1 \backslash \{e\} \cup \{e'\} \in \underline{B}$.

Condition 2 says, roughly speaking, two different things: First, different bases do not contain one another. Second, if B_1 and B_2 are two different bases, then one can exchange an arbitrary element x_1 from B_1 by an element x_2 from B_2 so that there will be again a basis. Speaking in terms of semiotics, the latter means, that in the general sign relation (3.a 2.b 1.c), where a, b, c ∈ {1, 2, 3}, we may choose any values for the variables a, b, c in order to get full triadic-trichotomic sign classes, and each triadic-trichotomic sign class is therefore a basis of the semiotic matroid $\underline{M}_{\underline{S}}$. Note that the conditions to be a basis of a semiotic matroid are not only fulfilled by the system of the 10 sign classes and their dual reality thematics, which are restricted by the trichotomic inclusion order ($a \leq b \leq c$), but by the whole system of all 27 possible sign classes, 17 out of which do not obey this trichotomic inclusion order.

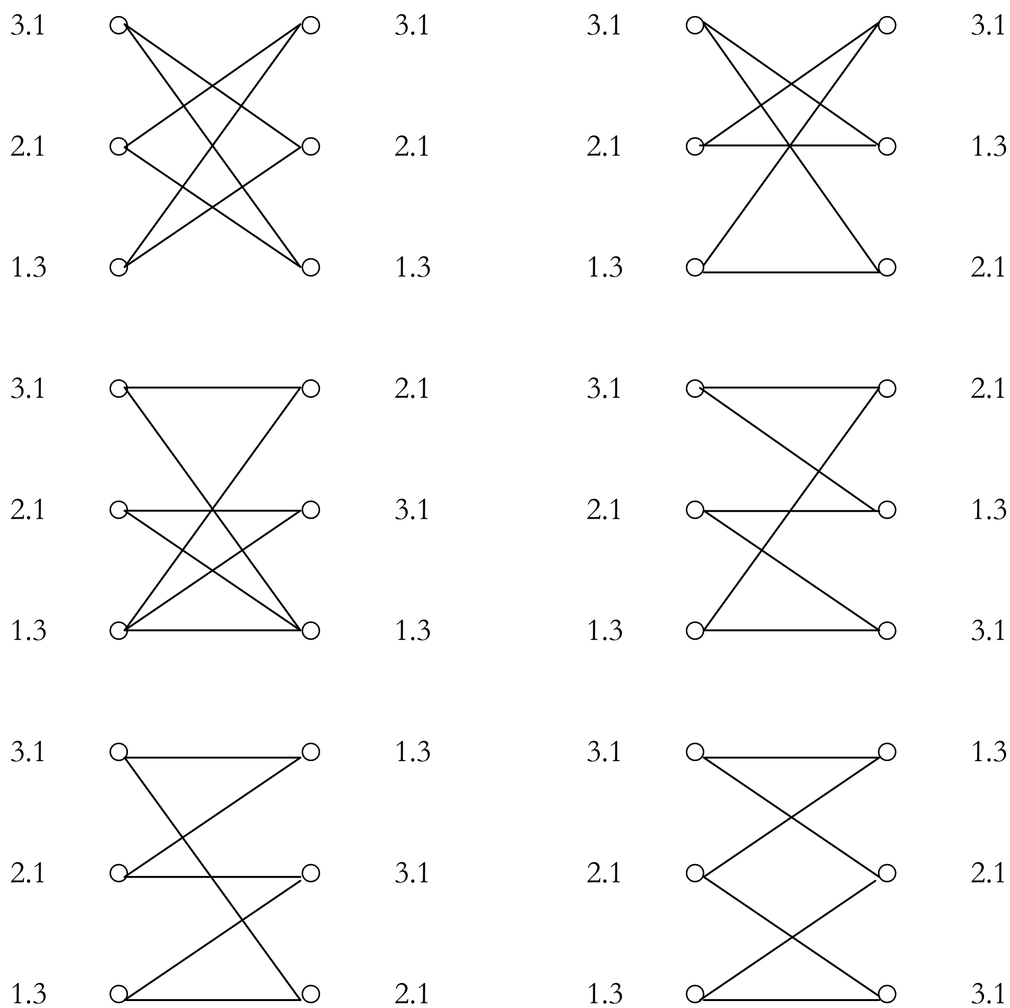
2. Since we are free, which sign class or reality thematic we choose as basis of the semiotic matroid, we will use the sign class (3.1 2.1 1.3) in order to show that the system of the 6 transpositions of each sign class

(3.a 2.b 1.c)   (2.b 1.c 3.a)
(3.a 1.c 2.b)   (1.c 3.a 2.b)
(2.b 3.a 1.c)   (1.c 2.b 3.a)

or reality thematic

(c.1 b.2 a.3) (a.3 c.1 b.2)
(b.2 c.1 a.3) (b.2 a.3 c.1)
(c.1 a.3 b.2) (a.3 b.2 c.1)

induces a special class of graphs which are won by displaying the semiotic matroid in the form of transversals (cf. also Toth 2008):



Thus, the 6 transversal graphs are the same for the system of transpositions of all 10 sign classes and their dual reality thematics. Of outmost interest is it that the part-systems of the first 4 transpositions, i.e the sets

TR_{SCl} = {(3.a 2.b 1.c), (3.a 1.c 2.b), (2.b 3.a 1.c), (2.b 1.c 3.a)}

TR_{RTh} ={(c.1 b.2 a.3), (b.2 c.1 a.3), (c.1 a.3 b.2), (a.3 c.1 b.2)}

are sufficient to create the 6 possible graphs of each semiotic dual system.

Bibliography

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