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Chirality in polycontextural sign relations

1. Perhaps the most exciting – or troublesome – feature that arises when inner semiotic environments are introduced in sign relations, is the disappearance one of the most central theories of semiotics: eigenreality (cf. Bense 1992). The problem is somewhat intricate:

1.1. In monocontextural semiotics, there is only 1 sign class amongst the 10 Peircean sign classes which is "identical" with it dual reality thematic[1]:

$$CS(3,1) = (3.1\ 2.2\ 1.3) \times (3.1\ 2.2\ 1.3)$$

1.2. However, already in CS(3,3)-systems, reality thematic and it corresponding sign class are no longer dual-invers:

$$CS(3,3) = (3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3)$$

1.3. While in CS(3,3)-systems, at least those sub-signs which have only one contextural index seem to be unchanged or "identical", this assumptions turns out to be wrong starting with CS(3,4)-systems:

$$CS(3,4) = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3})$$

1.4. Another very interesting observation is that dual sub-signs – as long as they appear in the same matrix – are really dual (and not complementary), i.e. they do change the order of their contextural indices; cf. the following (3,4)-matrix:

1 Following Kaehr (2008), but also v. supra, we do not speak any longer of "dual systems" (DS), but of "complementary systems" (CS), taking care of the fact that reality thematics are only then dual to their sign classes, when they are monocontextural. (Therefore, the term CS covers both mono- and polycontextural sign relations.) From the numbers in parenthesis the first one indicates the n-adicity, the second the m-contexturality of a sign relation.

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

Thus we have:

$(a.b_{i,j})° = (a.b_{i,j})$ for a, b ∈ {1, 2, 3} and i, j ∈ {1, 2, 3, 4}.

However, from this, it follows that polycontextural matrices cannot longer be considered transpositional vector spaces (cf. Toth 2007, 48 s.), since the transposed matrices do not give the sub-signs of the reality thematics anymore, which correspond to the sign classes as column-, row- or mixed column-row-vectors. In other words: Since $(a.b_{i,j})$ is not the corresponding reality thematic of $(a.b_{i,j})$, and since $(a.b_{i,j})° = (a.b_{i,j})$, we the complement-operator C, which turns $(a.b_{i,j})$ into $(a.b_{i,j})$ and thus a second matrix, hence totally two different semiotic matrices, one for sign relations and one for their corresponding reality relations:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix} \begin{pmatrix} 3.1_{4,3} & 2.1_{4,1} & 1.1_{4,3,1} \\ 3.2_{4,2} & 2.2_{4,2,1} & 1.2_{4,1} \\ 3.3_{4,3,2} & 2.3_{4,2} & 1.3_{4,3} \end{pmatrix}$$

As one can see easily, the two matrices are chiral, because their mirror pictures cannot be superimposed to one another (at least not in 3 dimensions).

2. Therefore, we are already in the center of our investigation. Thus, in order to look for chirality in polycontextural sign relations, it is necessary not only to look at the symmetry of the sub-signs, but also at the symmetry of their indices for any sign relation or reality thematic. However, the basic result from our earlier investigation (Toth 2009) is that there are no (formal or semantic) reasons to bind semiotic contextures either to specific sub-signs or to specific permutations or dualizations (complements, reflections) of sign relations. Therefore, it must be possible to put every sub-sign from a sign relation or reality thematic into any of n contextures and also in any n-tupels of contextures, whereby identitive morphisms (genuine sub-signs) alone receive the maximal number of contextural indices for a specifix contexture (the

diagonals on the above matrices). In order to visualize semiotic chirality, we use double arrows (⇒, ⇐) for semiosic or retrosemiosic relations between the sub-signs of sign classes or reality thematics

(3.a 2.b 1.c)	(⇒, ⇒)	(c.1 b.2 a.3)	(⇒, ⇒)
(3.a 1.c 2.b)	(⇒, ⇐)	(b.2 c.1 a.3)	(⇒, ⇐)
(2.b 3.a 1.c)	(⇐, ⇒)	(c.1 a.3 b.2)	(⇐, ⇒)
(2.b 1.c 3.a)	(⇒, ⇐)	(a.3 c.1 b.2)	(⇒, ⇐)
(1.c 3.a 2.b)	(⇐, ⇒)	(b.2 a.3 c.1)	(⇐, ⇒)
(1.c 2.b 3.a)	(⇐,⇐)	(a.3 b.2 c.1)	(⇐,⇐)

and simple arrows (→, ←) for the order relations in the contextural indices:

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$	((→, →), (→, →), (→, →))
$(3.a_{i,k,j}\ 2.b_{i,k,j}\ 1.c_{i,k,j})$	((→, ←), (→, ←), (→, ←))
$(3.a_{j,i,k}\ 2.b_{j,i,k}\ 1.c_{j,i,k})$	((←, →), (←, →), (←, →))
$(3.a_{j,k,i}\ 2.b_{j,k,i}\ 1.c_{j,k,i})$	((→, ←), (→, ←), (→, ←))
$(3.a_{k,i,j}\ 2.b_{k,i,j}\ 1.c_{k,i,j})$	((←, →), (←, →), (←, →))
$(3.a_{k,j,i}\ 2.b_{k,j,i}\ 1.c_{l,j,i})$	((←, ←), (←, ←), (←, ←))

Although the mappings of the arrows to the sign classes and to the indices, respectively, are not bijective, we still get the main types of semiotic symmetries and asymmetries and can reconstruct the homonymic ones easily. Then, we can represent the combinations of morphismic and contextural order for all sign classes by using the following 4 groups of each 6 possibilities:

Group 1:

(⇒, ⇒)	asymmetric
((→, →), (→, →), (→, →))	asymmetric
(⇒, ⇒)	asymmetric
((→, ←), (→, ←), (→, ←))	symmetric
(⇒, ⇒)	asymmetric
((←, →), (←, →), (←, →))	symmetric

$(\Rightarrow, \Rightarrow)$	asymmetric	
$((\rightarrow, \leftarrow), (\rightarrow, \leftarrow), (\rightarrow, \leftarrow))$	symmetric	
$(\Rightarrow, \Rightarrow)$	asymmetric	
$((\leftarrow, \rightarrow), (\leftarrow, \rightarrow), (\leftarrow, \rightarrow))$	symmetric	
$(\Rightarrow, \Rightarrow)$	asymmetric	
$((\leftarrow, \leftarrow), (\leftarrow, \leftarrow), (\leftarrow, \leftarrow))$	asymmetric	

Group 2:

$(\Rightarrow, \Leftarrow)$	symmetric	
$((\rightarrow, \rightarrow), (\rightarrow, \rightarrow), (\rightarrow, \rightarrow))$	asymmetric	non-chiral
$(\Rightarrow, \Leftarrow)$	symmetric	
$((\rightarrow, \leftarrow), (\rightarrow, \leftarrow), (\rightarrow, \leftarrow))$	symmetric	chiral
$(\Rightarrow, \Leftarrow)$	symmetric	
$((\leftarrow, \rightarrow), (\leftarrow, \rightarrow), (\leftarrow, \rightarrow))$	symmetric	chiral
$(\Rightarrow, \Leftarrow)$	symmetric	
$((\rightarrow, \leftarrow), (\rightarrow, \leftarrow), (\rightarrow, \leftarrow))$	symmetric	chiral
$(\Rightarrow, \Leftarrow)$	symmetric	
$((\leftarrow, \rightarrow), (\leftarrow, \rightarrow), (\leftarrow, \rightarrow))$	symmetric	chiral
$(\Rightarrow, \Leftarrow)$	symmetric	
$((\leftarrow, \leftarrow), (\leftarrow, \leftarrow), (\leftarrow, \leftarrow))$	asymmetric	non-chiral

Group 3:

$(\Leftarrow, \Rightarrow)$	symmetric	
$((\rightarrow, \rightarrow), (\rightarrow, \rightarrow), (\rightarrow, \rightarrow))$	asymmetric	non-chiral
$(\Leftarrow, \Rightarrow)$	symmetric	
$((\rightarrow, \leftarrow), (\rightarrow, \leftarrow), (\rightarrow, \leftarrow))$	symmetric	chiral

$(\Leftarrow, \Rightarrow)$	symmetric	
$((\leftarrow, \rightarrow), (\leftarrow, \rightarrow), (\leftarrow, \rightarrow))$	symmetric	chiral
$(\Leftarrow, \Rightarrow)$	symmetric	
$((\rightarrow, \leftarrow), (\rightarrow, \leftarrow), (\rightarrow, \leftarrow))$	symmetric	chiral
$(\Leftarrow, \Rightarrow)$	symmetric	
$((\leftarrow, \rightarrow), (\leftarrow, \rightarrow), (\leftarrow, \rightarrow))$	symmetric	chiral
$(\Leftarrow, \Rightarrow)$	symmetric	
$((\leftarrow, \leftarrow), (\leftarrow, \leftarrow), (\leftarrow, \leftarrow))$	asymmetric	non-chiral

Group 4:

$(\Leftarrow,\Leftarrow)$	asymmetric
$((\rightarrow, \rightarrow), (\rightarrow, \rightarrow), (\rightarrow, \rightarrow))$	asymmetric
$(\Leftarrow,\Leftarrow)$	asymmetric
$((\rightarrow, \leftarrow), (\rightarrow, \leftarrow), (\rightarrow, \leftarrow))$	symmetric
$(\Leftarrow,\Leftarrow)$	asymmetric
$((\leftarrow, \rightarrow), (\leftarrow, \rightarrow), (\leftarrow, \rightarrow))$	symmetric
$(\Leftarrow,\Leftarrow)$	asymmetric
$((\rightarrow, \leftarrow), (\rightarrow, \leftarrow), (\rightarrow, \leftarrow))$	symmetric
$(\Leftarrow,\Leftarrow)$	asymmetric
$((\leftarrow, \rightarrow), (\leftarrow, \rightarrow), (\leftarrow, \rightarrow))$	symmetric
$(\Leftarrow,\Leftarrow)$	asymmetric
$((\leftarrow, \leftarrow), (\leftarrow, \leftarrow), (\leftarrow, \leftarrow))$	asymmetric

So, chirality obviously exists only in combinations of order of morphisms and contextures under the condition that the order of morphisms is symmetric. If it is asymmetric, there is neither chirality nor non-chirality. However, chirality need symmetry of both the order of the morphisms and the order of the contextures, since, if the order of the contextures is asymmetric, then the type is non-chiral.

As a final remark, one could state that monocontextural semiotic systems are characterized by eigenreality, while polycontextural semiotic systems are characterized by chirality. Interestingly enough, from both concepts, there are strong connections to physics.

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